FRE6251
NUMERICAL AND SIMULATION TECHNIQUES IN FINANCE

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This half-semester course introduces the vast body of knowledge about how to actually implement various financial calculations on a digital computer. Much has been made about the enormous increases in calculating speed that have been achieved by computing hardware in the past decades, but what is much less widely known is that there have been improvements of similar magnitude in the numerical algorithms that run on that hardware.

Course Overview and Goals:

Only a small portion of this material can reasonably be presented in a half-semester course, so I have tried to focus on those aspects of it that are either directly relevant to financial engineering practice or helpful in understanding the directly relevant aspects. Thus, this course will include

- Solutions to non-linear algebraic equations found in finance
- Solution of ordinary differential equations and stochastic differential equations
- Solution of the partial differential equations of quantitative finance
- Numerical integration ("quadratures"), including the fast Fourier transform and an introduction to Monte Carlo methods
- Interpolation, most notably splines and "least squares"
- A survey of optimization techniques

Underlying these specifics will be the following "themes":

- Some methods are *much* better than others. In fact, sometimes methods fail altogether when it seems superficially that they should work, so it really pays to understand what is going on, even when using packages such as MatLab.
- The really good methods are often far from obvious.
- Mathematical analysis of numerical methods is often very revealing and is often the inspiration for the good, but non-obvious methods.

In the past, I presented this material in standard lecture format, augmented with frequent spreadsheet examples. However, in the Fall of 2021, when everything was done over Zoom, I recorded these zoom sessions and re-recorded those portions where I thought the presentation could be improved. The result is a set of recordings that is often better than the live lectures that I would likely give!

Meanwhile, in the Spring of 2022, I gave a "bonus lecture", in which I had the class work together with me in preparing a spreadsheet implementation of a Monte Carlo calculation. The enthusiastic reception of this "class participation" method of instruction has prompted me to do more of the same: rather than lecture about my spreadsheet examples, I propose to use class time to work with the class to re-create them!

Prerequisites

FRE6083 (Quantitative Methods) or equivalent and graduate standing. If you have these, you will have an understanding of multivariate calculus, a basic course in probability, and some prior study of both ordinary and partial differential equations.

Students will be expected to write programs in EXCEL/VBA (see "Grading" below for details). Computer professionals regard VBA as a "toy language", unfit for serious numerical work. However, our purpose here is not to do serious numerical work, but to study how such work is done, and, not incidentally, expose students to spreadsheets, the language of finance! Previous experience has shown that students who have written code in C, C++, java, or Python have little trouble picking up EXCEL/VBA. *However, students with no prior programming experience, or only experience running "canned" programs in languages such as MatLab, may well have trouble with this course.*

Required text

Numerical Recipes: The Art of Scientific Computing by Frank Press, *et. al.*, 3nd Ed., Cambridge University Press, 2007, ISBN-10: 0521880688. The 2nd edition is available free at http://www.nrbook.com/a/bookcpdf.php, but this is such an outstanding book that it's worth investing in the latest and greatest. References to chapters below assume the 3rd edition.

Recommended Reading

Options, Futures, and Other Derivatives, J. Hull, Prentice-Hall. Upper Saddle River, New Jersey. Any edition will do; hence the omission of ISBN numbers.

Introduction to Numerical Analysis, 3rd Ed., W. Stoer and R. Bulirsch, Springer-Verlag, New York. ISBN-10: 038795452X

This is the definitive book on much of numerical analysis. However, it is translated from the German, and it reads like it!

Numerical Methods in Finance and Economics: A MATLAB-Based Introduction, 2nd Edition. P. Brandimarte. Wiley. Hoboken, NJ. ISBN-10: 0471745030

Various books by Paul Wilmott and co-authors.

Course Materials and Resources

- **Access your course materials**: https://brightspace.nyu.edu/d2l/home/208906
- **Databases, journal articles, and more**: Bern Dibner Library (library.nyu.edu)
  NYU Virtual Business Library (guides.nyu.edu/vbl)
- **Obtain 24/7 technology assistance:** Tandon IT Help Desk (soehelpdesk@nyu.edu, 646.997.3123)
  NYU IT Service Desk (AskIT@nyu.edu, 212-998-3333)

Grading

My overall grading policy is

- Students showing evidence that they have mastered essentially everything I have taught by submitting all assignments correctly will get an "A".
- Students showing evidence that they have mastered much, but not all I have taught by submitting all assignments mostly correctly, but with significant imperfections, will get an "B".
- Students who are not in the room will get a "C".

"+" and "– " adjustments to grades are interpolations between the above.

Grades will have two components:

1. In-class spreadsheet exercises, selected from the list below, but subject to change, because this is an experiment! These must be submitted in working order by the beginning of the next class to receive credit.
2. The two programming assignments in EXCEL/VBA outside of class, as follows:
    a. Project I: Code and test a Visual Basic function to compute the implied volatility of an American vanilla put option that does not pay dividends.
    b. Project II: Code and test a Visual Basic function to perform a Value-at-Risk calculation of a given equities portfolio.

