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Partial Differential Equations for Finance

Finance and Risk Engineering

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Course description

The study of partial differential equations (PDE's) started in the 18th century as a central tool in the description of mechanics. The analysis of physical models has remained to the present day one of the fundamental concerns of the development of PDE's. Most of the natural laws of physics such as Maxwell equations, the Navier-Stokes equations, Newton equations of motion, and Schrodinger's equation of quantum mechanics, are stated in terms of PDE's by relating space and time derivatives. Derivatives occur in these equations because derivatives represent natural things (like velocity, acceleration, flux, current).

Beginning in the middle of the 19th century, PDE's became an essential tool in many branches of mathematics itself, such as differential geometry, analytic functions of complex variables, theory of Riemann surfaces, chaos theory and stochastic processes and stochastic differential equations, which is of interest to finance.

Therefore, it is no surprise that PDE are also very important for modeling and analysis in quantitative finance. For example, the famous Black Scholes equation represents the classical heat or diffusion equation, which has been studied for nearly two centuries. Most of the PDE encountered in financial applications are first and second order linear parabolic equations. The goal of this course is to introduce the main technique of solving parabolic PDE such as separation of variables, integral transform , Green function, perturbation methods, eigenfunction expansions with a specific goal of financial applications. We will illustrate these methods on particular derivatives pricing problems in fixed income, credit and commodities.

Agenda:

- Recall of technique for solving ODE and SDE equations
- The Feynman - Kac theorem on relations of SDE and PDE. Multidimensional Feynman - Fac theorem and Asian options.
- Boundary value problems, fundamental solution, Green function, integral transform methods and maximum principle for linear parabolic equations.
- Heat equation and BS formula.
- American Options as Variational Inequalities
- Interest Rate Models: Hull-White model, Vasicek model.
- PDE for credit derivatives: stochastic intensity models. CIR and Vasicek models, pricing of defaultable bonds and CDS.
- Pricing single and double barrier options by Fourier and Laplace transforms
- Heston model by method of characteristic function. Perturbation methods for SV models.

Target Audience

This course is intended for students of the Master Program in Science in Financial Engineering, who wants to deepen their knowledge in the PDE technique and their applications for derivatives pricing. It will prepare the students to apply for quantitative positions, as well a Ph.D. program in Financial Engineering or Mathematical Finance. The accent of the course is on using theoretical methods for solving important practical problems in major financial asset classes. Hence, the participation in this course involves several case studies by using analytical and in some cases numerical methods.

Final Project

In lieu of a final exam students will work on projects (team or individual) on derivatives pricing models using PDE methods. The project will involve all the aspects of creating new models in finance industry, such as calculation of Greeks, testing and calibration.

Prerequisites

The minimum course pre-requisites is FRE 6083. Some knowledge of stochastic calculus is desirable, some initial background in ODE and PDE is welcome but not required. Knowledge of a programming language is required for numerical applications.

Everyone with a good base and willing to devote time to immerse into a PDE world for seven weeks should be able to get a valuable knowledge