Polytechnic Tutoring Center Final Review - PH-UY 2023 Spring 2021 Solutions

1. The total capacitance of capacitors in series is given by

$$C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{10} + \frac{1}{21}\right)^{-1} = \frac{210}{31} = 6.77 \text{ nF}$$

   The charge is therefore

$$Q = CV = 6.77(12) = 81.29 \text{ nF}$$

   Since charge is the same across a series of capacitors, the voltage at 10 nF capacitor is therefore

$$V = \frac{Q}{C_1} = \frac{81.29}{10} = 8.13 \text{ V}$$

2. The energy from from each pair of particles is the same. So to remove one particle, the energy is simply 3 times the energy between a pair.

$$\sum_{i=1}^{3} U_{e,i} = 3U_e = -\frac{3kq^2}{r} = -\frac{3(8.98 \times 10^9)(2 \times 10^{-6})^2}{0.01} = -10.78 \text{ J}$$

3. First, we want to fix flow rate.

$$A_i v_i = A_f v_f \implies \pi r_i^2 v_i = \pi r_f^2 v_f \implies v_f = \left(\frac{r_i}{r_f}\right)^2 v_i$$

   Then, using Bernoulli's equation, we have

$$\Delta P = \frac{1}{2}\rho(v_f^2 - v_i^2) = \frac{1}{2}\rho\left(\left(\frac{r_i}{r_f}\right)^4 - 1\right)v_i^2 = \frac{1}{2}\rho\left(\left(\frac{r_i}{r_f}\right)^4 - 1\right)\left(\frac{dV/dt}{A_i}\right)^2$$
$$= \frac{1}{2}(1000)\left(\left(\frac{5}{1.2}\right)^4 - 1\right)\left(\frac{0.006}{0.05^2\pi}\right)^2 = 88 \text{ kPa}$$

   $P = P_{atm} + \Delta P = 189$ kPa.

4. The current in a LR system at a given time is given by

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$
$$I(3 \times 10^{-3}) = \frac{6}{8}\left(1 - e^{-8(10^{-3})/0.25}\right) = 0.0236 \text{ A}$$

   The voltage at the resister given current can be computed from Ohm's law

$$V = IR = 0.0236(8) = 0.189 \text{ V}$$

5. We can treat this system as a combined capacitor and a combined resistor. The total capacitance is then

$$\left((C+C)^{-1}+C^{-1}\right)^{-1} = \left(\frac{1}{2(3)}+\frac{1}{3}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2 \text{ F}$$

The resistance is then,

$$R + \left(\frac{1}{R}+\frac{1}{R}\right)^{-1} = \frac{3R}{2} = \frac{3(2)}{2} = 3\ \Omega$$

The time constant is given by

$$\tau = RC = 6$$

6. Recall the magnetic field magnitude around long straight current carrying wire is given by

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

We can compute the distance from the 3/4 point to each wire given by

$$r_1 = 3/4 \text{ and } r_2 = 1/4$$

The region between the two wires experiences opposite direction of magnetic field because magnetic fields run circularly around the wire. Therefore, the magnitude of the net field is

$$|B_{net}| = \frac{\mu_0 I}{2\pi}\left|\frac{1}{r_1}-\frac{1}{r_2}\right| = \frac{(4\pi\times 10^{-7})(30)(4)}{2\pi}\left(1-\frac{1}{3}\right) = 1.6\times 10^{-5} \text{ T}$$

7. Simply apply Ampere's Law as follows

$$\int B\cdot d\ell = \mu_0 I_{in} = (4\pi\times 10^{-7})(5\times 10^{-3}) = 2\pi\times 10^{-9} = 6.28 \text{ nHA/m}$$

8. The equation for magnetic field at the center of a current carrying ring is

$$B = \frac{\mu_0 I}{2R}.$$

Since the two currents are running in opposite direction, their magnetic fields subtracts. Therefore, the magnitude of the net magnetic field is

$$|B_{net}| = |B_5 - B_7| = \frac{\mu_0}{2}\left(\frac{5}{1}-\frac{7}{2}\right) = \frac{3\mu_0}{4} = 3\pi\times 10^{-7} \text{ T}$$

