Polytechnic Tutoring Center Midterm Review - PH-UY 2023 Spring 2021 Solutions

1. Given the mass of water and salt water is

$$5 \text{ L water} \times \frac{1 \text{ kg water}}{1 \text{ L water}} = 5 \text{ kg water}$$
$$5 \text{ L salt w.} \times \frac{1 \text{ L water}}{1 \text{ L salt w.}} \times \frac{1 \text{ kg water}}{1 \text{ L water}} \times \frac{1.035 \text{ kg salt water}}{1 \text{ kg water}} = 5.175 \text{ kg water}$$

The gauge pressure is the calculated as

$$P_{\text{gauge}} = \rho g h = \left(\frac{\sum m}{V}\right) g \left(\frac{V}{A}\right) = \frac{g \sum m}{A} = \frac{F_g}{A} = \frac{10(10.175)}{0.02} \approx 5078 \text{ Pa}$$

Equals 4985 if g is taken to be 9.8.

2. The additional pressure exerted by the water is

$$\Delta P = \rho g h = \frac{F_g}{A}$$
$$\Delta F = A_1 \Delta P = \frac{A_1}{A_0} \rho V g = 21.6 \text{ N}$$

3.

$$P_{\text{gauge}} = \rho g h = 13600 \frac{kg}{m^3} \times 9.8 \frac{m}{s^2} \times 0.31 m = 41316.8 \text{ Pa}$$
$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}} = 41316.8 + 101325 \approx 143 \text{ kPa}$$

4. The electric field at the origin cause the by the $+15n$ is

$$E_{+15n} = \frac{kq}{|\vec{r}|^2}\hat{r} = \frac{kq\vec{r}}{|\vec{r}|^3} = k\left(\frac{(15n)[(0\hat{i} - 0\hat{j}) - (1.5i + 0\hat{j})]}{1.5^3}\right) = -\frac{10nk}{1.5}\hat{i}$$

The electric field at the origin cause the by the $-20n$ is

$$E_{-20n} = \frac{kq}{|\vec{r}|^2}\hat{r} = \frac{kq\vec{r}}{|\vec{r}|^3} = k\left(\frac{(-20n)[(0\hat{i} - 0\hat{j}) - (0i - 2.0\hat{j})]}{2.0^3}\right) = -\frac{10nk}{2.0}\hat{j}$$

The magnitude of the added electric field is therefore

$$|E_{15n} + E_{-20n}| = 10nk\sqrt{\frac{1}{1.5^2} + \frac{1}{2,0^2}} = 74.1 N/C$$

5. The electric field near an uniform charged plane is given by

$$E = \frac{\sigma}{2\epsilon_0}$$

Using Newton's second law

$$a = \frac{F_{\text{net}}}{m_p} = \frac{Eq}{m_p} = \frac{\sigma q_e}{2\epsilon_0 m_p}$$

Since the electric field and electric force is constant, the acceleration is constant, which implies we can use the kinetic equations.

$$
\begin{aligned}
v^2 &= v_0^2 + 2ad \\
0 &= v_0^2 - 2ad \\
d &= v_0^2 \left(\frac{\epsilon_0 m_p}{\sigma q_e} \right) \approx 1.55 \text{ mm}
\end{aligned}
$$

6. Letting R_1 denote the outer radius and R_0 denote the inner radius. Assuming uniform charge density for the nonconducting shell, we have charge density given by

$$
\rho = \frac{Q_{\text{total}}}{V_{\text{total}}} = \frac{Q_{\text{total}}}{\frac{4}{3}\pi(R_1^3 - R_0^3)}
$$

Creating an Gaussian sphere of $r = 8$. Using Gauss's law, we have

$$
\begin{aligned}
\oint E \cdot \hat{n} dA &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
E \cdot A &= \frac{\rho V}{\epsilon_0} = \frac{\rho \frac{4}{3}\pi(r^3 - R_0^3)}{\epsilon_0} \\
E(4\pi r^2) &= \frac{Q_{\text{total}}}{\frac{4}{3}\pi(R_1^3 - R_0^3)} \frac{\frac{4}{3}\pi(r^3 - R_0^3)}{\epsilon_0} \\
E &= \frac{(r^3 - R_0^3)}{r^2(R_1^3 - R_0^3)} \frac{Q_{\text{total}}}{4\pi\epsilon_0} = \frac{(r^3 - R_0^3)}{r^2(R_1^3 - R_0^3)} Q_{\text{total}} k \\
&= 3106 \text{ kN/C}
\end{aligned}
$$

7. The electric field cause by the first wire at x is given by

$$
E_1 = \frac{2k\lambda_1}{x}
$$

The electric field cause by the second wire at x is given by

$$
E_2 = \frac{2k\lambda_2}{x - 9}
$$

Letting the sum equals to 0, we have

$$
\begin{aligned}
E_1 + E_2 &= 0 \\
\frac{2k\lambda_1}{x} &= \frac{2k\lambda_2}{x - 9} \\
\lambda_2 x &= \lambda_1 x - 9\lambda_1 \\
x &= \frac{9\lambda_1}{\lambda_1 - \lambda_2} \\
x &= 36 \text{ cm}
\end{aligned}
$$

8. Letting the fixed charge to be Q. The electrical potential energy at the beginning is given by

$$U_e = \frac{kQq}{r}$$

By conservation of energy, the maximum speed is reached when all potential energy is converted into kinetic energy. Then a work is down onto the charge by the electric field. We can then skip the kinetic energy step.

$$\begin{aligned} U_e &= W = Fd \\ \frac{kQq}{r} &= Eqd \\ \frac{kQ}{r} &= Ed \\ E &= \frac{kQ}{rd} = 4495 \text{ N/C} \end{aligned}$$

9. Recall that voltage is the distance integral of electric field. Since E only have x component in this case, we have

$$V(x) = -\int E\,dx = -12x$$

plug in

$$V_A - V_B = V(3) - V(6) = -36 + 72 = 36 \text{ V}$$

10. Integrating the path over electric field, with $x_1 > x_0$

$$\begin{aligned} \Delta V &= -\int_{x_0}^{x_1} E\,dx \\ &= -\int_{x_0}^{x_1} \frac{\sigma}{2\epsilon_0}\,dx \\ &= -\frac{\sigma}{2\epsilon_0}(x_1 - x_0) \\ x_1 - x_0 &= -\frac{2\epsilon_0 \Delta V}{\sigma} = -\frac{2(8.85 \times 10^{-12})(-98)}{80 \times 10^{-9}} = 2.16 \text{ cm (if } x_1 > x_0) \end{aligned}$$

For the value of travelling toward the plane

$$x_0 - x_1 = -2.16 \text{ cm}$$