FRE6233 Option Pricing and Stochastic Calculus

Professor	Keith A. Lewis
Course	FRE 6233 Section A
When	Thursday, 2pm-4:30pm
Where	Jacobs Academic Bldg Rm 777 Loc: Brooklyn Campus
Contact	kal12@nyu.edu
Phone	646-662-7113
Office hours	Thursdays, 1pm-2pm Jacobs 777 or by appointment
Pre-requisite	FRE 6083

Syllabus and course material based on notes graciously provided of Professor Agnes Tourin.

> This course provides the mathematical foundations of Option Pricing models. The techniques covered include arithmetic and geometric Brownian motion, first passage time, the reflection principle, the stochastic Ito integral, Ito differential Calculus, change of probability measure, martingales, Stochastic Differential Equations and Partial Differential Equations. Some of the pricing models considered are the European, Barrier, Asian and American options. These problems are either solved analytically by the martingale or Partial Differential Equations approach, or numerically, by applying approximation and simulation methods. Since the same techniques allow the treatment of more complex financial products, several advanced examples will be also presented.

Textbook Shreve, *Stochastic Calculus for Finance II: continuous-time models, 2nd edition*, 2004, Springer.

Course Objectives

- After taking this course, the students will be able to price any derivative security.
- This is a first course in Mathematical Finance and is a building block for more advanced courses.
- The material taught is part of the common knowledge shared by quants in the Financial industry and taking this course will prepare the students to apply for quantitative positions.
- This course also prepares the students who wish to apply to a Ph.D. program in Financial Engineering, Mathematical Finance, Operations Research, or Finance.

Additional Resources (optional)

- A Primer for the Mathematics of Financial Engineering, Dan Stefanica, Second Edition, 2011, FE Press New York.
- Monte Carlo Methods in Financial Engineering, Paul Glasserman, 2004, Springer.
- Financial Modelling With Jump Processes, Rama Cont and Peter Tankov, 2004, Chapman & Hall/CRC.
- Computational Methods in Finance, Ali Hirsa, 2013, Chapman & Hall/CRC.
- Introduction to Mathematical Finance, Discrete Time Models, Stanley R. Pliska, 1997, Blackwell Publishing.
- Stochastic Differential Equations: An introduction with Applications, Bernt Oksendal, Universitext, Third Printing, Sixth Edition, 2009, Springer.
- Probability Essentials, Jean Jacod and Philip Protter, Universitext, Second Printing, Second Edition, 2004, Springer.
- Numerical Partial Differential Equations: Finite Difference Methods, J.W. Thomas, Texts in Applied Mathematics, 22, 1995, Springer.

Recommended software for the homework:

Students will be required to use a programming language for prototyping, such as Python, Matlab, R, or Excel add-ins.

Course requirements:

- Students are expected to attend classes and participate actively. They should view read the textbooks ahead of time and come prepared to ask questions and discuss the weekly assignment.
- There will be weekly homework assignments and an in class final examination. Each counts for 50% of the final grade. Homework is due at the beginning of each class. No late homework will be accepted. The lowest homework grade will be discarded.
- There will be two types of homework assignments. The first type will consist of practice exercises designed to help the students assimilate the techniques taught in class and prepare them for the final. The second type will consist of implementing numerical and simulation techniques, to compute option prices that cannot be computed analytically.

Part I: Ito Stochastic Calculus

Week 1. Information and the Brownian motion Reading: Shreve, chapters 1-2.

- Information and conditioning.
- Brownian motion and filtrations.

Week 2. Ito Calculus Reading: Shreve, chapters 3.

- Ito formula.
- Reflections principal.
- Girsanov's Theorem.

Week 4. Application of stochastic calculus to the Black-Scholes mode Reading: Shreve, chapters 4.

- Black-Scholes/Merton partial differential equation.

Part II: The no arbitrage theory in continuous-time

Week 5. The martingale approach. Reading: Shreve, chapters 5.

Week 6. The Partial Differential Equations approach Reading: Shreve, chapters 6.

Week 7. Software implementation.

Week 8: The Asian option. Reading: Shreve, chapters 7.

Week 9: The martingale and PDE approaches

Week 10: American option pricing

weeks 11-12. Multidimensional market models: Reading: Shreve, chapters 5,9.

- Stochastic calculus in several dimensions.
- Multi-dimensional asset pricing models.
- Change of numeraire.
- Applications to FX derivatives.

Part III: Jump-diffusion processes and incomplete market models

Reading: Shreve, chapters 11.

Weeks 13. Unified Model Reading: Unified Model

- Fundamental Theorem of Asset Pricing.
- Discrete time hedging.
- Risk management.

Week 14. Review

> The NYU Tandon School values an inclusive and equitable environment for all our students. I hope to foster a sense of community in this class and consider it a place where individuals of all backgrounds, beliefs, ethnicities, national origins, gender identities, sexual orientations, religious and political affiliations, and abilities will be treated with respect. It is my intent that all students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength and benefit. If this standard is not being upheld, please feel free to speak with me.

NYU School of Engineering Policies and Procedures on Academic Misconduct (from the School of Engineering Student Code of Conduct)

A. Introduction: The School of Engineering encourages academic excellence in an environment that promotes honesty, integrity, and fairness, and students at the School of Engineering are expected to exhibit those qualities in their academic work. It is through the process of submitting their own work and receiving honest feedback on that work that students may progress academically. Any act of academic dishonesty is seen as an attack upon the School and will not be tolerated. Furthermore, those who breach the School's rules on academic integrity will be sanctioned under this Policy. Students are responsible for familiarizing themselves with the School's Policy on Academic Misconduct.

B. Definition: Academic dishonesty may include misrepresentation, deception, dishonesty, or any act of falsification committed by a student to influence a grade or other academic evaluation. Academic dishonesty also includes intentionally damaging the academic work of others or assisting other students in acts of dishonesty. Common examples of academically dishonest behavior include, but are not limited to, the following:

1. Cheating: intentionally using or attempting to use unauthorized notes, books, electronic media, or electronic communications in an exam; talking with fellow students or looking at another person's work during an exam; submitting work prepared in advance for an in-class examination; having someone take an exam for you or taking an exam for someone else; violating other rules governing the administration of examinations.

2. Fabrication: including but not limited to, falsifying experimental data and/or citations.

3. Plagiarism: intentionally or knowingly representing the words or ideas of another as one's own in any academic exercise; failure to attribute direct quotations, paraphrases, or borrowed facts or information.

4. Unauthorized collaboration: working together on work that was meant to be done individually.

5. Duplicating work: presenting for grading the same work for more than one project or in more than one class, unless express and prior permission has been received from the course instructor(s) or research adviser involved.

6. Forgery: altering any academic document, including, but not limited to, academic records, admissions materials, or medical excuses.

Accomodations

If you are a student with a disability who is requesting accommodations, please contact New York University's Moses Center for Students with Disabilities (CSD) at 212-998-4980 or mosescsd@nyu.edu. You must be registered with CSD to receive accommodations. Information about the Moses Center can be found at www.nyu.edu/csd. The Moses Center is located at 726 Broadway on the 2nd floor.