FRE6251
NUMERICAL AND SIMULATION TECHNIQUES IN FINANCE

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This half-semester course introduces the vast body of knowledge about how to actually implement various financial calculations on a digital computer. Much has been made about the enormous increases in calculating speed that have been achieved by computing hardware in the past few decades, but what is much less widely known is that there have been improvements of similar magnitude in the numerical algorithms that run on that hardware.

Course Overview and Goals:

Only a small portion of this material can reasonably be presented in a half-semester course, so I have tried to focus on those aspects of it that are either directly relevant to financial engineering practice or helpful in understanding the directly relevant aspects. Thus, this course will include

- Solutions to non-linear algebraic equations found in finance
- Solution of ordinary differential equations and (if time permits) stochastic differential equations
- Solution of the partial differential equations of quantitative finance
- Numerical integration ("quadratures"), including the fast Fourier transform and an introduction to Monte Carlo methods
- Interpolation, most notably splines and "least squares"
- A survey of optimization techniques

Underlying these specifics will be the following "themes":

- Some methods are *much* better than others. In fact, sometimes methods fail altogether when it seems superficially that they should work, so it really pays to understand what is going on, even when using packages such as MatLab.
- The really good methods are often far from obvious.
- Mathematical analysis of numerical methods is often very revealing and is often the inspiration for the good, but non-obvious methods.

My goal is to convey all of the above to my students.

Prerequisites

FRE6083 (Quantitative Methods) or equivalent and graduate standing. If you have these, you will have an understanding of multivariate calculus, a basic course in probability, and some prior study of both ordinary and partial differential equations.

Students will be expected to write programs in EXCEL/VBA (see "Grading" below for details). Previous experience has shown that students who have written code in C, C++, java, or Python have little trouble picking up EXCEL/VBA. *However, students with no prior programming experience, or only experience running "canned" programs in languages such as MatLab, may well have trouble with this course.*

Required text

Numerical Recipes: The Art of Scientific Computing by Frank Press*, et. al.*, 3rd Ed., Cambridge University Press, 2007, ISBN-10: 0521880688. The 2nd edition is available free at http://www.nrbook.com/a/bookcpdf.php, but this is such an outstanding book that it's worth investing in the latest and greatest. References to chapters below assume the 3rd edition.

Recommended Reading

Options, Futures, and Other Derivatives, J. Hull, Prentice-Hall. Upper Saddle River, New Jersey. Any edition will do; hence the omission of ISBN numbers.

Introduction to Numerical Analysis, 3rd Ed., W. Stoer and R. Bulirsch, Springer-Verlag, New York. ISBN-10: 038795452X

This is the definitive book on much of numerical analysis. However, it is translated from the German, and it reads like it!

Various books by Paul Wilmott and co-authors.

Course Materials and Resources

- **Access your course materials**: NYU Classes (nyu.edu/its/classes)
- **Databases, journal articles, and more**: Bern Dibner Library (library.nyu.edu)
  NYU Virtual Business Library (guides.nyu.edu/vbl)
- **Obtain 24/7 technology assistance:** Tandon IT Help Desk (soehelpdesk@nyu.edu, 646.997.3123)
  NYU IT Service Desk (AskIT@nyu.edu, 212-998-3333)

Grading:

Grades will be assigned based on the preparation of two programming assignments in EXCEL/VBA outside of class. The assignments are as follows:

Project I

Code and test a Visual Basic function to compute the implied volatility of an American option that does not pay dividends.

Project II

Code and test a Visual Basic function to perform a Value-at-Risk calculation of a given equities portfolio.

Detailed Course Outline

Introduction, by way of Root Finding and Ordinary Differential Equations

I "Machine numbers" vs standard mathematical numbers
- A) "short" and "long" integers
- B) Radix and mantissa of floating point numbers
- C) The "machine epsilon"
- D) Round-off error examples
- E) Machine independent measures of computational "work"

II Root finding in one dimension – implied volatility of European Call
- A) Bisection
- B) Secant method
- C) Newton's method
- D) Brent's method

III Newton's method in *N* dimensions

IV Ordinary Differential Equations (ODE's)
- A) Existence theorem
- B) Initial value problems vs boundary value problems
- C) Euler's method, explicit and implicit
- D) Runge-Kutta
- E) Variable Stepsizes

