Polytechnic Tutoring Center Midterm
Review – PH-UY 1213 Spring 2021 Solutions

1. First, convert 45 mph into m/s
$$\frac{45 \text{ miles}}{1 \text{ h}} \times \frac{1.61 \text{ km}}{1 \text{ mile}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 20.125 \text{ m/s}$$
The final velocity can be calculated using
$$v = v_0 + at = 20.125 + 3 \cdot 2 = 26.126 \text{ m/s} = 58.4 \text{ mph}$$

2. We denote north as the $\hat{\mathbf{j}}$ unit vector and east as $\hat{\mathbf{i}}$ unit vector. Then the velocity of the boat, separated in to components are
$$\vec{v}_b = 50/\sqrt{2}\,\hat{\mathbf{i}} - 50/\sqrt{2}\,\hat{\mathbf{j}},$$
and the velocity of the water is given by
$$\vec{v}_w = 10\,\hat{\mathbf{j}}.$$
Thus, the total velocity, in vector, is given by
$$\vec{v} = \vec{v}_w + \vec{v}_b = 50/\sqrt{2}\,\hat{\mathbf{i}} + (10 - 50/\sqrt{2})\,\hat{\mathbf{j}}.$$
The magnitude, i.e. the speed, of this velocity vector is computed as
$$||\vec{v}|| = \sqrt{\left(\frac{50}{\sqrt{2}}\right)^2 + \left(10 - \frac{50}{\sqrt{2}}\right)^2} = 43.5 \text{ m/s}$$

3. Since the ball is only subjected to uniform acceleration motion during vertical free fall, we can use the the 1D motion formula as follows
$$\begin{aligned} y &= y_0 + v_0 t + \frac{1}{2}at^2 \\ y - y_0 &= v_0 t - \frac{1}{2}gt^2 \\ 0 &= v_0 t - \frac{1}{2}gt^2 = v_0 - \frac{1}{2}gt \\ v_0 &= \frac{1}{2}gt = \frac{1}{2} \cdot 9.8 \cdot 10 = 49 \text{ m/s} \end{aligned}$$

4. Since the ice cube does not have any vertical velocity when it leaves the table, the time it takes for it to reach the ground can be computed using uniform acceleration formula as follows
$$\begin{aligned} y &= y_0 + v_0 t - \frac{1}{2}gt^2 \\ 0 &= 1 + 0 - \frac{1}{2}(9.8)t^2 \\ t &= \frac{1}{\sqrt{4.9}} = 0.45 \text{ s} \end{aligned}$$
During the 0.45 s of free fall, the ice cube would travel horizontally at a speed of 2 m/s. The total horizontal displace is therefore
$$x = v_x t = 2 * 0.45 = 0.90 \text{ m}.$$

5. The time it takes for the block to go from left to right is half of oscillation's period. Thus the full period is $T = 2 \cdot (1.2) = 2.4$ s. Recall that the frequency is given by, in terms of period,

$$f = \frac{1}{T} = \frac{1}{2.4} = 0.42 \text{ Hz}.$$

6. The tension in the rope is acting as a centripetal force. We can computed using the centripetal acceleration

$$T = F_c = a_c m = \frac{v^2}{r} m = \frac{10^2}{1}(1.5) = 150 \text{ N}$$

7. We first need to write the location of the three vertices in vector notation in order to add them. We set the center of the triangle as the origin. The vertices are then written as

$$\begin{aligned} \vec{r}_1 &= \frac{1}{\sqrt{3}}\,\hat{\mathbf{i}} \\ \vec{r}_2 &= -\frac{1}{2\sqrt{3}}\,\hat{\mathbf{i}} + \frac{1}{2}\,\hat{\mathbf{j}} \\ \vec{r}_3 &= -\frac{1}{2\sqrt{3}}\,\hat{\mathbf{i}} - \frac{1}{2}\,\hat{\mathbf{j}} \end{aligned}$$

Recall that the center of mass vector is the weighted averaged of the vectors. It is thus

$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{\left(\frac{2}{\sqrt{3}} - \frac{5}{2\sqrt{3}} - \frac{7}{2\sqrt{3}}\right)\,\hat{\mathbf{i}} + \left(\frac{5}{2} - \frac{7}{2}\right)\,\hat{\mathbf{j}}}{2+5+7} = -\frac{2}{7\sqrt{3}}\,\hat{\mathbf{i}} - \frac{1}{14}\,\hat{\mathbf{j}}$$

The magnitude of the center of mass vector, which is the offset of center of mass from the center of the triangle can be computed as

$$||\vec{r}_{cm}|| = \sqrt{\left(\frac{2}{7\sqrt{3}}\right)^2 + \left(\frac{1}{14}\right)^2} = 0.18 \text{ m}$$

8. The block will move horizontally and its acceleration is due to the net force. We need to figure out the normal force in order to compute the force due to friction. We can compute it using the balance of vertical forces.

$$\begin{aligned} F_{net,y} &= 0 = -F_g + F_N + F_{app,y} \\ F_N &= F_g - F_{app,y} = mg - F_{app}\sin(40) = 7(9.8) - 65\sin(40) = 26.82 \text{ N} \end{aligned}$$

The horizontal component of the net force is then

$$F_{net,x} = F_{app,x} - F_f = F_{app}\cos(40) - \mu F_N = 65\cos(40) - 0.25(26.82) = 43.09 \text{ N}.$$

We can find the acceleration of the block using Newton's Law

$$\begin{aligned} F &= ma \\ a &= \frac{F_{net,x}}{m} = \frac{43.09}{7} = 6.16 \text{m/s}^2 \end{aligned}$$

9. There are gravitational kinetic energy and elastic potential energy in this system. They are

$$U_g = mgh = 2 \cdot 20 \cdot 9.8 = 392 \text{ J} U_e \qquad = \frac{1}{2}kx^2 = \frac{1}{2}(1200)(0.5)^2 = 150 \text{ J}$$

The total potential energy is

$$U = U_g + U_e = 392 + 150 = 542 \text{ J}.$$

10. The 5 kg mass, $m_A$, will fall and 2 kg mass, $m_B$, will be lifted with the same magnitude in acceleration. We can model the net forces of the two blocks using tension of the string $T$:

$$\begin{aligned} F_{net,A} &= m_A a = m_A g - T \\ F_{net,B} &= m_B a = T - m_B g \end{aligned}$$

Equating the tension, we have

$$\begin{aligned} m_A g - m_A a &= m_B g + m_B a \\ a(m_A + m_B) &= (m_A - m_B)g \\ a &= \frac{m_A - m_B}{m_A + m_B} g = \frac{3}{7}(9.8) = 4.2 \text{ m/s}^2 \end{aligned}$$