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Leverage Effect, Volatility Feedback, and Self-Exciting Market Disruptions

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Abstract

Equity index volatility variation and its interaction with the index return can come from three distinct channels. First, index volatility increases with the market's aggregate financial leverage. Second, positive shocks to systematic risk increase the cost of capital and reduce the valuation of future cash flows, generating a negative correlation between the index return and its volatility, regardless of financial leverage. Finally, large negative market disruptions show self-exciting behaviors. This article proposes a model that incorporates all three channels and examines their relative contribution to index option pricing and stock option pricing for different types of companies.

I. Introduction

Equity index returns interact negatively with return volatilities. This article proposes a model tracing the negative interaction to three distinct economic channels, examines the relative contribution of the three channels to index option pricing, and explores how the three channels show up differently in companies with different business types and capital structure behaviors.

First, equity index return volatility increases with the market's aggregate financial leverage. Financial leverage can vary either passively as a result of stock market price movement or actively through dynamic capital structure management. With the amount of debt fixed, financial leverage increases when

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the stock market experiences downward movements due to shrinkage in market capitalization. Black (1976) first proposes this *leverage effect* to explain the negative correlation between equity returns and return volatilities for companies holding their debt fixed. Adrian and Shin (2010) show, however, that some companies also proactively vary their financial leverage based on variations in market conditions. Both types of variations in financial leverage generate variations in the index return volatility.

Second, the volatility of the market's risky asset portfolio can also vary because the risk of a particular business can change over time, and so can the composition of businesses in an aggregate economy. With fixed future cash flow projections, an increase in the market's systematic business risk increases the cost of capital and reduces the present value of the asset portfolio, generating a *volatility feedback effect* (Campbell and Hentschel (1992), Bekaert and Wu (2000)). This effect can show up as a negative correlation between the index return and its volatility, regardless of the market's financial leverage level.

Third, financial crises have shown a disconcerting pattern that worries both policymakers and financial managers: A large negative financial event often increases the chance of more such events to follow. Researchers (e.g., Azizpour, Giesecke, and Schwenkler (2017), Ding, Giesecke, and Tomecek (2009), and Aït-Sahalia, Cacho-Diaz, and Laeven (2015)) label this phenomenon as *self-exciting behavior*. Cross-sectionally, the default of or large negative shock to one company has been found to increase the likelihood of default or large downward movements of other companies. In aggregation, such cross-sectional propagation leads to an intertemporal self-exciting pattern in the market: One market turmoil increases the chance of another to follow.

This article proposes a model for the equity index dynamics that captures all three channels of economic variation. The model separates the dynamics of the market's risky asset portfolio from the variation of the market's aggregate financial leverage. Returns on the risky asset portfolio generate stochastic volatilities from both a diffusion risk source with volatility feedback effect and a jump risk source with self-exciting behavior. The market's aggregate financial leverage, conversely, can vary both via unexpected random shocks and through proactive target adjustments based on market risk conditions.

We first estimate the model on the Standard & Poor's (S&P) 500 index options to examine the relative contribution of each channel to equity index option pricing. Estimation shows that the volatility feedback effect reveals itself mainly in the variation of short-term options, the self-exciting behavior affects both short-term and long-term option variations, and the financial leverage variation has its largest impact on long-dated options. Thus, by using option observations across a wide range of strikes and maturities, we can effectively disentangle the three sources of volatility variation.

The disentangling reveals economic insights on important empirical observations. An important recent finding in the index options market is that market-level variance risk generates a strongly negative risk premium (Bakshi and Kapadia (2003a), (2003b), Carr and Wu (2009)). Several studies provide interpretations and explore its relations with other financial phenomena (see, e.g., Bakshi and Madan (2006), Bollerslev, Tauchen, and Zhou (2009), Zhou (2010), Bollerslev, Gibson, and Zhou (2011), Drechsler and Yaron (2011), and Baele, Driessen, Londono, and Spalt (2014)). Our structural decomposition shows that the volatility risk premium can come from several different sources, including the market prices of financial leverage risk, asset volatility risk, and self-exciting market crashes. Estimation shows that the negative variance risk premium comes mainly from the latter two sources.

The disentangling also allows us to answer important capital structure questions based on option price variations. Model estimation shows that contrary to common wisdom, the aggregate financial leverage in the U.S. market does not always decline with increased business risk. Instead, the market responds differently to different types of risks. The aggregate financial leverage can actually increase with increasing business risk if the business risk is driven by small, diffusive market movements. Only when the perceived risk of self-exciting market disruptions increases does the market become truly concerned and start the deleveraging process.

By separating the variation in business risk and capital structure decisions, our model also has important implications for individual stock option pricing. The three economic channels show up differently for companies with different business types and capital structure behaviors, thus leading to different individual stock option pricing behaviors. In particular, we expect companies with more systematic shocks to experience a stronger volatility feedback effect and companies with more passive capital structure policies to experience stronger leverage effects. We also expect the self-exciting behavior to be more of a market behavior through aggregation of cross-sectional propagation of negative shocks among structurally related companies.