Reading: Chapter 1.1, 9, and 17 of *Numerical Recipes*

Partial Differential Equations – One Space/Price Dimension

I Intro. to partial differential equations (PDE's)
- A) Types of PDE's
  - i) Cauchy problems vs boundary value problems
  - ii) Hyperbolic, Elliptic, and Parabolic
  - iii) Free boundary problems
- B) Heat/diffusion equation as prototypical parabolic PDE
- C) Analytic solutions
- D) Standard finite difference approaches
  - i) Forward Time, Centered Step (FTCS)
  - ii) Von Neumann stability and lack thereof
  - iii) Fully implict methods
  - iv) Crank Nicholson
- F) Dealing with sparse matricies

G) Binary and Trinary Trees – Accuracy vs Computational Effort
  i) Binary tree for European/American option w/ error estimate
    (1) Hull method
    (2) CRR method
    (3) Discrete dividends
    (4) Computing Greeks
  ii) Trinary trees

Reading: Chapter 20, Sections 0, 2, and 3 of *Numerical Recipes*
(Covers both lectures on partial differential equations)
Numerical methods chapters of Hull (Chapter number varies by edition)

<u>Partial Differential Equations – Multiple Space/Price Dimensions</u>

I Example of how multiple space/price dimensions arise in finance
II Discretization of the multiple dimensional diffusion operator
III Finite difference approaches and Von Neumann Stability revisited
  A) Forward Time, Centered Step (FTCS)
  B) Fully implict methods
  C) Crank Nicholson
IV Jacobi method and Successive Over-Relaxation (SOR)
V Operator splitting methods

<u>Numerical Integration (Numerical Quadratures)</u>

I Basic methods, inc. Gaussian quadrature

Reading: *Numerical Recipes*, Chapter 4, though the Wikipedia account of Gaussian quadrature, http://en.wikipedia.org/wiki/Gaussian_quadrature, is at least as helpful

II The Fast Fourier Transform (Reading: *Numerical Recipes*, Chapter 12)

Reading: *Numerical Recipes*, Chapter 12, though the Wikipedia account of the Fast Fourier transform, http://en.wikipedia.org/wiki/Fast_Fourier_transform, is at least as helpful. I will be presenting the Cooley-Tukey version, so pay special attention to that.

<u>Monte Carlo Simulation</u> (as much of the following as time permits)

I Random number generation: An Oxymoron, but a Useful One
II Monte Carlo with Clever Tricks for Variance Reduction
  A) The efficient market hypothesis as a rationale for Monte Carlo
  B) Finding the area of a circle: a simple Monte Carlo calculation
    i) Statistical analysis
    ii) Random (or not) number generation
  C) Non-uniform random numbers
  D) Generating correlated random variables

E) Variance reduction techniques
   i) Importance sampling
   ii) Antithetic variance applied to Black Scholes European Call
   iii) Control variates and stratified sampling
F) Monte Carlo methods for American options

III Low discrepancy sequences
A) The most basic low discrepancy sequence is the Halton sequence
B) The more sophisticated Sobol' sequence seems to work better

Reading: *Numerical Recipes*, Chapter 7.0-7.3, 7.6, though *Probability, Random Variables, and Stochastic Processes* by Athanasios Papoulis (a Poly prof!) has a better explanation of how non-uniform random variables can be generated from uniform ones.

Linear and Spline Interpolation

I Why polynomial and linear interpolation don't cut it
II Splines
A) "Natural" splines
B) B-splines
III Limitations of splines
IV Two ways of improving on standard splines
A) Rational interpolation
B) Splines with tension
V "Least squares" a.k.a. multiple regression

Numerical Recipes Chapter 3 of *Numerical Recipes* has a discussion of splines, but I find both the Wikipedia article on splines, http://en.wikipedia.org/wiki/Spline_interpolation, and the Wolfram math world article on splines, http://mathworld.wolfram.com/CubicSpline.html easier to understand.

