New York University Tandon School of Engineering
Department of Finance and Risk Engineering
Course Outline FRE6233 Option Pricing and Stochastic Calculus
Fall 2019
Professor Agnes Tourin
Thursday; 2pm-4:30pm, location RH 603

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Course Pre-requisites: FRE 6083

Course Description: This course provides the mathematical foundations of Option Pricing models. The techniques covered include arithmetic and geometric Brownian motion, first passage time, the reflection principle, the stochastic Ito integral, Ito differential Calculus, change of probability measure, martingales, Stochastic Differential Equations and Partial Differential Equations. Some of the pricing models considered are the European, Barrier, Asian and American options. These problems are either solved analytically by the martingale approach or numerically, by applying approximation and simulation methods. Since the same techniques allow the treatment of more complex financial products, several advanced examples will be also presented.

Course Objectives:

- After taking this course, the students will be able to price any derivative security.
- This is a first course in Mathematical Finance and is a building block for more advanced courses such as "Continuous time Finance".
- The material taught is part of the common knowledge shared by quants in the Financial industry and taking this course will prepare the students to apply for quantitative positions.
- This course also prepares the students who wish to apply to a Ph.D. program in Financial Engineering or Mathematical Finance.

Course Structure:
Most weeks, a lecture will be delivered, followed by a discussion and a Q&A session. There will be two problem solving sessions, on weeks 6 and 14.

Readings:
The required texts for the course are:

- Tomas Bjork*, Arbitrage Theory in continuous time*, Oxford University press, 3rd edition, 2009, ISBN-10: 019957474X.

- S. Shreve, *Stochastic Calculus for Finance II: continuous-time models,* 2nd edition, 2004, Springer.

Location of books: NYU bookstore. The second book can also be downloaded for free from the Springer online service.

Slides based on the textbooks and several additional sources will be posted on NYU classes.

Additional Resources (optional):

1. A Primer for the Mathematics of Financial Engineering, Dan Stefanica, Second Edition, 2011, FE Press New York.
2. Monte Carlo Methods in Financial Engineering, Paul Glasserman, 2004, Springer.
3. Financial Modelling With Jump Processes, Rama Cont and Peter Tankov, 2004, Chapman & Hall/CRC.
4. Computational Methods in Finance, Ali Hirsa, 2013, Chapman & Hall/CRC.
5. Introduction to Mathematical Finance, Discrete Time Models, Stanley R. Pliska, 1997, Blackwell Publishing.
6. Stochastic Differential Equations: An introduction with Applications, Bernt Oksendal, Universitext, Third Printing, Sixth Edition, 2009, Springer.
7. Probability Essentials, Jean Jacod and Philip Protter, Universitext, Second Printing, Second Edition, 2004, Springer.
8. Numerical Partial Differential Equations: Finite Difference Methods, J.W. Thomas, Texts in Applied Mathematics, 22, 1995, Springer.
9. Heard on the street: Quantitative Questions from Wall Street Job Interviews, Timothy Falcon Crack, revised 15th Edition, 2014.
10. 150 Most Frequently Asked Questions on Quant Interviews, Dan Stefanica, Rados Radoicic and Tai-Ho Wang, 2013, FE Press New York.

Recommended software for the homework:

Students will be required to use a programming language for prototyping, such as Matlab, R (http://www.r-project.org), or Python.

Course requirements:
The students are expected to attend classes and participate actively. They should read the lecture notes ahead of time and come prepared. There will be a midterm examination, a final examination, and weekly homework assignments.

Midterm examination, 30% of final grade.
This examination will be held in the classroom, at the scheduled class time, on week 7. The students will be required to solve four or five problems by using the computational techniques taught during the first 6 weeks.

Final examination, 40% of final grade.
This examination will be held in the classroom, at the scheduled class time, on week 15. The students will be required to solve four or five problems, by using the computational techniques taught throughout this course.

Homework assignments, weekly, due on weeks 2,3,4,5,6,8,9,10,11,12,13 count for 30% of the final grade. There will be two types of homework assignments. The first type will consist of practice exercises designed to help the students assimilate the techniques taught in class and prepare them for the examinations. The second type will consist of implementing some numerical or simulation techniques, to compute option prices that cannot be computed analytically.