As a guidance for future comprehensive analysis, we perform a preliminary examination of 5 companies selected from 5 distinct business sectors. Model estimation on the selected companies shows that the volatility feedback effect is the strongest for energy companies, the shocks to which tend to have fundamental impacts on the aggregate economy. The leverage effect is the strongest for manufacturing companies, which tend to hold their debt fixed for a long period of time and thus exacerbate the leverage effect described by Black (1976). By contrast, bank holding companies show a much weaker leverage effect but much stronger mean-reverting behavior in their financial leverage variation because they tend to actively manage their financial leverage to satisfy regulatory requirements.

The current literature often indiscriminately labels the observed negative correlation between equity or equity index returns and return volatilities as a "leverage effect," causing confusion about the exact economic origins of the correlation. Hasanhodzic and Lo (2010) highlight the inappropriateness of the terminology by showing that the negative correlation between stock returns and return volatilities is just as strong for all-equity-financed companies, which are by definition absent of any financial leverage. Figlewski and Wang (2000) also raise questions about whether the so-called "leverage effect" is really caused by financial leverage variation. In this article, we provide a careful distinction of the different economic channels that can all generate a negative relation between returns and volatilities. In particular, even in the absence of financial leverage, our model can still generate a negative relation between returns and volatilities through the diffusive volatility feedback effect and the self-exciting jump-propagation behavior.

Our modeling approach provides a balance between richness in economic structures and tractability for option pricing. The former is a prerequisite for addressing economic questions, and the latter is a necessity for effectively extracting information from the large amount of option observations. The current literature largely stays at the two ends of the spectrum. On the one end is the capital structure literature pioneered by Merton (1974), which is rich in economic structures but remains both too stylized and too complicated to be a working solution for equity options pricing. Cremers, Driessen, and Maenhout (2008) specify a jump-diffusion stochastic volatility dynamics for the asset value and compute the equity option values as compound options on the asset value. Through this specification, they are able to calibrate the average credit spreads on corporate bonds to the average variance and jump risk premiums estimated from equity index options. Their resolution of the average credit spread puzzle highlights the virtue of exploiting information in equity options, but their stylized calibration exercise also highlights the inherent difficulty in making the structural approach a feasible solution for capturing the time variation of equity options.

On the other end of the spectrum is the reduced-form option pricing literature, which can readily accommodate multiple sources of stochastic volatilities with analytical tractability. Prominent examples include Heston (1993), Bates (1996), (2000), Bakshi, Cao, and Chen (1997), Heston and Nandi (2000), Duffie, Pan, and Singleton (2000), Pan (2002), Carr and Wu (2004), Eraker (2004), Huang and Wu (2004), Broadie, Chernov, and Johannes (2007), Christoffersen, Jacobs, Ornthanalai, and Wang (2008), Christoffersen, Heston, and Jacobs (2009), Santa-Clara and Yan (2010), and Andersen, Fusari, and Todorov (2015). Nevertheless, these models are often specified as linear combinations of purely statistical factors, without any direct linkage to economic sources. The absence of economic linkage prevents these models from addressing economic questions. For example, many option pricing models allow a negative correlation between return and volatility. The negative correlation is often labeled as the "leverage effect," without further distinction on whether it is really coming from financial leverage variation or simply a volatility feedback effect that has nothing to do with financial leverage, thus causing confusion in the interpretation of the estimation results. Furthermore, the generic factor structure often poses identification issues that limit most empirical estimations to one or two volatility factors.

Our specification retains the flexibility and tractability of reduced-form option pricing models but incorporates economic structures motivated by the capital structure literature. As a result, the model can use the rich information in equity index options to show how capital structure decisions vary with different types of economic risk. Furthermore, by applying the economic structures, we also obtain a rich and yet parsimonious 3-volatility-factor specification that can be well identified from option observations.

Also related to our work is the increasing awareness of the rich information content in options in addressing economically important questions. For example, Birru and Figlewski (2012) and Figlewski (2009) provide insights into the recent financial crisis in 2008 by analyzing the risk-neutral return densities extracted from the S&P 500 index options. Bakshi and Wu (2010) infer how the market prices of various sources of risks vary around the National Association of Securities Dealers Automated Quotations (NASDAQ) bubble period using options on the NASDAQ 100 tracking stock. Backus and Chernov (2011) use equity index options to quantify the distribution of consumption growth disasters. Bakshi, Panayotov, and Skoulakis (2011) show that appropriately formed option portfolios can be used to predict both real activities and asset returns. Bakshi, Carr, and Wu (2008) extract the pricing kernel differences across different economies using options on exchange rates that form a currency triangle. Ross (2015) proposes a recovery theorem that separates the pricing kernel and the natural probability distribution from the state prices extracted from option prices. In this article, we show that variations in financial leverage, asset diffusion risk, and asset crash risk contribute differently to options price behaviors at different strikes and maturities. Thus, we can rely on the large cross section of options to disentangle these different sources of volatility variations.

The rest of the article is organized as follows: Section II specifies the equity index dynamics and discusses how the model incorporates the three sources of volatility variation through separate modeling of asset dynamics and financial leverage decisions. The section also discusses how the three sources of market variations can show up differently in individual companies with different business types and different capital structure behaviors. Section III describes the data sources and summarizes the statistical behavior of alternative financial leverage measures and option-implied volatilities on both the S&P 500 index and 5 individual companies selected from 5 distinct business sectors. The section also elaborates the model estimation strategy with equity and equity index options. Based on the estimation results, Section IV discusses the historical behavior of the three economic sources of variation, their relative contribution to equity index option pricing, and the cross-sectional variation in the 5 selected individual companies. Section V concludes. The Internet Appendix (available at http://faculty.baruch.cuny.edu/lwu/) provides the technical details on option valuation under our model specification, the model estimation methodology, and a discussion of the model's option pricing performance.

