New York University Tandon School of Engineering
Department of Finance and Risk Engineering
Course Outline FRE6233 Option Pricing and Stochastic Calculus
Flipped teaching version
Spring 2021
Professor Agnes Tourin
Section A: Monday, 2pm-4:30pm, RH 214
Section I: Thursday: 5:30pm-8pm, online

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Office hours: Mondays, 8am-10am (online), or by appointment (online or in person)

Course Pre-requisite: FRE 6083

Course Description: This course provides the mathematical foundations of Option Pricing models. The techniques covered include arithmetic and geometric Brownian motion, first passage time, the reflection principle, the stochastic Ito integral, Ito differential Calculus, change of probability measure, martingales, Stochastic Differential Equations and Partial Differential Equations. Some of the pricing models considered are the European, Barrier, Asian and American options. These problems are either solved analytically by the martingale or Partial Differential Equations approach, or numerically, by applying approximation and simulation methods. Since the same techniques allow the treatment of more complex financial products, several advanced examples will be also presented.

Course Objectives:

- After taking this course, the students will be able to price any derivative security.
- This is a first course in Mathematical Finance and is a building block for more advanced courses.
- The material taught is part of the common knowledge shared by quants in the Financial industry and taking this course will prepare the students to apply for quantitative positions.
- This course also prepares the students who wish to apply to a Ph.D. program in Financial Engineering, Mathematical Finance, Operations Research, or Finance.

Course Structure:
This course will be delivered in a flipped classroom format. The lessons, composed of pre-recorded videos will be posted online, as well as weekly assignments. The instructor and students will then meet at the designated class times on Albert (in the classroom or online depending on the section). The instructor will answer students questions about the

material, provide help with and feedback about the weekly assignments, and will also hold active learning activities such as problem solving sessions.

Readings:
The required texts for the course are:

- Tomas Bjork*, Arbitrage Theory in continuous time*, Oxford University press, Reprint, ISBN-0191525103, 9780191525100.

- Shreve, *Stochastic Calculus for Finance II: continuous-time models,* 2nd edition, 2004, Springer.

Location of books: NYU bookstore. Slides based on the textbooks and other additional sources will also be posted on NYU classes.

Additional Resources (optional):

1. A Primer for the Mathematics of Financial Engineering, Dan Stefanica, Second Edition, 2011, FE Press New York.
2. Monte Carlo Methods in Financial Engineering, Paul Glasserman, 2004, Springer.
3. Financial Modelling With Jump Processes, Rama Cont and Peter Tankov, 2004, Chapman & Hall/CRC.
4. Computational Methods in Finance, Ali Hirsa, 2013, Chapman & Hall/CRC.
5. Introduction to Mathematical Finance, Discrete Time Models, Stanley R. Pliska, 1997, Blackwell Publishing.
6. Stochastic Differential Equations: An introduction with Applications, Bernt Oksendal, Universitext, Third Printing, Sixth Edition, 2009, Springer.
7. Probability Essentials, Jean Jacod and Philip Protter, Universitext, Second Printing, Second Edition, 2004, Springer.
8. Numerical Partial Differential Equations: Finite Difference Methods, J.W. Thomas, Texts in Applied Mathematics, 22, 1995, Springer.

Recommended software for the homework:

Students will be required to use a programming language for prototyping, such as Python, Matlab, or R (http://www.r-project.org).

Course requirements:
- Students are expected to attend classes and participate actively. They should view the videos and read the textbooks ahead of time and come prepared to ask questions and discuss the weekly assignment.
- There will be a take home midterm examination, a take home final examination and weekly homework assignments.

The Midterm examination will be due on week 7: students will be required to solve four or five problems by using the computational techniques taught during the first 6 weeks.

- The final examination will be due on week 15.
- There will be two types of homework assignments. The first type will consist of practice exercises designed to help the students assimilate the techniques taught in class and prepare them for the examinations. The second type will consist of implementing numerical or simulation techniques, to compute option prices that cannot be computed analytically.

Grading of Assignments:

The grade for this course will be determined according to the following formula:

Assignments/Activities	% of Final Grade
Participation (as recorded in NYU classes under *statistics*)[1]	10%
Average weekly homework assignment score (the worst grade will be dropped)	30%
Take home midterm exam	30%
Take home final exam	30%

Letter Grades

Letter grades for the entire course will be assigned as follows:

Letter Grade	Points	Percent
A	4.00	95% and higher

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[1] The participation grade is the average of all the weekly participation grades. The weekly participation grade is computed as follows: you will get
-10% if you were active that week in *lessons* and *resources* on NYU classes
-0% if you weren't

A-	3.67	90.0 – 94.99%
B+	3.33	87% - 89.99%
B	3.00	83% - 86.99%
B-	2.67	80% - 82.49%
C+	2.33	77% - 79.99%
C	2.00	70.0% - 76.99%
F	.00	69.99% and lower

Lessons

Part I: Ito Stochastic Calculus

Preparation: Textbook by Shreve, chapter 1.