9. The area of the coil is given by $A_0 = \pi r^2 = 2.5\pi\times 10^{-4}$ m^2. Since the area that is perpendicular to magnetic flux is changing due to rotation, we can model the perpendicular area or the projection as

$$A(t) = A_0\cos(\omega t), \quad \text{where } \omega \text{ is the angular velocity.}$$

The instantaneous change of the area is computed by taking the derivative

$$A'(t) = -A_0\,\omega\sin(\omega t)$$

Recall formula for induced voltage and formula for magnetic flux

$$V = N\frac{d}{dt}(\Phi) = N\frac{d}{dt}(BA(t)) = NBA'(t) = NBA_0\,\omega\cos(\omega t)$$

Using Ohm's Law, we can find the current. Also, we can neglect the sign since we are only interested in the magnitude of the current.

$$\begin{aligned} I &= \frac{V}{R} = \frac{NBA_0\,\omega\sin(\omega t)}{R} \\ &= \frac{(1000)(0.2)(2.5\pi\times 10^{-4})(200)\sin(200\cdot 5.236\times 10^{-3})}{2} = 136\text{ A} \end{aligned}$$

10. To compute the potential difference, We only need to integrate over the z axis since the x and y directions are unaffected by the field. Using the formula for electric field for large plane, we obtain

$$V = -\int_7^5 E\,dz = \int_5^7 \frac{\sigma}{2\epsilon_0}dz = \frac{(10\times 10^{-9})(7-5)}{2(8.85\times 10^{-12})} = 1130\text{ V}$$

11. The speed of the deuteron after the acceleration can be computed from conservation of energy, where the electric potential energy is converted into kinetic energy.

$$U_e = Vq = KE = \frac{1}{2}mv^2 \implies v = \sqrt{\frac{2Vq}{m}}$$

Charged particle orbits in magnetic field due to magnetic force acting as a centripetal force. We can then equating these two forces to find the radius of the orbit

$$ma_c = \frac{mv2}{r} = qvB.$$

Hence,

$$r = \frac{mv}{qB} = \frac{1}{B}\sqrt{\frac{2Vm}{q}} = \frac{1}{1.6}\sqrt{\frac{2(10^4)(3.34\times 10^{-27})}{1.60\times 10^{-19}}} = 0.0128\text{ m}$$

12. We can computer number of turns N from dividing length by circumference. i.e.

$$N = \frac{13}{2\pi(0.23)} \approx 9$$

Setting up the equation for magnetic field, we have

$$B = \mu_0 NI \implies I = \frac{B}{\mu_0 N} = \frac{35\times 10^{-6}}{(4\pi\times 10^{-7})(9)} = 3.09A$$

13. Since they orbiting in perfect circles, we can view one as fixed, and view one particle orbiting the other with radius equals to diameter of their path, that is $2r$. The velocity of that particle is twice as the velocity on the circle path due to same angular velocity.

$$\frac{kq^2}{(2r)^2} = \frac{m(2v)^2}{2r} \implies v = q\sqrt{\frac{k}{8mr}}$$

We can view $2qv$ as a loop current around the center of their path. The magnetic field is therefore

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 qv}{r} = \mu_0 q^2\sqrt{\frac{k}{8mr^3}}$$
$$= (4\pi \times 10^{-7})(3 \times 10^{-3})^2\sqrt{\frac{8.99 \times 10^9}{8(0.02)(1^3)}} = 2.68 \times 10^{-6} \text{ T}$$

14. Since induced voltage depends on the change in flux. Knowing the electric flux of a loop, we have

$$|\mathcal{E}| = N\left|\frac{d\phi_B}{dt}\right| = N\left|\frac{d}{dt}B(t)A\right| = N(\pi R^2)\frac{d}{dt}\frac{\mu_0|I(t)|}{2R} = \frac{N\pi R\mu_0}{2}\left|\frac{dI}{dt}\right|$$

We can isolate the magnitude for the change in current.

$$\left|\frac{dI}{dt}\right| = \frac{2|\mathcal{E}|}{N\pi R\mu_0} = \frac{2(10)}{100\pi(0.05)(4\pi \times 10^{-7})} = 1013200 \text{ A/s}$$

15. Potential energy is defined using dot product. Thus

$$U = \mu \cdot B = \mu B\cos\theta = 200\cos(30) = 173 \text{ J}$$