II. Model Specification

We fix a filtered probability space $\{\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t\geq 0}\}$ and assume no-arbitrage conditions in the economy. Under certain technical conditions, there exists a risk-neutral probability measure $\mathbb{Q}$, absolutely continuous with respect to $\mathbb{P}$, such that the gains process associated with any admissible trading strategy deflated by the risk-free rate is a martingale.

A. Separating Leverage Effect from Volatility Feedback and Self-Exciting Jumps

Let F_t denote the time t forward level of the equity index over some fixed time horizon. We separate the dynamics of the risky asset portfolio from the variation in the market's financial leverage via the following multiplicative decomposition:

$$F_t = X_t A_t, \tag{1}$$

where A_t denotes the time t forward value of the risky asset, and $X_t = F_t / A_t$ denotes the ratio of equity to risky assets. Intuitively, one can decompose a balance sheet into equity and debt on the one side and risky assets and riskless assets (e.g., cash) on the other side. The riskiness of the equity is determined by the riskiness of the risky asset investments and the ratio of the equity to the risky assets. In this classification, one can think of the cash position as a reduction in debt and hence reduction in leverage, noting that the equity can be safer than the risky asset if the company holds a large amount cash such that the ratio of equity to risky assets X_t is greater than 1. The decomposition in equation (1) is a mere tautology, but it allows us to disentangle the impact of financing leverage decisions from decisions regarding business investment and operation in the economy.

1. Asset Value Dynamics with Volatility Feedback Effects and Self-Exciting Jumps

We model the forward value dynamics for the risky asset A_t under the physical measure $\mathbb{P}$ as follows:

$$dA_t/A_{t-} = \xi_t\,dt + \sqrt{v_t^Z}\,dZ_t + \int_{-\infty}^{0} (e^x - 1)\left(\mu(dx,dt) - \pi^{\mathbb{P}}(x)\,dx\,v_t^J\,dt\right), \tag{2}$$

$$dv_t^Z = \kappa_Z^{\mathbb{P}}\left(\theta_Z^{\mathbb{P}} - v_t^Z\right)dt + \sigma_Z\sqrt{v_t^Z}\,dZ_t^v, \quad \mathbb{E}\left[dZ_t^v\,dZ_t\right] = \rho dt < 0, \tag{3}$$

$$dv_t^J = \kappa_J^{\mathbb{P}}\left(\theta_J^{\mathbb{P}} - v_t^J\right)dt - \sigma_J\int_{-\infty}^{0} x\left(\mu(dx,dt) - \pi^{\mathbb{P}}(x)\,dx\,v_t^J\,dt\right), \tag{4}$$

where ξ_t denotes the instantaneous risk premium on the asset return, determined by the market pricing specification on various risk sources; Z_t and Z_t^v denote two standard Brownian motions; $\mu(dx,dt)$ denotes a counting measure for jumps; $\pi^{\mathbb{P}}(x)v_t^J$ denotes the time t arrival rate of jumps of size x in the natural logarithm of asset value, $\ln A_t$, with

$$\pi^{\mathbb{P}}(x) = e^{-|x|/v_J^{\mathbb{P}}}|x|^{-1}; \tag{5}$$

and A_{t-} denotes the asset value at time t just prior to a jump.

Equation (2) decomposes the asset value variation into a diffusion component with stochastic variance v_t^Z and a discontinuous component with stochastic jump intensity v_t^J. A negative correlation between Z_t and Z_t^v in equation (3) generates the *volatility feedback effect*: A positive shock to the market business risk increases the cost of capital and reduces the asset value.

The negative jump component in equation (2) captures the impact of market turmoil. The intensity of market turmoil is stochastic and follows the dynamics specified in equation (4), where a downside jump in the asset value is associated with an upside jump in the jump intensity. The coefficient $\sigma_J^{\mathbb{P}} > 0$ captures the proportional scale on the jump size in the intensity per each jump in the log asset value, and the negative sign in front of $\sigma_J^{\mathbb{P}}$ highlights the opposite effect of the jump on the asset value and the jump intensity. The specification captures the *self-exciting behavior*: The occurrence of a downside jump event increases the intensity of future downside jump events.

