New York University Tandon School of Engineering

Department of Finance and Risk Engineering

Course Outline FRE6233 Option Pricing and Stochastic Calculus

Fall 2024, Professor Nizar Touzi: Monday, 8am-10:31am

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Course Pre-requisite: FRE 6083

Course Description:

This course provides the mathematical foundations of Option Pricing models. The techniques covered include the properties of the Brownian motion and the corresponding stochastic calculus, Ito differential Calculus, change of probability measure, martingales, Stochastic Differential Equations, Partial Differential Equations as the heat equation and the Fokker-Planck equation. Some of the pricing models considered are the European, Barrier, Asian and American options. These problems are either solved analytically by the martingale and the change of numéraire technique, the Partial Differential Equations approach, or numerically, by applying approximation and simulation methods. Since the same techniques allow the treatment of more complex financial products, several advanced examples will be also presented.

Course Objectives:

● After taking this course, the students will be able to price any derivative security.

● This is a first course in Mathematical Finance and is a building block for more advanced courses.

● The material taught is part of the common knowledge shared by quants in the Financial industry and taking this course will prepare the students to apply for quantitative positions.

● This course provides the first steps for a Ph.D. program in Financial Engineering, Mathematical Finance, Operations Research, or Finance.

Readings:

Reference texts for the course are:

● Tomas Bjork, *Arbitrage Theory in continuous time*, Oxford University press, Reprint, ISBN-0191525103, 9780191525100.

● Steve E. Shreve, *Stochastic Calculus for Finance II: continuous-time models*, 2nd edition, 2004, Springer.

A more advanced reference text is:

● Ioannis Karatzas & Steve E. Shreve, *Brownian Motion and Stochastic Calculus*, Graduate Texts in Mathematics, 113, 2nd edition, 2004, Springer.

Location of books: NYU bookstore. Slides and other additional sources will be posted on NYU classes.

Recommended software for the homework: Students will be required to use a programming language for prototyping, such as Python, Matlab, or R (http://www.r-project.org).

Course requirements:

● Students are expected to attend classes and participate actively. They should view the videos and read the textbooks ahead of time and come prepared to ask questions and discuss the weekly assignment.

● There will be a take home midterm examination, a take home final examination and weekly homework assignments. The Midterm examination will be due on week 7: students will be required to solve four or five problems by using the computational techniques taught during the first 6 weeks.

● The final examination will be due on week 15.

● There will be two types of homework assignments. The first type will consist of practice exercises designed to help the students assimilate the techniques taught in class and prepare them for the examinations. The second type will consist of implementing numerical or simulation techniques, to compute option prices that cannot be computed analytically.

Grading of Assignments:

The grade for this course will be determined according to the following formula:

Assignments/Activities	% of Final Grade
Average weekly homework assignment score (the worst grade will be dropped)	50%
Take home midterm exam	20%
Take home final exam	30%

Lessons

Weeks 1-2: The Brownian motion

Weeks 3-4: Stochastic integral and Itô's formula

Weeks 5-6: Risk neutral pricing and the Black-Scholes model

Weeks 7-8: Connection with PDEs and martingale representation

Weeks 9-10: Change of measure and Martingale derivatives pricing

Weeks 11: Fokker-Planck equation and implied volatility surface

Weeks 12: Stochastic interest rates

Weeks 13: Asian and Barrier options

Weeks 14: American options

Week 15: Final exam

The NYU Tandon School values an inclusive and equitable environment for all our students. I hope to foster a sense of community in this class and consider it a place where individuals of all backgrounds, beliefs, ethnicities, national origins, gender identities, sexual orientations, religious and political affiliations, and abilities will be treated with respect. It is my intent that all students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength and benefit. If this standard is not being upheld, please feel free to speak with me.

NYU School of Engineering Policies and Procedures on Academic Misconduct *(from the School of Engineering Student Code of Conduct)*

A. Introduction: The School of Engineering encourages academic excellence in an environment that promotes honesty, integrity, and fairness, and students at the School of Engineering are expected to exhibit those qualities in their academic work. It is through the process of submitting their own work and receiving honest feedback on that work that students may progress academically. Any act of academic dishonesty is seen as an attack upon the School and will not be tolerated. Furthermore, those who breach the School's rules on academic integrity will be sanctioned under this Policy. Students are responsible for familiarizing themselves with the School's Policy on Academic Misconduct.

B. Definition: Academic dishonesty may include misrepresentation, deception, dishonesty, or any act of falsification committed by a student to influence a grade or other academic evaluation. Academic dishonesty also includes intentionally damaging the academic work of others or assisting other students in acts of dishonesty. Common examples of academically dishonest behavior include, but are not limited to, the following:

1. Cheating: intentionally using or attempting to use unauthorized notes, books, electronic media, or electronic communications in an exam; talking with fellow students or looking at another person's work during an exam; submitting work prepared in advance for an in-class examination; having someone take an exam for you or taking an exam for someone else; violating other rules governing the administration of examinations.

2. Fabrication: including but not limited to, falsifying experimental data and/or citations.

3. Plagiarism: intentionally or knowingly representing the words or ideas of another as one's own in any academic exercise; failure to attribute direct quotations, paraphrases, or borrowed facts or information.

4. Unauthorized collaboration: working together on work that was meant to be done individually.

5. Duplicating work: presenting for grading the same work for more than one project or in more than one class, unless express and prior permission has been received from the course instructor(s) or research adviser involved.

6. Forgery: altering any academic document, including, but not limited to, academic records, admissions materials, or medical excuses.

Accommodations

If you are a student with a disability who is requesting accommodations, please contact New York University's Moses Center for Students with Disabilities (CSD) at 212-998-4980 or mosescsd@nyu.edu. You must be registered with CSD to receive accommodations. Information about the Moses Center can be found at www.nyu.edu/csd. The Moses Center is located at 726 Broadway on the 2nd floor.