Module 1 (weeks 1-2): Information and the Brownian motion

- Textbook by Shreve, chapters 2-3
- Section 1
  - Information and conditioning: videos 1.1, 1.2
  - Textbook by Shreve, Chapter 2.
- Section 2
  - The Brownian motion and filtrations, Chapter 3,
  - videos 1.3-1.7
- Week 1: videos 1.1-1.3
- Week 2: videos 1.4-1.7
- First assignment is due on week 2 (problem set)

Module 2 (weeks 2-3): Ito Calculus

- week 2**:** videos 2.1-2.3
- week 3: videos 2.4-2.5
- Textbook by Shreve, chapter 4
- Complements: Textbook by Bjork, chapter 4.
- Second assignment is due (problem set)

Module 3 (week 4): Application of stochastic calculus to the Black-Scholes model

- videos 3.1-3.5
- Textbook by Shreve, chapter 4.
- Complements: Textbook by Bjork, chapters 5,6,7,9.
- Third assignment is due (problem set)

Part II: The no arbitrage theory in continuous-time

Module 4 (week 5) : The martingale approach

- videos 4.1-4.8
- Textbook by Shreve, chapter 5.
- Complements: Textbook by Bjork, chapters 10, 11,12.
- Assignment #4 is due (problem set)

Module 5 (week 6): The Partial Differential Equations approach

- videos 5.1-5-7
- Textbook by Shreve, chapter 6.
- Textbook by Bjork, chapter 18.
- Assignment #5 is due (problem set).

Week 7 The take home midterm examination is due 48 hours after the class meeting time

- During the class meeting times, the instructor will be available to answer questions about the midterm examination

Module 6 (week 8): The Asian option

- videos 6.1-6.6
- Textbook by Shreve, chapter 7.
- assignment #6 is due (problem set).

Module 7 (weeks 9-10): The American option

Week 9: The martingale and PDE approaches

- videos 7.1-7.7
- Textbook by Bjork, chapter 21.
- Assignment #7 is due (implementation of Monte Carlo simulations for the As
- ian option)

Week 10: Finite Difference scheme for the American option

- Videos 7.8-7.13
- Lecture notes
- Assignment #8 is due (problem set).

Module 8 (weeks 11-12) Multidimensional market models: stochastic calculus in several dimensions, multi-dimensional asset pricing models , introduction to the change of Numeraire, applications to FX derivatives.

- Videos 8.1-8.7
- Textbook by Shreve, Chapters 5,9.
- Complements: Textbook by Bjork, chapter 14,17, 26.
- Assignment #9 is due on week 11 and assignment #10 is due on week 12.

Part III: Jump-diffusion processes and incomplete market models

Module 9 (weeks 13-14): Stochastic Calculus for jump-diffusion processes

- **Section 1 (week 13): Stochastic Calculus with jumps**
    - videos 9.1-9.6
    - Textbook by Shreve, chapter 11
- **Section 2 (week 14): Option pricing in a jump-diffusion model**
    - Videos 9.7-9.15
    - Textbook by Shreve, Chapter 11
- Assignment #11 is due on week 12 and assignment #12 is due on week 14

Week 15
- **Final examination is due at the end of the class!**
- **The instructor will be available to answer questions about the exam.**

The NYU Tandon School values an inclusive and equitable environment for all our students. I hope to foster a sense of community in this class and consider it a place where individuals of all backgrounds, beliefs, ethnicities, national origins, gender identities, sexual orientations, religious and political affiliations, and abilities will be *treated with respect. It is my intent that all students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength and benefit. If this standard is not being upheld, please feel free to speak with me.*

NYU School of Engineering Policies and Procedures on Academic Misconduct *(from the School of Engineering Student Code of Conduct)*

A. Introduction: The School of Engineering encourages academic excellence in an environment that promotes honesty, integrity, and fairness, and students at the School of Engineering are expected to exhibit those qualities in their academic work. It is through the process of submitting their own work and receiving honest feedback on that work that students may progress academically. Any act of academic dishonesty is seen as an attack upon the School and will not be tolerated. Furthermore, those who breach the School's rules on academic integrity will be sanctioned under this Policy. Students are responsible for familiarizing themselves with the School's Policy on Academic Misconduct.

B. Definition: Academic dishonesty may include misrepresentation, deception, dishonesty, or any act of falsification committed by a student to influence a grade or other academic evaluation. Academic dishonesty also includes intentionally damaging the academic work of others or assisting other students in acts of dishonesty. Common examples of academically dishonest behavior include, but are not limited to, the following:

    1. Cheating: intentionally using or attempting to use unauthorized notes, books, electronic media, or electronic communications in an exam; talking with fellow students or looking at another person's work during an exam; submitting work prepared in advance for an in-class examination; having someone take an exam for you or taking an exam for someone else; violating other rules governing the administration of examinations.
    2. Fabrication: including but not limited to, falsifying experimental data and/or citations.
    3. Plagiarism: intentionally or knowingly representing the words or ideas of another as one's own in any academic exercise; failure to attribute direct quotations, paraphrases, or borrowed facts or information.
    4. Unauthorized collaboration: working together on work that was meant to be done individually.
    5. Duplicating work: presenting for grading the same work for more than one project or in more than one class, unless express and prior permission has been received from the course instructor(s) or research adviser involved.
    6. Forgery: altering any academic document, including, but not limited to, academic records, admissions materials, or medical excuses.

Accomodations

If you are a student with a disability who is requesting accommodations, please contact New York University's Moses Center for Students with Disabilities (CSD) at 212-998-4980 or mosescsd@nyu.edu. You must be registered with CSD to receive accommodations. Information about the Moses Center can be found at www.nyu.edu/csd. The Moses Center is located at 726 Broadway on the 2nd floor.