To derive the risk-neutral $\mathbb{Q}$ dynamics for option pricing, we perform the following decomposition on the diffusion asset value risk:

$$Z_t = \rho Z_t^v + \sqrt{1-\rho^2}\widetilde{Z}_t, \tag{6}$$

and assume proportional market prices (γ^Z, γ^v) on the independent diffusion return risk $\widetilde{Z}_t$ and the diffusion variance risk Z^v, respectively. The proportional market price (γ^Z) of the independent diffusion return risk $(\widetilde{Z}_t)$ generates an instantaneous risk premium on the asset return of $\gamma^Z\sqrt{1-\rho^2}v_t^Z$. The proportional market price (γ^v) of the diffusion variance risk (Z_t^v) generates a drift adjustment term $\gamma^v\sigma_Z v_t^Z$ for v_t^Z. It also generates an instantaneous asset return risk premium $\rho\gamma^v v_t^Z$. We assume constant market price (γ^J) on the jump return risk, which generates an exponential tilting on the jump arrival rate under the risk-neutral measure $\mathbb{Q}$:

$$\pi(x) = e^{-\gamma^J x}e^{-|x|/v_J^{\mathbb{P}}}|x|^{-1} = e^{-|x|/v_J}|x|^{-1}, \tag{7}$$

with $v_J = v_J^{\mathbb{P}}/(1-\gamma^J v_J^{\mathbb{P}})$. Under these specifications for the market price of risk, we can write the forward asset value dynamics under the risk-neutral measure $\mathbb{Q}$ as follows:

$$dA_t/A_{t-} = \sqrt{v_t^Z}dZ_t + \int_{-\infty}^0 (e^x-1)\left(\mu(dx,dt) - \pi(x)dx v_t^J dt\right), \tag{8}$$

$$dv_t^Z = \kappa_Z\left(\theta_Z - v_t^Z\right)dt + \sigma_Z\sqrt{v_t^Z}dZ_t^v, \tag{9}$$

$$dv_t^J = \kappa_J\left(\theta_J - v_t^J\right)dt - \sigma_J\int_{-\infty}^0 x\left(\mu(dx,dt) - \pi(x)dx v_t^J dt\right), \tag{10}$$

where $\kappa_Z = \kappa_Z^{\mathbb{P}} + \gamma^v\sigma_Z$, $\theta_Z = \kappa_Z^{\mathbb{P}}\theta_Z^{\mathbb{P}}/\kappa_Z$, $\kappa_J = \kappa_J^{\mathbb{P}} + \sigma_J(v_J^{\mathbb{P}} - v_J)$, and $\theta_J = \kappa_J^{\mathbb{P}}\theta_J^{\mathbb{P}}/\kappa_J$.

Although we can readily accommodate both positive and negative jumps in the risky asset dynamics, equation (5) incorporates only negative jumps for parsimony. When we incorporate positive jumps in the specification, estimation on equity index options often results in a positive jump size that is not significantly different from zero.

Many option pricing models add jumps to the equity index dynamics. Bakshi et al. (1997) allow a jump with constant intensity. Pan (2002) specifies the jump intensity as a function of the diffusion variance rate. Du (2011) builds a general equilibrium model for equity index options pricing that includes a Poisson jump component in the consumption growth rate and time-varying risk aversion induced by habit formation. Eraker, Johannes, and Polson (2003), Eraker (2004), and Broadie et al. (2007) use synchronized finite-activity jumps to model the equity index return and volatility. Still, the diffusion variance and jump intensity are governed by one process. These models do not capture the observation that small market movements and large market turmoils can be driven by completely different forces. Huang and Wu (2004) and Santa-Clara and Yan (2010) allow diffusion and jumps to generate separate stochastic volatilities, but they do not accommodate the self-exciting behavior. Our specification for the asset value dynamics is the first in the literature that allows small market movements with volatility feedback effect and large market turmoils with self-exciting behavior as separate sources of stochastic volatility.

2. Active Capital Structure Decisions and Dynamic Financial Leverage Variation

We propose a dynamic for the ratio of equity to risky assets X_t that accommodates the market's active capital-structure-targeting decisions and the impacts of financial leverage shocks on equity return volatility. Formally, under the physical measure $\mathbb{P}$, we specify the following:

$$(11) \qquad dX_t/X_t = X_t^{-p}\left(a_X - \kappa_{XX}X_t - \kappa_{XZ}v_t^Z - \kappa_{XJ}v_t^J\right)dt + \delta X_t^{-p}\,dW_t,$$

where W_t denotes a standard Brownian motion, δ is a positive quantity capturing the volatility scale of the financial leverage shocks, and p is a power coefficient that determines how the equity index return volatility varies with the level of financial leverage. With $p > 0$, the process captures the *leverage effect*: Conditional on a fixed level for the risky asset, a decline in X increases the financial leverage and reduces the equity value by definition, and it also raises the equity volatility via the power term X_t^{-p} in equation (11). Hurd and Li (2008) show that under certain parametric conditions, the Leland (1994) capital structure model, where equity is modeled as a barrier option on the asset value, implies that the equity return volatility is a power function of the equity-to-asset ratio, with the power being $p = 1/2$. Our specification can be regarded as a generalization of the Leland structural model by allowing the power dependence on the leverage ratio to be a free parameter and by accommodating much more sophisticated and realistic asset-value dynamics.

The drift specification in equation (11) captures the market's active capital-structure-targeting decision: The market adjusts the capital structure target based on the current levels of financial leverage (X_t), the business diffusion risk v_t^Z, and the business jump risk v_t^J. The constant term a_X allows the market to set a long-run target on the equity-to-asset ratio.

Traditional capital structure models such as those by Merton (1974) and Black and Cox (1976) often fix the notional amount of debt. In such models, capital structure variations are completely passive because they are driven purely by variations in the asset value. Collin-Dufresne and Goldstein (2001) specify a simple mean-reverting process for the leverage ratio. Adrian and Shin (2010) show that a company often proactively varies its financial leverage target based on variations in market conditions. In particular, commercial banks, in an effort to comply with regulation requirements, strive to maintain a stable leverage ratio despite market variations. Investment banks go one step further. They not only manage their leverage ratios proactively but also pro-cyclically by raising leverage during economic booms and deleveraging during recessions. These actions generate the exact opposite effect of what is described in the traditional models, where leverage would go down when asset values go up.