Detailed Course Outline

<u>Introduction, by way of Root Finding and Ordinary Differential Equations</u>

I "Machine numbers" vs standard mathematical numbers
- A) "short" and "long" integers
- B) Radix and mantissa of floating point numbers
- C) The "machine epsilon"
- D) Round-off error examples
- E) Machine independent measures of computational "work"

II Root finding in one dimension – implied volatility of European Call:
- A) Bisection
- B) Secant method
- C) Newton's method
- D) **SPREADSHEET EXAMPLE OF ABOVE METHODS**
- E) Brent's method

III Newton's method in *N* dimensions

IV Ordinary and Stochastic Differential Equations
- A) Existence theorem for ODE's
- B) Initial value problems vs boundary value problems
- C) Euler's method, explicit and implicit
- D) SDE's
- E) **SPREADSHEET EXAMPLE OF EULER METHOD FOR SDE'S**
- F) Runge-Kutta
- G) Variable Stepsizes

Reading: Chapter 1.1, 9, and 17 of *Numerical Recipes*

Partial Differential Equations – One Space/Price Dimension

I Intro. to partial differential equations (PDE's)
- A) Types of PDE's
  - i) Cauchy problems vs boundary value problems
  - ii) Hyperbolic, Elliptic, and Parabolic
  - iii) Free boundary problems
- B) Heat/diffusion equation as prototypical parabolic PDE
- C) Analytic solutions
- D) Standard finite difference approaches
  - i) Forward Time, Centered Step (FTCS)
  - ii) **SPREADSHEET EXAMPLE OF FTCS FOR 1 D HEAT EQUATION**
  - iii) Von Neumann stability and lack thereof

iv) Fully implict methods
v) Crank Nicholson
H) Dealing with sparse matricies
I) Binary and Trinary Trees – Accuracy vs Computational Effort
i) Binary tree for European/American option w/ error estimate
(1) Hull method
(2) CRR method
(3) Discrete dividends
(4) Computing Greeks
ii) Trinary trees

Reading: Chapter 20, Sections 0, 2, and 3 of *Numerical Recipes*
(Covers both lectures on partial differential equations)
Numerical methods chapters of Hull (Chapter number varies by edition)

Partial Differential Equations – Multiple Space/Price Dimensions

I Example of how multiple space/price dimensions arise in finance
II Discretization of the multiple dimensional diffusion operator
III Finite difference approaches and Von Neumann Stability revisited
A) Forward Time, Centered Step (FTCS)
B) **SPREADSHEET EXAMPLE OF FTCS FOR 2D HEAT EQUATION**
C) Fully implict methods
D) Crank Nicholson
IV Jacobi method and Successive Over-Relaxation (SOR)
V Operator splitting methods

Numerical Integration (Numerical Quadratures)

I Basic methods, inc. Gaussian quadrature **SPREADSHEET EXAMPLE OF BOTH**

Reading: *Numerical Recipes*, Chapter 4, though the Wikipedia account of Gaussian quadrature, http://en.wikipedia.org/wiki/Gaussian_quadrature, is at least as helpful

II The Fast Fourier Transform (Reading: *Numerical Recipes*, Chapter 12) **WITH SPREADSHEET EXAMPLE**

Reading: *Numerical Recipes*, Chapter 12, though the Wikipedia account of the Fast Fourier transform, http://en.wikipedia.org/wiki/Fast_Fourier_transform, is at least as helpful. I will be presenting the Cooley-Tukey version, so pay special attention to that.

Monte Carlo Simulation (as much of the following as time permits)

I Random number generation: An Oxymoron, but a Useful One
II Monte Carlo with Clever Tricks for Variance Reduction
A) The efficient market hypothesis as a rationale for Monte Carlo
B) Finding the area of a circle: a simple Monte Carlo calculation
i) Statistical analysis

- ii) Random (or not) number generation
- C) Non-uniform random numbers
- D) Generating correlated random variables
- E) Variance reduction techniques
  - i) Importance sampling with **SPREADSHEET EXAMPLES**
  - ii) Antithetic variance applied to Black Scholes European Call
  - iii) Control variates and stratified sampling
- F) Monte Carlo methods for American options

III Low discrepancy sequences
- A) The most basic low discrepancy sequence is the Halton sequence
- B) The more sophisticated Sobol' sequence seems to work better

Reading: *Numerical Recipes*, Chapter 7.0-7.3, 7.6, though *Probability, Random Variables, and Stochastic Processes* by Athanasios Papoulis (a Poly prof!) has a better explanation of how non-uniform random variables can be generated from uniform ones.

