Polytechnic Tutoring Center Final Review - PH-UY 2033 Spring 2021 Solutions

1. If two polarizing lens are perpendicular to another, the final intensity is 0 because $I = \frac{1}{2}\cos^2(90)I_0 = 0$. If another polarization lens is placed in the middle, the final intensity is given by

$$I = \frac{1}{2}\cos^2(\theta)\cos^2(90-\theta)I_0 = \frac{1}{2}\cos^4(45)20 = 2.5\ \mathrm{W/m^2} \tag{1}$$

2. The wall they share has area $A = 4.5^2 = 20.25\ m^2$. The heat transfer, provided by n number of light bulbs is given by

$$\begin{aligned} NP &= \left|\frac{dQ}{dt}\right| = kA\frac{\Delta T}{\Delta x} \\ N &= \frac{(0.84)(20.25)(35-12)}{(100)(0.13)} = 30 \end{aligned}$$

3. This question asks for the critical angle, which happens when you take ($n_{air} = 1$, $\theta_{air} = 90°$. Recall the index of refraction in an material is defined as $n_i = c/v_i$. The critical angle in the fiber cable can be derived as

$$\begin{aligned} n_a \sin\theta_a &= n_f \sin\theta_f = \frac{c}{v}\sin\theta_f \\ \theta_f &= \sin^{-1}\left(\frac{vn_a\sin\theta_a}{c}\right) = \sin^{-1}\left(\frac{v}{c}\right) = \sin^{-1}\left(\frac{2.16\times 10^8}{3.00\times 10^8}\right) = 46.05° \end{aligned}$$

While 46.05 is the normal angle, we want to take $90 - 46.05 = 43.95°$ as the angle to the wall.

4. We can compute the equilibrium temperature by equating the heat.

$$\begin{aligned} Q_1 + Q_2 &= Q_{eq} \\ m_1T_1 + m_2T_2 &= (m_1+m_2)T_{eq} \\ T_{eq} &= \frac{m_1T_1 + m_2T_2}{m_1+m_2} = \frac{9(273+3)+5(273+38)}{14} = 288.5\ \mathrm{K} \end{aligned}$$

5. Recall critical angle given by $\theta_c = \sin^{-1}(n_1/n_2)$,

$$\begin{aligned} \theta_{c,air} &= 38 = \sin^{-1}(n_{air}/n) \\ n &= n_{air}/\sin(38) \\ \theta_{c,water} &= \sin^{-1}(n_{water}/n) = \sin^{-1}\left(\frac{n_{water}}{n_{air}/\sin(38)}\right) = 55° \end{aligned}$$

6. Using lensmakers equation, we can establish the relationship

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

As $R_1 \to \infty$, $1/R_1 \to 0$, we have

$$\frac{1}{f} = (n-1)\frac{1}{R_2} \implies n = \frac{45}{75} + 1 = 1.6$$

7. The intensity of the light bulb at distance r is given by $I = P/A$. Since the bulb radiates uniformly and spherically, we take the area to be $4\pi r^2$. The intensity at 3, 4 m are given by

$$I_3 = \frac{60}{(4)3^2\pi} \quad \text{and} \quad I_4 = \frac{60}{(4)4^2\pi}$$

Their difference is then given by

$$I_3 - I_4 = \frac{15}{\pi}\left(\frac{1}{9} - \frac{1}{16}\right) = 0.232 \text{ W/m}^2.$$

8. The distance between the central diffraction peak and the mth dark spot is given by

$$\frac{dy}{L} \approx m\lambda.$$

We can then compute

$$y \approx \frac{Lm\lambda}{d} = \frac{1.5(3)(650 \times 10^{-9})}{1 \times 10^{-4}} = 0.02925 \text{ m} = 29.25 \text{ mm}$$

9. Using

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

and $f = R/2$, by taking negative f for convex mirrors, we have d_i computed as

$$-\frac{2}{0.30} = \frac{1}{10} + \frac{1}{d_i} \implies d_i = \left(-\frac{1}{0.15} - \frac{1}{10}\right)^{-1} = -0.148$$

The magnification is then given by

$$h_i = Mh_0 = \frac{d_i}{d_o}h_0 = \frac{0.1478}{10}(2) = -0.0296 \text{ m}$$

10. For a two lens system, we can build the following relationship for each lens

$$\frac{1}{d_{o,1}} + \frac{1}{d_{i,1}} = \frac{1}{f}, \quad \frac{1}{d_{o,2}} + \frac{1}{d_{i,2}} = \frac{1}{f}$$

where we can use $d_{o,2} = L - d_{i,1}$ to relate the two. L is the distance between the two lenses. Solve the above system for $d_{i,2}$:

$$d_{i,1} = \left(\frac{1}{f} - \frac{1}{d_{o,1}}\right)^{-1} = \left(\frac{1}{20} - \frac{1}{30}\right)^{-1} = 60 \text{ cm}$$
$$d_{i,2} = \left(\frac{1}{f} - \frac{1}{d_{o,2}}\right)^{-1} = \left(\frac{1}{f} - \frac{1}{L - d_{i,1}}\right)^{-1} = \left(\frac{1}{20} - \frac{1}{10 - 60}\right)^{-1} = 14.3 \text{ cm}$$

11. Using

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

and $f = R/2$, by taking positive f for concave mirrors, we have d_i computed as

$$\frac{2}{50} = \frac{1}{70} + \frac{1}{d_i} \implies d_i = \left(\frac{1}{25} - \frac{1}{70}\right)^{-1}$$

The one dimensional magnification is then given by

$$M = \frac{d_i}{d_o} = \frac{1}{70\left(\frac{1}{25} - \frac{1}{70}\right)} = 0.56$$

When we talk about ratio of two dimensional area, we take $M^2 = 0.31$.

12. Linear expansion from heat is given by

$$\Delta l = \alpha l_0 \Delta T$$

By setting up the expanded length, we have

$$35.280 = l_0 + \Delta l = l_0 + (2.4 \times 10 - 5)(40 - 20) l_0$$
$$l_0 = \frac{35.280}{1 + (2.4 \times 10 - 5)(20)} = 35.263 \text{ m}$$

Their difference is

$$35.280 - 35.263 = 0.017 \text{ m}.$$

13. Monoatonic gas' expansion is modeled using $C_V = 3R/2$, and

$$\gamma = \frac{C_P}{C_V} = \frac{5R/2}{3R/2} = 5/3.$$

We have

$$P_i V_i^{5/3} = P_f V_f^{5/3}.$$

Using ideal gas law, we can substitute $P_f V_f / P_i V_i$ with T_f / T_i. Hence,

$$T_i V_i^{2/3} = T_f V_f^{2/3}$$
$$V_i = \left(\frac{T_f}{T_i}\right)^{3/2} V_f = \left(\frac{273 - 22}{273 + 35}\right)^{3/2} (1.2) = 0.883 \ m^3$$

14. The efficiency of the engine is

$$e = \frac{W}{Q_H} = \frac{1800 - 940}{1800} = 0.48$$

The Carnot engine efficiency is given by

$$e_c = 1 - \frac{T_L}{T_H} = 1 - \frac{38 + 273}{1300 + 273} = 0.80$$

Their ratio is thus

$$\frac{0.48}{0.80} = 0.60$$

15. When we model simple harmonic motion with $A\sin(\omega t)$ or $A\cos(\omega t)$. The amplitude of second derivative is $A\omega^2$. Since angular velocity is given by $\omega = 2\pi/T$. The result is

$$A\omega^2 = \frac{A4\pi^2}{T^2} = \frac{4\pi^2(10)}{9} = 43.86\ cm/s^2$$