Based on such evidence, we take a completely new approach by directly modeling the leverage ratio (X_t) variation. Our dynamics specification in equation (11) captures the proactive leverage decision rules through the drift specification and captures the random, unexpected shocks in the leverage variation through the Brownian motion. We not only allow mean-reverting financial-leverage-targeting behaviors but also allow the market to target different levels of financial leverage based on different market risk conditions.

We assume independence between the Brownian shock in financial leverage (dW_t) and the Brownian shocks in the asset value (dZ_t) and volatility (dZ_t^v). The independence assumption allows us to model A_t and X_t as separate martingales under the risk-neutral measure, facilitating the pricing of equity index options. The risk-neutral dynamics for the equity-to-asset ratio can thus be written as follows:

$$dX_t/X_t = \delta X_t^{-p} dW_t. \tag{12}$$

The Internet Appendix provides the technical details on how options can be priced tractably under our model specification.

B. An Alternative Representation

Combining the specifications in equations (8)–(11), we can write the risk-neutral dynamics for the forward equity index as follows:

$$\begin{aligned} dF_t/F_{t-} &= \delta\left(\frac{F_{t-}}{A_{t-}}\right)^{-p} dW_t + \sqrt{v_t^Z}\, dZ_t \\ &\quad + \int_{-\infty}^{0} (e^x - 1)\left(\mu(dx,dt) - \pi(x)\,dx\,v_t^J\,dt\right), \end{aligned} \tag{13}$$

where the variation of the equity index return volatility comes from three distinct sources: the financial leverage ($X_t = F_t/A_t$), the variance of the asset diffusion movement (v_t^Z), and the arrival rate of the self-exciting jumps (v_t^J).

By performing a change of variable $v_t^X = \delta^2 X_t^{-2p}$, we can rewrite the risk-neutral equity index dynamics in equation (13) in the form of a 3-factor stochastic volatility model,

$$\begin{aligned} dF_t/F_{t-} &= \sqrt{v_t^X}\, dW_t + \sqrt{v_t^Z}\, dZ_t \\ &\quad + \int_{-\infty}^{0} (e^x - 1)\left(\mu(dx,dt) - \pi(x)\,dx\,v_t^J\,dt\right), \end{aligned} \tag{14}$$

where the stochastic variance from the leverage effect v_t^X follows a 3/2 process:

$$dv_t^X = \kappa_X (v_t^X)^2 dt - \sigma_X (v_t^X)^{3/2} dW_t, \tag{15}$$

where $\kappa_X = p(2p+1)$, $\sigma_X = 2p$, and the innovation is perfectly negatively correlated with the corresponding return innovation component. Under the specification in equation (14), the index return is driven by two Brownian motion components and a jump component. The instantaneous variances on the 2 Brownian motions v_t^X and v_t^Z and the jump-intensity process v_t^J are all stochastic and are driven by three separate dynamic processes, specified in equations (15), (3), and (4), respectively.

The alternative representation reveals several new insights. First, equation (15) makes it explicit that financial leverage variation can be one of the three contributors to stochastic volatility in the equity index return. Indeed, in classic capital structure models, such as those by Merton (1974), Black and Cox (1976), Leland (1994), and Leland and Toft (1996), financial leverage variation is the only source of variation in equity return volatility because these models assume constant asset return volatility.

Second, the particular 3/2 volatility of volatility dependence due to the leverage effect in equation (15) is interesting. The behaviors of 3/2 processes have been studied by several authors, including Heston (1997), Lewis (2000), and Carr and Sun (2007). Within the context of 1-factor diffusion, several empirical studies find that a 3/2 specification on the variance-rate dynamics performs better than the square-root specification.[1] Thus, by including a 3/2-volatility component in addition to the square-root dynamics for the asset-diffusion variance rate v_t^Z and the self-exciting jump dynamics for the jump intensity v_t^J, our model has the potential to generate better pricing performance than existing affine specifications in the literature.

Third, equations (14) and (15) reveal that from index options alone, we cannot identify the volatility scale (δ) of the process for the equity-to-asset ratio. Instead, we can identify a standardized version of the equity-to-asset ratio, $\widetilde{X}_t = \delta^{-1/p} X_t$, or the corresponding leverage-induced variance rate $v_t^X = \delta^2 X_t^{-2p} = \widetilde{X}_t^{-2p}$. The standardized equity-to-asset ratio shows a unit volatility scaling for its dynamics:

$$(16) \qquad d\widetilde{X}_t / \widetilde{X}_t \quad = \quad \widetilde{X}^{-p}\left(\widetilde{a}_X - \widetilde{\kappa}_{XX}\widetilde{X}_t - \widetilde{\kappa}_{XZ} v_t^Z - \widetilde{\kappa}_{XJ} v_t^J\right) dt + \widetilde{X}_t^{-p}\, dW_t,$$

with $\widetilde{a}_X = a_X \delta^{-1}$, $\widetilde{\kappa}_{XX} = \delta^{(1-p)/p}\kappa_{XX}$, $\widetilde{\kappa}_{XZ} = \delta^{-1}\kappa_{XZ}$, and $\widetilde{\kappa}_{XJ} = \delta^{-1}\kappa_{XJ}$. Thus, the impact of financial leverage on equity index option pricing comes only through its standardized form. To the extent that the actual equity-to-asset ratio can show stochastic volatility, with δ_t following a stochastic process, our identified variation on the standardized leverage measure $\widetilde{X}_t$ will reflect the combined effect of the variations in the raw financial leverage and its volatility. A separate identification of the two would need inclusion of actual financial leverage data.