Optimization in one and several dimensions

I Why optimization important in finance
II Example: Max. likelihood estimation of GARCH(1,1) model
III Some unconstrained optimization problems and techniques
A) Markowitz optimization and the CAPM
B) "Lin min"
C) Nelder & Mead's Simplex method
D) Fletcher Powell
IV Constrained optimization
A) Linear programming
B) Constrained quadratic optimization and the Black-Litterman model
V Combinatorial optimization (*e.g.* The Traveling Salesman Problem; Markowitz optimization with constraints)
VI The Levenberg Marquant method

Reading: *Numerical Recipes*, Chapter 10, esp. 10.1 thru 10.5

Simulating Stochastic Differential Equations (SDE's) (time permitting)

I What SDE's actually are - stochastic calculus background
- A) Modes of stochastic convergence
- B) Ito's lemma
- C) The Ito integral as the l.i.m. of a stochastic sum

II Example SDE: The lognormal stock price process

III The Euler-Maruyama method

IV Convergence modes of method
- A) Strong convergence: convergence of "mean of error"
- B) Weak convergence: convergence of "error of mean"
- C) Long term stability

VI The Milstein method

Reading: Hingam's Introduction to Numerical Solution of SDE's,

Departmental/School-Wide Policies (Comments specific to the projects **in bold**, below)

Academic Misconduct

A. Introduction: The School of Engineering encourages academic excellence in an environment that promotes honesty, integrity, and fairness, and students at the School of Engineering are expected to exhibit those qualities in their academic work. It is through the process of submitting their own work and receiving honest feedback on that work that students may progress academically. Any act of academic dishonesty is seen as an attack upon the School and will not be tolerated. Furthermore, those who breach the School's rules on academic integrity will be sanctioned under this Policy. Students are responsible for familiarizing themselves with the School's Policy on Academic Misconduct.

B. Definition: Academic dishonesty may include misrepresentation, deception, dishonesty, or any act of falsification committed by a student to influence a grade or other academic evaluation. Academic dishonesty also includes intentionally damaging the academic work of others or assisting other students in acts of dishonesty. Common examples of academically dishonest behavior include, but are not limited to, the following:

    1. Cheating: intentionally using or attempting to use unauthorized notes, books, electronic media, or electronic communications in an exam; talking with fellow students or looking at another person's work during an exam; submitting work prepared in advance for an in-class examination; having someone take an exam for you or taking an exam for someone else; violating other rules governing the administration of examinations.
    2. Fabrication: including but not limited to, falsifying experimental data and/or citations.
    3. Plagiarism: Intentionally or knowingly representing the words or ideas of another as one's own in any academic exercise; failure to attribute direct quotations, paraphrases, or borrowed facts or information. **Submitting code that implements the same methodology as another student is not plagiarism; submitting the same code is plagiarism. It is surprisingly easy to tell the difference.**
    4. Unauthorized collaboration: working together on work that was meant to be done individually. **You are encouraged to discuss the project with others. However, since I need to assign grades individually, I hold each of you individually responsible for the quality of the project you submit. Submitting code "borrowed" from another student that is not fully understood therefore runs two risks: First, that I will punish you for plagiarism, and, second, that you will not detect problems in the code because you don't understand it!**
    5. Duplicating work: presenting for grading the same work for more than one project or in more than one class, unless express and prior permission have been received from the course instructor(s) or research adviser involved.
    6. Forgery: altering any academic document, including, but not limited to, academic records, admissions materials, or medical excuses.

Disability Disclosure Statement

Academic accommodations are available for students with disabilities. Please contact the **Moses Center for Students with Disabilities** (212-998-4980 or mosescsd@nyu.edu) for further information. Students who are requesting academic accommodations are advised to reach out to the Moses Center as early as possible in the semester for assistance.

Inclusion Statement

The NYU Tandon School values an inclusive and equitable environment for all our students. I hope to foster a sense of community in this class and consider it a place where individuals of all backgrounds, beliefs, ethnicities, national origins, gender identities, sexual orientations, religious and political affiliations, and abilities will be treated with respect. It is my intent that all students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed resource, strength and benefit. If this standard is not being upheld, please feel free to speak with me.

One of the ways that I try to maintain an equitable environment is by devising grading standards that are fair to all students. I therefore cannot arbitrarily raise a student's grade simply because failure to do so will "spoil their GPA" or cause them to lose a scholarship.