

FRE 6931: Algorithmic Differentiation and Adjoint Methods in Finance

Spring 2022

Instructor Information

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Zoom Office Hours: Fridays 6 p.m. (by appointment)

Class Information

Dates: On Thursday, from January 27th to March 17th

Time: 6.10 p.m. - 8.40 p.m.

Classroom: TBD

Special Dates

Special dates will be announced.

Course Description

Adjoint Algorithmic Differentiation (AAD) is one of the principal innovations in risk management of the recent times, revolutionizing the way practitioners compute price sensitivities (the so-called ‘Greeks’) of derivatives portfolios and reducing by orders of magnitude the cost associated with their computation.

Despite being a well-established mathematical approach, the potential of Algorithmic Differentiation (AD) has remained largely untapped until very recently in many areas of natural and applied sciences. In particular, it has been only recently rediscovered in financial engineering where it has opened a new important chapter in risk management, by making possible the calculation of the risk borne by large portfolios of securities accurately, in real time and with limited computational costs, ultimately boosting profitability through better risk management practices.

Being a relatively recent development in financial engineering, AD and Adjoint Methods are still not generally part of Master of Financial Engineering curricula, and the aim of this course is to contribute to bridge this gap and provide a comprehensive and practical introduction to this topic.

The theory of AD will be discussed starting from simple, easy to understand, examples illustrating the workings of the different ‘modes’ of AD and the reason behind the remarkable computational efficiency of adjoint methods. Focus will be placed on its application to Monte Carlo methods for Stochastic Differential Equations and finite difference methods for Partial Differential Equations which will be reviewed putting emphasis on practical examples and applications. With several use cases, drawing from different asset classes both in Equities and Fixed Income, the course will illustrate the workings of AD and demonstrate how it can be straightforwardly implemented to reduce the computation time of the risk of any portfolio by order of magnitudes. Each example will be preceded by a self-contained and practical overview of the relevant option pricing theory/methods.

This course can be used as a complement to modules on numerical methods in financial engineering.

Materials

Slides/Lecture Notes will be made available as the course progresses.

Outline

- Module 1: Monte Carlo Methods in Finance
 - General Concepts: Quadrature vs Stochastic Methods
 - Simulating Stochastic Differential Equations by Monte Carlo
 - Discretization Schemes
 - Multivariate Processes
 - Correlation and Choleski's decomposition.
- Module 2: Computing Risk in Monte Carlo
 - The Finite Difference Method
 - The Likelihood Ratio Method
 - The Pathwise Derivative Method
 - Algebraic Adjoint Approaches
- Lecture 2: Algorithmic Differentiation
 - Computational Graphs
 - Tangent Mode
 - Adjoint Mode (AAD)
 - Computing Greeks in Monte Carlo
 - Smoking Adjoints for the Libor Market Model
 - Automated Tool (Operator Overloading/Source Code Transformation)
- Lecture 3: AAD and Monte Carlo I
 - AAD and the Pathwise Derivative Method
 - Case Study: Adjoint Greeks for the Libor Market Model
 - Correlation Greeks
 - Case Study: Correlation Risk for Basket Default Contracts
- Lecture 4: AAD and Monte Carlo II
 - Case Study: Real Time Counterparty Credit Risk Management
 - Case Study: Longstaff and Schwartz and AAD
 - Second Order Greeks
 - Payout Regularization
- Lecture 5: AAD and Partial Differential Equations (PDE)

- Option Pricing and PDEs
 - Discretization of Forward and Backward parabolic PDEs
 - Adjoint Forward parabolic PDEs
 - Adjoint Backward parabolic PDEs
- Lecture 6: AAD and Calibration
 - Calibration problems
 - AAD and the Implicit Function Theorem
 - AAD Source Risk and the Implicit Function Theorem
 - Case Study: Calibration of Interest Rate and Default Intensity short-rate models