Finally, the literature often makes a dichotomous distinction between the local volatility models of Dupire (1994) and stochastic volatility models such as that by Heston (1993). The local volatility models are popular in the industry,[2] but the academic option pricing literature focuses almost exclusively on scale-free stochastic volatility specifications. The two representations of our model show that the gap between the two strands of literature is not as big as is generally perceived. We can represent our model as either a pure 3-factor stochastic volatility model as in equation (14) or as having a local-volatility component as in equation (13). Compared with Dupire's local volatility specification as a function of the index level, we scale the index level by the risky asset level to build a more

[1]Favorable evidence from time-series returns includes Chacko and Viceira (2003), Ishida and Engle (2002), Javaheri (2005), and Jones (2003). Supporting evidence from equity index options include Jones (2003), Medvedev and Scaillet (2007), and Bakshi, Ju, and Ou-Yang (2006). Christoffersen, Jacobs, and Mimouni (2010) provide further empirical support from a joint analysis of stock index returns, realized volatilities, and options.

[2]Several earlier articles specify the equity index as following a pure constant elasticity of variance process (e.g., Beckers (1980), Cox (1996), Emanuel and MacBeth (1982), and Schroder (1989)). Dupire (1994) specifies the equity return volatility as a generic function of the equity level and shows that this function can be identified via a forward partial differential equation. Dumas, Fleming, and Whaley (1998) investigate the empirical performance of various specifications for local volatility functions.

fundamentally stable local volatility function in terms of the unitless equity-to-asset ratio (F_t/A_t).

C. Application to Individual Companies

We use three economic channels to model the equity index volatility variation and its interactions with the equity index return. These channels act differently and show up to different degrees in individual stocks for companies with different business types and different capital structure behaviors.

First, in modeling the aggregate financial leverage behavior, we allow the market to target the aggregate financial leverage variation according to the market's levels of diffusive and jump risk. When an individual company determines its financial leverage target, the company may consider both the aggregate market conditions and its own unique situation. Furthermore, different types of companies can show drastically different capital structure behaviors. For example, Adrian and Shin (2010) show that manufacturing companies tend to hold their debt level fixed for a long period of time and are therefore more likely to experience the leverage effect described by Black (1976). In contrast, bank holding companies tend to target a fixed financial leverage level in accordance with regulatory requirements, whereas investment firms tend to be even more proactive in their capital structure management, often reducing financial leverage during market recessions while increasing financial leverage during market booms. Such proactive financing activities can significantly reduce the leverage effect and, accordingly, the negative relation between the company's stock returns and volatilities.

Second, the volatility feedback effect in the equity index is generated based on classic asset pricing arguments that an increase in systematic business risk raises the cost of capital and accordingly lowers the valuation of the business, with the cash flow projection held fixed. For an individual company, the volatility feedback effect can be weaker or even nonexistent if shocks to this company's business are largely idiosyncratic or even countercyclical. Only companies with a large proportion of systematic business shocks show strong volatility feedback effects. Conversely, due to diversification, shocks to the equity index are mostly systematic and should thus generate the strongest feedback effect. This diversification effect suggests that the volatility feedback effect on the equity index is stronger than the average volatility feedback effect on the individual companies that constitute the index.

Third, the self-exciting jump behavior on the aggregate index can also be stronger than the average behavior of individual companies because a cross-sectional self-exciting propagation or contagion across companies can lead to a strong intertemporal self-exciting pattern on the equity index. A large negative shock or, in the extreme, a default event in one company can propagate and trigger a large negative shock for another related company, either through their structural connection (e.g., a supplier–customer relationship) or due to shared business activities and markets. In aggregation, such propagation generates the intertemporal self-exciting jump behavior that our model captures. When confined to one individual company, the analysis does not reveal as much about the cross-company propagation and mainly captures the clustering of large negative shocks for one particular company.

Taken together, different types of companies can have different stock price and stock option behaviors, and these behaviors can differ from those on the equity index. In particular, even for a company without financial leverage, its return and volatility can show a strong negative correlation if shocks to the company have fundamental impacts on the aggregate economy and induce strong volatility feedback effects. Conversely, even for a company with a large amount of debt, if the company has the capability and financing environment to either actively rebalance its financial leverage around a fixed target level or move against its business shocks, its stock price dynamics will show very little of the leverage effect described by Black (1976). Finally, we expect to see a stronger volatility feedback effect on the equity index than on an average individual company due to the diversification effect, and we may also see stronger self-exciting jump behavior on the equity index because of the index's aggregation of cross-company risk propagations.

