Non-Locality in Intrinsic Topologically Ordered Systems

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Abstract

Recent work in condensed matter physics has sought to define the notion of "intrinsic topological order" (ITO) [1,2]. ITO systems are characterized by two types of non-locality. The first type is a non-locality associated with topological properties. The second type is associated with a particular kind of quantum entanglement, referred to as long-range entanglement (LRE). LRE characterizes the ability for a system to retain entanglement under certain local operators. Recent work in quantum information theory has sought to use ITO systems to topologically protect information against decoherence from local error. The Toric Code is one example of this. Finally, the relationships between systems that may be able to exhibit such non-locality and traditional quantum mechanical systems is considered.

Intrinsic Topological Order

Intrinsic Topological Order (ITO) *(e.g., Wen 2013)*

A physical system possesses ITO just when:

(a) It exhibits a *ground state degeneracy* that depends on its topology.

(b) It exhibits *anyonic low-energy excitations*.

(c) Its ground states exhibit a *finite energy gap*.

(d) Its ground states satisfy the Knill-Laflamme Condition.

In condensed matter physics, Landau Symmetry Breaking has been the most useful theory in describing condensed matter systems. However, recently, it has been realized that symmetry breaking fails in completely describing the states of certain systems, such as those of Fractional Quantum Hall (FQH) systems. To fully describe these systems, a characterization of intrinsic topological order was introduced to the theory [2]. ITO systems possess global topological properties (a, b), as well as quantum entanglement propeties d. Both types of properties are expected to be invariant under local unitary evolutions. This invariance is called long-range entanglement. A local operator is an operator which acts non-trivially only on a local region of the system.

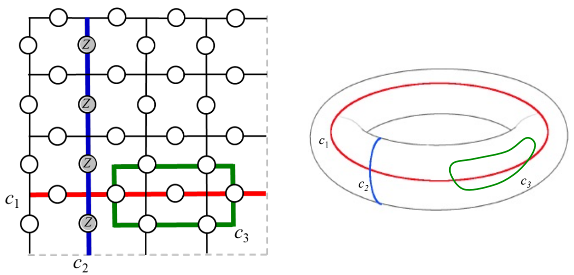

Figure 1. In the toric code , information is stored in non trivial elements c_1 and c_2 which cannot be disturbed by trivial element c_3 which is generated by local operators on the space.

The Toric Code

The Toric Code is a topological quantum error correction code that is an example of ITO [3]. Begin with a square lattice on a torus, and place qubits on each edge (figure 1). The local operators on this space can be interpreted as either constructing loops at points, or pushing and pulling existing loops. Single non-contractible loops going around a handle of the torus such as c_1 and c_2 of figure 1 cannot arise from these operators, and likewise cannot be destroyed by them. Thus we say the "codespace" (the space that encodes information in the non-contractible loops c_1, c_2) is not perturbed the local operators.

The Toric Codespace:

$$|\psi_{ee}\rangle,\ |\psi_{eo}\rangle,\ |\psi_{oe}\rangle,\ |\psi_{oo}\rangle$$

Each basis vector of the codespace of the Toric Code corresponds to either an even (*e*) or odd (*o*) number of each non-trivial element c_1 or c_2.

A Physical Approach

The Toric code provides a theoretical framework that suggests how information can be stored in the topological order of states. However, it remains purely mathematical in the sense that it is not concerned with any physical quantities. One can give it a physical meaning by interpreting loop operators as observables that commute with a system's Hamiltonian, but not with themselves [3]. This means the ground state of the Hamiltonian is degenerate, and one can encode information in a non-local way in the space of such ground states. The algebra generated by these loop operators is a subalgebra of the system's full operator algebra, and its dimension depends on the system's topology. Physical systems that exhibit these features have additional properties, such as a finite energy gap between the ground states and excited states, and low-energy excitations that obey fractional statistics.

Future Work

The only realized system that exhibits ITO is the Fractional Quantum Hall system. The feasibility of using ITO in realized quantum computation depends on whether or not effective ITO systems can be produced.

Works Cited

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