# 1 Synopsis

In this course, we investigate the mathematically known strengths and limiations of machine learning. To motivate the discussion, we focus on the problem of portfolio selection. We use as a jumping off point, Chapters 1, 2 and 5 of [LGW12] supplemented with online readings. We will focus on the following questions:

1. How can we choose a machine learning approach which will perform well for portfolio selection?
2. What are the performance limitations that we can demonstrate of machine learning?
3. Which techniques can be shown to perform optimally?

The answers to these questions sometimes require results from mathematics that many students will not have had exposure to. We will teach topics from measure-theoretic probability theory, the theory of stochastic processes (including ergodic stochastic processes), metric spaces and other topics as needed.

# 2 Prerequisite

The prerequisite for this course is an introductory course on machine learning.

# 3 Grading

1. 40% based on homework assignments
2. 30% based on a midterm exam
3. 30% based on a final exam

# 4 Preliminary Syllabus

The following is preliminary syllabus and is likely to be subject to many changes.

1. Introduction (Weeks 1 and 2):
   (a) What is **machine learning**?
   (b) **The St. Petersburg paradox**
   (c) **Elementary probability theory:**
      i. The axioms of elementary probability
      ii. Independence
      iii. Expectation

   (d) **Utility theory**
      i. Bernoulli's solution of the St. Petersburg paradox
      ii. Modern utility theory

2. Discrete-time Growth Optimal Portfolios (Weeks 2, 3 and 4).
   (a) **Laws of large numbers**
      i. IID returns and the importance of sums of random variables
      ii. Limitations of finite additivity
      iii. Cantor diagonalization
      iv. The axioms of modern probability
      v. The strong law of large numbers
      vi. Lebesgue integration
      vii. Ergodic stochastic processes
   (b) **Finer analysis**
      i. Rates of convergence and big O notation
      ii. The central limit theorem
      iii. The law of the iterated logarithm
   (c) **Comparison with utility theory**

3. Nearest-neighbor learning (Weeks 5-6)
   (a) Universality
   (b) Metric spaces
   (c) Nearest neighbor in portfolio selection

4. Midtern (Week 7)

5. The limitations of machine learning (Weeks 8-10)
   (a) The nonexistence of a rate of convergence in one dimension
   (b) The curse of dimensionality
   (c) Stone's results

6. Uniform laws of large numbers (Weeks 11-12)
   (a) The Glivenko-Cantelli theorem
   (b) Packing and covering numbers
   (c) The VC dimension

7. Additional techniques (Week 13)

8. Dependent observations (Week 14)

# 5 Course Materials

## References


[LGW12] György Ottucsák Lázló Györfi and Harro Walk. *Machine Learning for Financial Engineering*. Imperial College Press, 2012.