<u>Linear and Spline Interpolation</u>

I Why polynomial and linear interpolation don't cut it

II Splines
- A) "Natural" splines
- B) B-splines

III Limitations of splines

IV Two ways of improving on standard splines
- A) Rational interpolation
- B) Splines with tension

V "Least squares" a.k.a. multiple regression

Numerical Recipes Chapter 3 of *Numerical Recipes* has a discussion of splines, but I find both the Wikipedia article on splines, http://en.wikipedia.org/wiki/Spline_interpolation, and the Wolfram math world article on splines, http://mathworld.wolfram.com/CubicSpline.html easier to understand.

<u>Optimization in one and several dimensions</u>

I Why optimization important in finance

II Example: Max. likelihood estimation of GARCH(1,1) model

III Some unconstrained optimization problems and techniques
- A) Markowitz optimization and the CAPM
- B) "Lin min"
- C) Nelder & Mead's Simplex method
- D) Fletcher Powell

IV Constrained optimization
- A) Linear programming

B) Constrained quadratic optimization and the Black-Litterman model

V Combinatorial optimization (*e.g.* The Traveling Salesman Problem; Markowitz optimization with constraints)

VI The Levenberg Marquant method

Reading: *Numerical Recipes*, Chapter 10, esp. 10.1 thru 10.5

<u>Simulating Stochastic Differential Equations (SDE's)</u> (time permitting)

I What SDE's actually are - stochastic calculus background
  A) Modes of stochastic convergence
  B) Ito's lemma
  C) The Ito integral as the l.i.m. of a stochastic sum
II Example SDE: The lognormal stock price process
III The Euler-Maruyama method
IV Convergence modes of method
  A) Strong convergence: convergence of "mean of error"
  B) Weak convergence: convergence of "error of mean"
  C) Long term stability
VI The Milstein method

Reading: Hingam's Introduction to Numerical Solution of SDE's

Departmental/School-Wide Policies (Comments specific to the projects **in bold**, below)

Academic Misconduct

A. Introduction: The School of Engineering encourages academic excellence in an environment that promotes honesty, integrity, and fairness, and students at the School of Engineering are expected to exhibit those qualities in their academic work. It is through the process of submitting their own work and receiving honest feedback on that work that students may progress academically. Any act of academic dishonesty is seen as an attack upon the School and will not be tolerated. Furthermore, those who breach the School's rules on academic integrity will be sanctioned under this Policy. Students are responsible for familiarizing themselves with the School's Policy on Academic Misconduct.

B. Definition: Academic dishonesty may include misrepresentation, deception, dishonesty, or any act of falsification committed by a student to influence a grade or other academic evaluation. Academic dishonesty also includes intentionally damaging the academic work of others or assisting other students in acts of dishonesty. Common examples of academically dishonest behavior include, but are not limited to, the following:

   1. Cheating: intentionally using or attempting to use unauthorized notes, books, electronic media, or electronic communications in an exam; talking with fellow students or looking at another person's work during an exam; submitting work prepared in advance for an in-class examination; having someone take an exam for you or taking an exam for someone else; violating other rules governing the administration of examinations.
   2. Fabrication: including but not limited to, falsifying experimental data and/or citations.
   3. Plagiarism: Intentionally or knowingly representing the words or ideas of another as one's own in any academic exercise; failure to attribute direct quotations, paraphrases, or borrowed facts or information. **Submitting code that implements the same methodology as another student is not plagiarism; submitting the same code is plagiarism. It is surprisingly easy to tell the difference.**
   4. Unauthorized collaboration: working together on work that was meant to be done individually. **You are encouraged to discuss the projects with others. However, since I need to assign grades individually, I hold each of you individually responsible for the quality of the projects you submit. Submitting code "borrowed" from another student that is not fully understood therefore runs two risks: First, that I will punish you for plagiarism, and, second, that you will not detect problems in the code because you don't understand it!**
   5. Duplicating work: presenting for grading the same work for more than one project or in more than one class, unless express and prior permission have been received from the course instructor(s) or research adviser involved.
   6. Forgery: altering any academic document, including, but not limited to, academic records, admissions materials, or medical excuses.

Disability Disclosure Statement

Academic accommodations are available for students with disabilities. Please contact the **Moses Center for Students with Disabilities** (212-998-4980 or mosescsd@nyu.edu) for further information. Students who are requesting academic accommodations are advised to reach out to the Moses Center as early as possible in the semester for assistance.

Inclusion Statement

The NYU Tandon School values an inclusive and equitable environment for all our students. I hope to foster a sense of community in this class and consider it a place where individuals of all backgrounds, beliefs, ethnicities, national origins, gender identities, sexual orientations, religious and political affiliations, and abilities will be treated with respect. It is my intent that all students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed resource, strength and benefit. If this standard is not being upheld, please feel free to speak with me.

One of the ways that I try to maintain an equitable environment is by devising grading standards that are fair to all students. I therefore cannot arbitrarily raise a student's grade simply because failure to do so will "spoil their GPA" or cause them to lose a scholarship.