NYU POLYTECHNIC SCHOOL OF ENGINEERING

What explains the Spin-Statistics Connection?

Faculty: **Jonathan Bain, Dept. of Technology, Culture and Society (NYU-Poly).**

Research assistants: **Hind Al-Tantawi (NYUAD), Shearyar Khan (NYU-Poly).**

Abstract

In quantum mechanics, particles are divided into two categories: fermions and bosons. Fermions obey Fermi-Dirac statistics and bosons obey Bose-Einstein statistics. This entails that an exchange of two fermions in a group of fermions results in a change of the many-particle state, whereas an exchange of two bosons does not affect the many-particle state of a group of bosons (this implies that while bosons can be in the same single-particle state, fermions cannot). Quantum mechanical particles can also possess either integer spin or half-integer spin. In relativistic quantum field theory, there is a fundamental connection between spin and statistics that is derived from the theory's axioms: fermions must possess half-integer spin and bosons must possess integer spin. However, there is no similar explanation in non-relativistic quantum mechanics. In simpler terms there is no simple explanation for why two half-integer spin particles (such as electrons) cannot exist in the same state, whereas integer spin particles can. This raises the question: What explains the spin-statistics connection in nonrelativistic quantum mechanics?

In this project, we conduct a literature review of attempts to answer this question. We sort each attempt in terms of the type of philosophical explanation it provides: causal-mechanical, structural, or unifying. We also assess its strengths and weaknesses. It is important to take this philosophical approach because each explanation purports to describe reality in a different way. For example, a causal-mechanical explanation always requires a cause while a structural one says the phenomenon is a consequence of imposing mathematical constraints.

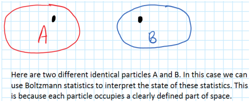
Introduction

"An explanation has been worked out by Pauli from complicated arguments of quantum field theory and relativity... we have not been able to find a way of reproducing his arguments on an elementary level... This probably means we do not have a complete understanding of the fundamental principle involved." (Feynman 1965, pg. 4–3.)

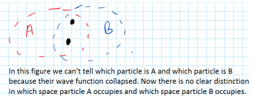
- ❖Boltzmann statistics: imagine modelling all the particles in a mole!
- ❖Fermi-Dirac Statistics Vs. Bose-Einstein statistics
- ❖What explains the spin statistics connection on an elementary level?

Background information

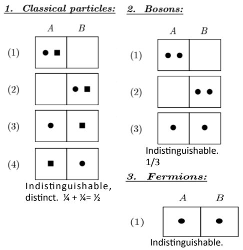
What is the probability that a particle is in state A and a particle is in state B?



Here are two different identical particles A and B. In this case we can use Boltzmann statistics to interpret the state of these statistics. This is because each particle occupies a clearly defined part of space.



In this figure we can't tell which particle is A and which particle is B because their wave function collapsed. Now there is no clear distinction in which space particle A occupies and which space particle B occupies.



Indistinguishable, distinct. ¼ + ¼= ½

Indistinguishable. 1/3

Indistinguishable. 1

Spin in particles

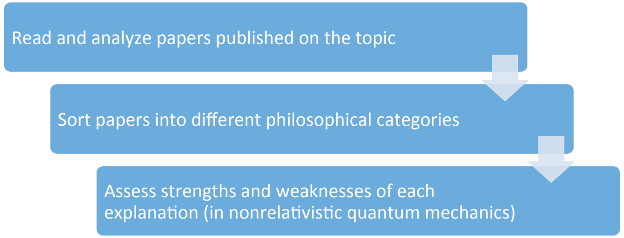
- ❖ Spin is an intrinsic property of quantum particles like mass and charge are.
- ❖ Quantum particles can possess two types of spin: integer spin and half-integer spin.
- ❖ Spin is related to its analogue in classical mechanics but is also quite distinct.
- ❖ By exhaustive empirical evidence it has been found that particles that have integer spin are bosons while those that have half-integer spin are fermions

What are the philosophical categories of scientific explanations?

Deductive – Nomological	Explains a phenomenon by showing how it can be derived from a law of nature (in conjunction with appropriate initial/boundary conditions).
Unification	Explains a phenomenon by showing how it can be derived from a unifying theoretical framework (i.e., a theory) that maximizes scope, simplicity and stringency.
Causal	Explains a phenomenon by identifying its causes.
Structural	Explains a phenomenon by showing how it is a result of constraints imposed by the mathematical structure of a theory.

Table 1. Four Types of Explanation.

Methodology



Read and analyze papers published on the topic

Sort papers into different philosophical categories

Assess strengths and weaknesses of each explanation (in nonrelativistic quantum mechanics)