Given the distinct behaviors of the stock index and individual stocks, it is important for future research to develop theoretical models for individual stocks that accommodate both market-wide and firm-specific shocks. It is also important to perform a comprehensive empirical analysis of individual stock options and their linkage to the company's fundamental characteristics and aggregate market conditions. Although such a comprehensive analysis is beyond the scope of this article, we apply our equity index model to a selected number of individual companies to gain a preliminary understanding of how individual stock options behave differently across companies that are in different business sectors and pursue different capital structure policies.

III. Data and Estimation on Equity Index and Selected Single Names

We estimate the model on the S&P 500 index (SPX) options and also on individual stock options for the 5 selected companies. Estimating the model on the index options extracts the three market risk factors and identifies the values of the structural model parameters that govern the market risk dynamics. Estimating the model on individual stock options reveals how the three economic channels show up differently in different types of companies.

A. Data Sources and Sample Choices

The SPX index options are both listed on the Chicago Board of Options Exchange (CBOE) and traded actively over the counter (OTC). Options transactions on the listed market are concentrated at short maturities, whereas activities on the OTC market are more on long-dated contracts. We estimate the model using OTC SPX options data from a major bank. The data are in the form of Black and Scholes (1973) implied volatility quotes from Jan. 8, 1997 to Oct. 29, 2014. At each date, the quotes are on a matrix of 8 fixed time to maturities at 1, 3, 6, 12, 24, 36, 48, and 60 months and 5 relative strikes at each maturity at 80%, 90%, 100%, 110%, and 120% of the spot index level. The OTC quotes are constructed to match listed option prices at short maturities and OTC transactions at long maturities. We choose the OTC quotes for model estimation mainly because they cover a much wider span of maturities than do the listed options. The wider maturity span helps us achieve a better disentanglement of the different mechanisms that our model incorporates. The data are available daily, but we sample the data weekly every Wednesday to avoid weekday effects. The sample contains 40 implied volatility series over 930 weeks, a total of 37,200 observations.

For application to individual companies, we choose 5 sectors where we expect to see distinct behaviors in terms of both business types and capital structure policies. Within each sector, we select one company that has actively traded stock options over the same sample period. The 5 chosen companies are as follows:

1. General Electric Company (GE), one of the companies with the most actively traded stock options throughout the sample period within the "Industrials" sector. GE is a large infrastructure company that operates in many different segments, including power and water, oil and gas, energy management, aviation, health care, transportation, and appliances and lighting. It also has a capital segment offering financial services.

2. Wal-Mart Stores Inc. (WMT), one of the companies with the most actively traded stock options throughout the sample period within the "Staples" sector. The company operates retail stores in various formats worldwide.

3. JPMorgan Chase & Co. (JPM), one of the largest bank holding and financial service companies worldwide.

4. Duke Energy Corporation (DUK), one of the companies with the most actively traded stock options throughout the sample period within the "Utilities" sector. The company operates through three segments: regulated utilities, international energy, and commercial power.

5. Exxon Mobil Corporation (XOM), one of the companies with the most actively traded stock options throughout the sample period within the "Energy" sector. The company explores for and produces crude oil and natural gas in the United States, Canada, South America, Europe, Africa, Asia, and Australia/Oceania.

Manufacturing companies and bank holding companies tend to show different behaviors in managing their capital structures in response to market conditions (Adrian and Shin (2010)). We use GE as a representative manufacturing company and JPM as a representative bank holding company. We are also interested in knowing how a large retail store like WMT differs in behavior from a manufacturing company. In addition, we expect distinct behaviors from a company operating in a regulated utility market such as DUK, which tend to have stable profits and stable financial leverage. We also include an oil and gas exploration company (XOM). Energy companies like XOM tend to have long and risky investment horizons for their projects and lower financial leverage due to the inherent business risk. Furthermore, shocks to energy prices not only affect the profitability of such energy exploration companies but also have reverberating impacts on the whole economy.

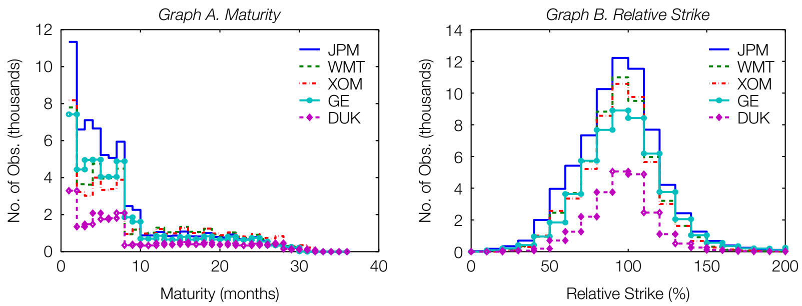
To estimate the model on each selected company, we obtain listed stock options data from OptionMetrics. Listed options on individual stocks are American-style options. OptionMetrics uses a binomial tree to estimate the option-implied volatility that accounts for the early exercise premium. We estimate our model based on the implied volatility estimates. At each maturity and strike, we take the implied volatility quote of the out-of-the-money option (call option when the strike is higher than the spot; put option when strike is lower than the spot) and convert it into a European option value based on the Black and Scholes (1973) pricing formula.[3] We further filter the data by requiring that i) the time to maturities of the chosen options are greater than 21 days, and ii) the log strike deviation from the log forward is within 2 standard deviations of its mean.

