New York University Tandon School of Engineering
Department of Finance and Risk Engineering
Course outline FRE6233 Option Pricing and Stochastic Calculus
Spring 2020
Professor Roza Galeeva, rg63@nyu.edu
Monday, 5.30pm – 8 pm, location Rogers Hall, 339
TA: TBA

Course Pre-requisites: FRE 6083

Course Description: The course provides the mathematical foundation of Option Pricing models. The techniques covered include arithmetic and geometric Brownian motion, first passage time, the reflection principle, the stochastic Ito Integral, Ito differential Calculus, change of probability measure, martingales, SDE and PDE. Some of the pricing models considered are the European, Barrier, Asian and American Options. These problems are solved either analytically or numerically, by applying approximations and simulation methods.

Course Objective:

- This is the first course in Mathematical Finance, the material taught is a part of the common knowledge shared by quants in the Financial Industry. It provides the students the necessary skills to work in the field of quantitative finance.
- The course teaches students skills and techniques in stochastic calculus, indispensable for any professional working in the finance industry

- After taking the course, the students will be able to price standard types of derivatives securities, understand the principles of risk neutral pricing.
- It is imperative for students aiming to apply for Ph.D. program in Financial Engineering or Mathematical Finance. It provides a foundation for more advanced courses such as "Continuous time Finance"
- The considered practical problems should also prepare you for interviews for internships and jobs.

Course Structure

Most weeks, a lecture will be delivered, followed by a discussion of the previous homework and Q&A. I will give occasional short quizzes in the class.

Readings

The required text for the course is:

1. S. Shreve, *Stochastic calculus for Finance II: continuous-time series*, 2 edition, 2004, Springer (can be found online for free)

Additional Resources

1. Tomas Bjork, *Arbitrage Theory in Continuous time*, Oxford University Press, 3 edition, ISBN-10:019957474X
2 A. Shiryaev, *Probability.*, 1996, Springer.
3 P. Glasserman, *Monte Carlo Methods in Financial Engineering*, 2004, Springer
4 B. Oksendal, *Stochastic Differential Equations: An introduction with Applications*. Springer 2009.
5 R. Cont and P. Tankov, *Financial Modeling with Jump Processes*, 2004, Chapman & Hall.

6 I will provide additional references to research papers during the course.

Recommended software for classwork and homework:

Students will be required to use a programming language for prototyping, such as Python, Matlab or R,(http://www.r-project.org), C.

Course requirements:
The students are expected to attend the classes and participate actively. You need to bring your laptops to the class, as we will be doing often work in the class. If for any reason you can't come to lecture, please inform me by email in advance. There will a midterm exam, final exam, home assignments and quizzes.

Midterm exam will count for 30% of the grade.
The exam will be held in the classroom, March 9 on the scheduled class time.

Final exam (or project) will count for 35% of the grade, will be held May 18 at the usual scheduled class time.

Participation will count for 5%.

In both exams, the students will be required to solve questions, based on the previous material, homework questions and class discussions.

Homework assignments, weekly, due on weeks 2,3,5,6, 9,10,11,13,14 count for 30% of the final grade. There will be two types of homework assignments. The first type will consist of practice exercises designed to help the students assimilate the techniques taught in class and prepare them for the examinations. The second type will consist of implementing some numerical or simulation techniques, to compute option prices that cannot be computed analytically

Participation and quizzes will count for 5%.

Calendar

We will have 12 lectures + 2 exams. No lecture on February 17 (Presidents Day) : NYU calendar

Contact

The best way to contact me is by email, I monitor my emails regularly. You can always talk to me before and after the class. Most of the weeks, I should be on the campus on Mondays after 4pm.

Tentative program:

Part I: Ito Stochastic Calculus

Week 1-2 The Brownian motion and filtrations

- Lecture notes for week 1,2
- Textbook by Shreve, chapters 1-3

Week 3 – 4 The Ito integral and Ito's lemma, techniques on stochastic differentiation and integration, SDE

- Lecture notes for week 3-5
- Textbook by Shreve, chapter 4, lots of exercises in classes

Week 5 Application of stochastic calculus to the Black-Scholes model

- Lecture notes for week 3
- Textbook by Shreve, chapter 4, 5

Week 6 Midterm examination

Part II: The no arbitrage theory in continuous-time

Week 7 The martingale approach for option pricing

- Lecture notes
- Textbook by Shreve, chapter 5.

Week 8-9 The Partial Differential Equations approach, heat equation, application to barrier options

- Lecture notes
- Textbook by Shreve, chapter 6.

Week 10-11 Multidimensional market models: *stochastic calculus in several dimensions, multi-dimensional asset pricing models , introduction to the change of Numeraire, applications to FX and commodity derivatives*

- Lecture notes
- Textbook by Shreve, Chapters 5,9.

Week 12 The Asian option

- Lecture notes
- Textbook by Shreve, chapter 7.

Week 13 The American option

- Lecture notes
- Textbook by Shreve, chapter 8.

Week 14 Introduction to Jump-diffusion processes and option pricing under jump-diffusion

- Lecture notes
- Textbook by Shreve, chapter 11

Week 15 Final Exam

This is only a *tentative* schedule, subject to revisions and changes, depending on the speed of the process and other factors.

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