Data & results

Approach	Explanation of SSC	Type of Explanation	Limitations
Real Forces (Mullin & Blaylock 2003; Holland 1993)	FD stats are due to repulsive forces and BE stats are due to attractive forces.	Causal	Not an attempt to explain SSC. Only works for spinless systems.
Topology and Kinks (Finkelstein & Rubinstein 1968)	3-dim exchange of two indistinguishable particles is topologically equivalent to a twist (kink) in one of them and the identity on the other (first rubberband theorem).	Causal (in the appeal to kink-antikink pair creation and annihilation) and Structural (in the appeal to topological features of the configuration space).	To be applicable to non-relativistic QM, must assume the existence of nonlinear kink-antikink pairs in non-relativistic many particle systems.
Topology and Antiparticles (Balachandran et a. 1991)	3-dim exchange of indistinguishable particles is accompanied by the corresponding particle-antiparticle pair creation and annihilation in a way that underwrites Finkelstein's first rubberband theorem.	Causal (in the appeal to particle-antiparticle pair creation and annihilation) and Structural (in the appeal to topological features of the configuration space).	To be applicable to non-relativistic QM, must assume the existence of particle-antiparticle pairs in non-relativistic many particle systems.
Geometric phase (Berry & Robbins 1997, 2000; Twamley 1997)	Transported spin basis part of wavefunction picks up a phase of $(-1)^K$ upon exchange, where K = 2 × (spin).	Structural	The condition K = 2 × (spin) does not hold in general.
Continuity (Peshkin 2006)	Truncated configuration space for 2 impenetrable, indistinguishable spin zero particles imposes a constraint on the wavefunction that forces it to be even.	Structural	Limited to two spin zero particles in 3-dim.
Algebraic (Kuckert 2004)	Given a 2-dim 2-particle quantum system, if the total angular momentum of the center of mass is equal to twice the total angular momentum of either particle, then the spin-statistics connection holds.	Structural	Limited to two particles in 2-dim. Constraint is a classical mechanical result, but does not hold in general in quantum mechanics.
Composite Particle (Aitchison & Mavromatos 1991; Wilczek 1982)	For a 2-dim charged composite particle (particle with flux tubes attached), statistics determines orbital angular momentum L_z (i.e., FD stats entails half-integer L_z, and BE stats entails integer L_z).	Causal (Aharonov–Bohm effect causally influences composite particle exchange statistics).	Limited to 2-dim. Only establishes a connection that goes from stats to angular momentum, and not from angular momentum to stats (so leaves open the possibility of anyonic stats in 2-dim). Only establishes a connection between statistics and orbital angular momentum, not spin.

Table 2. Attempts to Explain the Spin-Statistics Connection in Non-Relativistic Quantum Mechanics.

Analysis

From the table one can observe that all the explanations go into two categories: causal or structural. This is probably because none of these explanations are unifying in the theory of non-relativistic quantum mechanics. It also seems to be the case that all of these explanations only prove the SSC for very specialized and unique cases and fail to provide a general theory. For example, the last three articles have excellent mathematical derivations for 2-dimensional spaces that cannot manifest on the "elementary level" Feynman seeks. It can also be understood from the table that most papers use assumptions and constraints that only serve their purpose in relativistic quantum mechanics and not in non-relativistic frames of reference.

Conclusion

The purpose of this project was to answer Feynman's question about an elementary (non-relativistic) explanation for the spin statistics connection. After conducting the literature review and sorting present literature into different categories we conclude that none of the explanations present to this day are flawless. Most explanations are limited by their assumptions and constraints. Some of them assume constraints that only apply to relativistic quantum mechanics, while others mathematically derive the connection in two dimensions which does not represent our physical world and isn't elementary enough. While these theories are all fascinating and creative, we would need a physical experiment to verify them.

References

1. Aitchison, J. and N. Mavromatos (1991) 'Anyons', Contemporary Physics 32, 219–233.
2. Bain, J. Quantum particles, Identity and individuality, 5.
3. Balachandran, A. P., A. Daughton, Z-C Gu, G. Marmo, R. D. Sorkin, and A. M. Srivastava. "A Topological Spin-statistics Theorem or a Use of the Antiparticle." Phys. Scr. Physica Scripta T36 (1991): 253-57.
4. Berry, M. and J. Robbins (1997) 'Indistinguishability for Quantum Particles: Spin, Statistics and the Geometric Phase', Procedings of the Royal Society of London A 453, 1771–1790.
5. Berry, M. and J. Robbins (2000) 'Quantum Indistinguishability: Spin–statistics without Relativity or Field Theory?', in Hilborn, R. and G. Tino (eds.) CP545, Spin–Statistics Connection and Commutation Relations, American Institute of Physics, 3–15.
6. Finklestein, J. and D. Rubinstein (1968) 'Connection Between Spin, Statistics, and Kinks', Journal of Mathematical Physics 9, 1762–1779.
7. Holland, P. (1993) The Quantum Theory of Motion, Cambridge: Cambridge University Press.
8. Kuckert, B. (2004) 'Spin and Statistics in Nonrelativsitic Quantum Mechanics, I', Physics Letters A 322, 47–53.
9. Mullin, W. and G. Blaylock (2003) 'Quantum Statistics: Is there an Effective Fermion Repulsion or Boson Attraction?', American Journal of Physics 71, 1223–1231.
10. Peshkin, M. (2006) 'Spin-Zero Particles must be Bosons: A New Proof within Nonrelativistic Quantum Mechanics', Foundations of Physics 36, 19–29.
11. Peshkin, Murray. "Reply to "Comment on 'Spin and Statistics in Nonrelativistic Quantum Mechanics: The Spin-zero Case' "." Phys. Rev. A Physical Review A 68.4 (2003).
12. Peshkin, Murray. "Spin and Statistics in Nonrelativistic Quantum Mechanics: The Spin-zero Case." Phys. Rev. A Physical Review A 67.4 (2003).
13. Twamley, J. (1997) 'Statistcs Given a Spin', Nature 389, 127–129.