# MELANCOLIA AND MAGIC SQUARES

David and Gregory Chudnovsky Discuss Math, Art, and More

WHEN someone is particularly passionate about a given topic, it's not uncommon to hear them being described as "eating, breathing, and sleeping" it. In the case of David and Gregory Chudnovsky, it would be fitting to say that they, quite literally, stand on it. The brothers, mathematicians at the New York University Polytechnic School of Engineering, share an office whose floor features a large reproduction of Albrecht Dürer's *Melencolia I* (a 1514 engraving that depicts a winged being holding a caliper and surrounded by various other tools associated with geometry). The figure's dejected mien, Gregory Chudnovsky quips, is one often seen on mathematicians trying to solve seemingly unsolvable problems.

Despite the joke, the brothers, who are interested in the branch of pure mathematics known as number theory, are generally far from dejected. "The harder the problem, the more fun," they assert. "In mathematics, no one wants to focus on the low-hanging fruit. They want to work on the problems that have stumped others for ages." The two are perhaps best known to the general public for their work on one such problem: calculating the mathematical constant known as pi, the ratio of a circle's circumference to its diameter. By the early 1990s, they had computed pi, referred to by most laypeople and non-mathematicians using only two decimal places (3.14), to more than four billion places—at the time an astonishing, record-setting number. They did so with the help of Mo, a supercomputer they designed

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and built in Gregory Chudnovsky's apartment, using mail-order parts and PVC piping from Home Depot.

The Chudnovsky brothers are still involved in supercomputing and their smallest computers are at their Institute for Mathematics and Advanced Supercomputing (IMAS), which was endowed for them at the NYU School of Engineering in 1998. (A portion of the floor displays, in addition to the Dürer reproduction, a large digital image showing more than 100 different identities, many discovered by the Chudnovskys themselves.) They explain that because of Moore's Law—the prediction that the number of transistors incorporated in a chip will increase exponentially at a steady rate—chip design keeps them exceptionally busy. (The two work on a process called very-large-scale integration or VLSI design, creating integrated circuits by combining many billions of transistors into a single chip.) Each of their projects takes some two-and-a-half years, from conception to manufacturing, and each chip goes through several iterations and a grueling process of formal verification. Because of the intricacies of the development and manufacturing process, many things can go wrong; even a speck of dust can stymie the entire process. Pointing out that visitors to IMAS have to don only surgical booties over their shoes in order to protect the artwork on the floor, they say, "If you were to go near one of the cleanrooms used in the semiconductor industry, you would have to wear a full hazmat suit."

They see the chips they are designing as part of an important continuum. "Think about life now and compare it to life 50 years ago," they say. "Almost any major difference you can think of is due to semiconductors and computer chips." Designing such complex and powerful chips is akin to planning a city—in miniature—they explain: "The smaller the features, the more complicated the mathematics gets. You must optimize the wiring, where components are placed, and the whole thing." A computer, they remind listeners, is simply a hardwired set of mathematical rules.

Mathematics, the Chudnovskys enthusiastically declare, is a universal language required not just for computing but almost everything. "If you know math, you can do absolutely anything," they both assert. The brothers are fond of the tale of the great 19th-century mathematician Josiah Willard Gibbs, who famously objected when administrators at Yale decided to stop requiring the study of mathematics in favor of Latin. "Mathematics is the language," Gibbs had protested, and the brothers concur. "You use number theory every day, even if you don't realize it," David Chudnovsky says. "A cell phone requires number theory and algorithms. So does the television. It's all mathematics in the form of digital signal processing." The pair are quick, however, to praise the work of engineers, and they single out the NYU School of Engineering's Department of Electrical Engineering, which they call "the best in the entire region."

In addition to their admiration for engineers, the brothers hold artists in exceptionally high esteem, as evidenced by their love of Dürer, who was also, not at all incidentally, a mathematician. In honor of the 500th anniversary of *Melencolia I*, the Chudnovskys organized a conference, "500 Years of Melancholia in Mathematics," which took place on May 17 at the school. The event—which was made possible by a generous grant from the Alfred P. Sloan Foundation, as the brothers very gratefully acknowledge—featured the world's leading experts, including John Conway and Sergiu Klainerman (Princeton University), Jeffrey Lagarias (University of Michigan), John W. Morgan (Stony Brook University), Richard Schroeppel (Sandia National Lab), Richard Stanley (MIT), and Günter Ziegler (Free University of Berlin).



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Many of their talks included discussions of magic squares and polyhedra, items featured prominently within Dürer's iconic etching. (The engraving itself is displayed at New York City's Metropolitan Museum of Art, and the day after the conference, the museum hosted a Sunday at the Met program, "Spotlight on a Masterpiece: Albrecht Dürer's *Melencolia I*," which featured a series of lectures by curators and historians.) *Melencolia I* depicts what is thought to be the first example of a 4x4 magic square ever published in Europe. Magic Squares consist of a series of numbers arranged in a square in such a way that the sum of each row, each column and both the corner diagonals adds up to the same sum, which is called the magic constant—in this case, 34—Dürer's age when he did the engraving. (The two numbers in the center of Dürer's bottom row, 15 and 14, represent the date of the engraving.) The engraving's eight-sided polyhedron (consisting of two equilateral triangles and six irregular pentagons) bears the faint image of a human skull and has spawned dozens of scholarly treatises examining not only its geometrical significance but its possible artistic symbolism.

Dürer's dual interest in mathematics and art led to his 1525 publication of the seminal book *Underweysung der Messung* ("Painter's Manual"), one of the first ever to teach the methods of perspective, which he had learned in Italy. (The book featured the earliest known examples of polyhedral nets—polyhedra unfolded flat for printing.) David and Gregory Chudnovsky believe that more students, even those who don't consider themselves "good at math," should try to incorporate mathematics into their studies in a similar manner. "It's schizophrenic," Gregory Chudnovsky says, "this separation between the humanities and mathematics and the sciences. Young people should try to emulate Dürer." ■

## Art and Math Meld Yet Again

In 1998 the seven intricate tapestries that make up the famed "Hunt of the Unicorn" series were moved from their longtime home at the Cloisters, New York City's museum of medieval art, which was being renovated. Sent to the Metropolitan Museum of Art, which runs the Cloisters, they were cleaned and then painstakingly photographed, section by section, for archival preservation. Curators later tried to assemble those digital files, which filled more than 200 CDs, into a complete image, but were unable to match up the thousands upon thousands of fibers within the sections. Frustrated, they declared the problem unsolvable and shelved the project.

In 2003, however, a Met curator happened to meet David Chudnovsky at a social gathering. David and brother Gregory subsequently agreed to take on the challenge. They discovered that there had been movement in the 500-year-old fibers of the tapestry they were examining—which had apparently begun to relax and shift during and after the cleaning—and created a vector displacement map (with some 15,000 arrows) to track the motion. They then devised a set of equations to optimize the position of all 240 million pixels in the images. As portions of the tapestry were entered into the supercomputer, it performed some 300 million operations per pixel to digitally "weave" the fibers back together. Finally, after four months of work, the Chudnovskys entered the final data into the system, and after 30 hours of continuous running, their supercomputer had successfully performed the 7.7 *quadrillion* calculations needed to put together a flawless multi-gigabyte image of "Unicorn in Captivity," the tapestry that depicts the mythical creature loosely tethered to a tree behind a low circle of fencing.

A copy of the artwork currently hangs in the Bern Dibner Library at MetroTech Center, reinforcing the connection between the arts and engineering.