Grading of Assignments:

The grade for this course will be determined according to the following formula:

Assignments/Activities	% of Final Grade
Homework	30%
Midterm exam	30%
Final Exam	40%

Letter Grades

Letter grades for the entire course will be assigned as follows:

Letter Grade	Points	Percent
A	4.00	95% and higher
A-	3.67	90.0 – 94.99%
B+	3.33	87% - 89.99%

B	3.00	83% - 86.99%
B-	2.67	80% - 82.49%
C+	2.33	77% - 79.99%
C	2.00	70.0% - 76.99%
F	.00	69.99% and lower

Lecture schedule:

Part I: Ito Stochastic Calculus

Week 1 The Brownian motion and filtrations

- Lecture notes for week 1
- Textbook by Shreve, chapters 1-2

Week 2 The Ito integral and Ito's lemma in several dimensions

- Lecture notes for week 2
- Textbook by Shreve, chapter 3
- Textbook by Bjork, chapter 4.
- First assignment is due (problem set)

Week 3 Application of stochastic calculus to the Black-Scholes model

- Lecture notes for week 3
- Textbook by Shreve, chapter 4.
- Textbook by Shreve, chapters 5,6,7,9.
- Second assignment is due (problem set)

Part II: The no arbitrage theory in continuous-time

Week 4 The martingale approach

- Lecture notes for week 4

- Textbook by Shreve, chapter 5.
- Textbook by Bjork, chapters 10, 11,12.
- Assignment 3 is due (problem set)

Week 5 The Partial Differential Equations approach

- Lecture notes for week 5
- Textbook by Shreve, chapter 6.
- Textbook by Bjork, chapter 18.
- Assignment 4 is due (problem set).

Week 6 Completion of lecture 5, review and problem solving session

Week 7 Midterm examination

Week 8 The Asian option

- Lecture notes for week 8
- Textbook by Shreve, chapter 7.

- Fifth assignment is due (problem set).

Week 9 The American option

- Textbook by Bjork, chapter 21.
- Lecture notes for week 9
- Sixth assignment due (implementation of Monte Carlo simulations for the Asian option)

Week 10 Finite Difference scheme for the American option

- Lecture notes for week 10
- Seventh Assignment due (problem set).

Week 11 Multidimensional market models: stochastic calculus in several dimensions, multi-dimensional asset pricing models , introduction to the change of Numeraire, applications to FX derivatives.

- Lecture notes for week 11
- Textbook by Shreve, Chapters 5,9.
- Texbook by Bjork, chapter 14,17, 26.

- Assignment 8 is due

Part III: Jump-diffusion processes and incomplete market models

Week 12 Stochastic Calculus for jump-diffusion processes

- Lectures notes for week 12
- Textbook by Shreve, chapter 11
- Assignment 9 is due

Week 13 Option pricing under jump-diffusion models

- Lecture notes for week 13
- Textbook by Shreve, chapter 11
- Assignment 10 is due

Week 14 Review and problem solving session

Week 15 Final examination

Policies

Academic Misconduct

A. Introduction: The School of Engineering encourages academic excellence in an environment that promotes honesty, integrity, and fairness, and students at the School of Engineering are expected to exhibit those qualities in their academic work. It is through the process of submitting their own work and receiving honest feedback on that work that students may progress academically. Any act of academic dishonesty is seen as an attack upon the School and will not be tolerated. Furthermore, those who breach the School's rules on academic integrity will be sanctioned under this Policy. Students are responsible for familiarizing themselves with the School's Policy on Academic Misconduct.

B. Definition: Academic dishonesty may include misrepresentation, deception, dishonesty, or any act of falsification committed by a student to influence a grade or other academic evaluation. Academic dishonesty also includes intentionally damaging the academic work of others or assisting other students in acts of dishonesty. Common examples of academically dishonest behavior include, but are not limited to, the following:

   1. Cheating: intentionally using or attempting to use unauthorized notes, books, electronic media, or electronic communications in an exam; talking with fellow students or looking at another person's work during an exam; submitting work prepared in advance for an in-class examination; having someone take an exam for you or taking an exam for someone else; violating other rules governing the administration of examinations.
   2. Fabrication: including but not limited to, falsifying experimental data and/or citations.

3. Plagiarism: intentionally or knowingly representing the words or ideas of another as one's own in any academic exercise; failure to attribute direct quotations, paraphrases, or borrowed facts or information.
4. Unauthorized collaboration: working together on work that was meant to be done individually.
5. Duplicating work: presenting for grading the same work for more than one project or in more than one class, unless express and prior permission has been received from the course instructor(s) or research adviser involved.
6. Forgery: altering any academic document, including, but not limited to, academic records, admissions materials, or medical excuses.

Disability Disclosure Statement

Academic accommodations are available for students with disabilities. Please contact the **Moses Center for Students with Disabilities** (212-998-4980 or mosescsd@nyu.edu) for further information. Students who are requesting academic accommodations are advised to reach out to the Moses Center as early as possible in the semester for assistance.