Polytechnic Tutoring Center Midterm Review – PH-UY 2033 Spring 2021 Solutions

1. Total energy of the system is given by

$$E_{\text{max}} = U_{\text{s,max}}(x) = \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}kA^2$$

And the spring potential energy is given by, as a function of displacement from the center x,

$$U_{\text{s}}(x) = \frac{1}{2}kx^2$$

Lastly, the displacement is calculated from the ratio

$$\frac{U_{\text{s}}(x)}{E_{\text{max}}} = \frac{kx^2}{kA^2} = \left(\frac{x}{0.02}\right)^2 = 0.18$$

$$x = 0.02\sqrt{0.18} \approx \pm 0.00849 \text{ m} = \pm 0.85 \text{ cm}$$

2. The linear mass density μ of the string can be computed as

$$\mu = \frac{m}{L} = \frac{0.0015}{0.65} \approx 0.0023kg/m$$

The relation between velocity of of the wave and tension is

$$v = \sqrt{\frac{T}{\mu}}$$

With wavelength of a fundamental frequency given by $\lambda = 2L$, we can set up the equation

$$f = \frac{v}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$330 = \frac{1}{2(0.65)}\sqrt{\frac{T}{0.0023}}$$

$$T = 424.71 \text{ N}$$

3. The relation between intensity can magnetic field can be defined by

$$I = u_{av}c = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\varepsilon_0 c^3 B_0^2$$

$$B_0 = \sqrt{\frac{2I}{\varepsilon_0 c^3}} = \sqrt{\frac{2(2.0 \times 10^7)}{8.85 \times 10^{-12}(2.99 \times 10^8)^3}} = 4.1 \times 10^{-4}$$

4. The frequency that the prey will hear is

$$f_1 = f_0\left(\frac{v_{snd}}{v_{snd} \mp v_{src}}\right) = 100\left(\frac{343}{333}\right) = 103\text{kHz}$$

And the wave bounced back to the bat is given by

$$f_2 = f_1\left(\frac{v_{snd} \pm v_{obs}}{v_{snd}}\right) = 103\left(\frac{353}{343}\right) = 106\text{kHz}$$

5. Recall that upon passing through the first polarizer, light will be polarized and the intensity will be halved.

$$I_1 = \frac{1}{2} I_0$$

After that, for each polarizer added, the intensity will be multiplied by $cos^2(20°)$

$$\begin{aligned}
I_n &= \frac{1}{2}(\cos^2(20°))^{n-1} I_0 \\
0.1 I_0 &\geq \frac{1}{2}(\cos^2(20°))^{n-1} I_0 \\
0.2 &\geq (\cos^2(20°))^{n-1} \\
\frac{\ln 0.2}{\ln(\cos^2(20°))} &\leq n - 1 \\
n &\geq \frac{\ln 0.2}{\ln(\cos^2(20°))} + 1 = 14
\end{aligned}$$

6. The period of an mass-spring harmonic oscillator is given by

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{20}{810}} = 0.99 \text{ s}$$

The process of compressing the string at $x = 0$ and bounces back is in fact experiencing half of period of a harmonic oscillation.

$$t = \frac{1}{2}T = 0.49 \text{ s}$$

7. By parallel angles, we first see that the angle from which the fish is viewing (measured from vertical line) is the same angle as the angle of incidence at surface of water. And we are interested in the critical angle which is essentially

$$\begin{aligned}
n_{\text{water}} \sin(\theta_c) &= n_{\text{air}} \sin(90°) \\
1.33 \sin(\theta_c) &= 1 \\
\theta_c &= \sin^{-1}\left(\frac{1}{1.33}\right) \approx 48.8°
\end{aligned}$$

8. From sin function's dependency on kx, we see that the velocity of the wave is travelling to the x direction. Thus the electric field will also be a wave travelling to the x direction but perpendicular to B. Thus it has to be

$$E_y = E_0 \sin(kx - \omega t)$$

9. Recall that dB is defined as

$$\beta = 10 \log\left(\frac{I}{I_0}\right) = 75 \text{ dB}$$

If we substitute $I \to 4I$

$$\beta = 10 \log\left(\frac{4I}{I_0}\right) = 10\left[\log\left(\frac{I}{I_0}\right) + \log(4)\right] = 75 + 6.02 = 81.0 \text{ dB}$$

10.

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\begin{aligned}
\vec{S} &= \frac{1}{\mu_0}\vec{E} \times \vec{B} \\
&= \frac{1}{\mu_0}\det\left(\begin{array}{ccc} \hat{i} & \hat{j} & \hat{z} \\ \hline 0 & E & 0 \\ 0 & 0 & B \end{array}\right) \\
&= \frac{E_0 B_0}{\mu}\sin^2(kx + \omega t)\hat{i}
\end{aligned}
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