Figure 1 summarizes the distribution of the selected data sample across different brackets of maturities and relative strikes via histogram plots. The legends in the graph show the tickers of the 5 selected companies in descending order in terms of the total number of selected data observations, which are 71,344 for JPM, 54,859 for WMT, 52,918 for XOM, 52,592 for GE, and 22,785 for DUK. The maturity histograms in the left panel show that the listed options are concentrated at short maturities. The number of observations declines rapidly when option maturities are longer than 12 months. The longest maturities are less than 3 years. The relative strike histograms in the right panel show that the strikes center around the spot level but spread far apart and can be 50% below or above the spot level.

FIGURE 1

Maturity and Relative Strike Distribution of Individual Stock Options Sample

Graph A of Figure 1 plots the histogram of the time to maturities of the selected options data sample for each company. Graph B plots the histogram of the relative strikes of the selected options data sample for each company. The legends list the companies in descending order based on the total number of selected option observations.



B. Summary Statistics of Different Financial Leverage Measures

Because the model decomposes equity value F into asset value A and the equity-to-asset ratio X, model identification would be stronger if we could include financial leverage as an observed series. The issue is that it is difficult to obtain accurate and timely estimates of financial leverage. In Table 1, we construct three alternative measures, A_1–A_3, for the equity-to-asset ratio based on accounting

[3]See Carr and Wu (2009) for a detailed discussion on the various considerations involved in processing American-style individual stock options.

and financial market data, and we report the summary statistics for both the 5 individual companies and the S&P 500 index. The associated financial data are obtained from Bloomberg.

Panel A of Table 1 reports the statistics on the A_1 measure, which is constructed based on book value of total debt (TD) and total assets (TA), $A_1 = 1 - \mathrm{TD}/\mathrm{TA}$. Book values can deviate significantly from market values; nevertheless, the data are readily available from quarterly balance sheet statements, and the summary statistics provide us with a general picture of the company's level of financial leverage. Among the 5 companies, GE has the lowest mean estimate at 0.48 and hence the highest book leverage. By contrast, the mean estimate for XOM is very high at 0.94, suggesting that the company has very little debt. WMT, DUK, and JPM have similar average levels from 63% for JPM to 73% for WMT, close to the average estimate on the equity index at 66%. The largest time variation comes from the company with the highest debt ratio (GE), with a standard deviation estimate of 7%, whereas the smallest standard deviation comes from the retail store WMT at 2%.

Panel B of Table 1 reports the statistics on the A_2 measure, constructed based on book value of total debt and market capitalization (MC), $A_2 = \mathrm{MC}/(\mathrm{MC} + \mathrm{TD})$. Whereas the market value of a company's debt is not always readily observable, the market capitalization on the company's equity is readily available. The measure A_2 uses the market capitalization to represent the equity value but retains the book value of total debt as the debt amount. In terms of market capitalization, JPM now has the lowest average estimate and thus the highest financial leverage. By contrast, WMT's average financial leverage looks much lower in terms of market capitalization than its book-value counterpart. XOM's financial leverage looks even smaller in terms of market capitalization. The standard deviation

TABLE 1

Summary Statistics of Alternative Financial Leverage Measures

Table 1 reports summary statistics on three alternative financial leverage measures, A_1–A_3, constructed using financial and accounting data to proxy the equity-to-asset ratio X that we use in our model. The statistics are computed over the sample period from Jan. 1997 to Oct. 2014. Data are from Bloomberg.

Statistics	GE	WMT	JPM	DUK	XOM	SPX
Panel A. A_1 in Book Value of Total Debt and Total Asset						
Mean	0.48	0.73	0.63	0.69	0.94	0.66
Median	0.51	0.73	0.63	0.70	0.94	0.63
Standard deviation	0.07	0.02	0.04	0.04	0.03	0.05
Minimum	0.34	0.68	0.52	0.60	0.87	0.61
Maximum	0.56	0.81	0.73	0.78	0.97	0.77
Panel B. A_2 in Book Value of Total Debt and Market Capitalization						
Mean	0.49	0.86	0.20	0.60	0.96	0.61
Median	0.49	0.84	0.20	0.59	0.96	0.60
Standard deviation	0.14	0.05	0.05	0.09	0.02	0.07
Minimum	0.12	0.75	0.09	0.33	0.90	0.44
Maximum	0.75	0.96	0.32	0.78	0.98	0.71
Panel C. A_3 in Merton's Standardized Distance to Default						
Mean	1.48	3.05	0.60	2.28	4.39	2.83
Median	1.39	2.95	0.40	2.62	4.32	2.57
Standard deviation	1.06	1.23	1.16	1.11	1.38	1.21
Minimum	−1.08	0.81	−1.99	−0.33	1.38	0.19
Maximum	3.52	5.81	2.64	4.26	6.00	5.36

estimate remains the largest for GE but the lowest for XOM, which has very little debt compared with its market capitalization. The average level for the index is similar to that for DUK but with a lower standard deviation.

Even with fixed debt principal, the market value of debt can fluctuate, not only with the market capitalization but also with the riskiness of the investment. Merton (1974) proposes to value the company by treating the equity as a call option on the company's asset and by measuring the credit risk of the company with a standardized financial leverage measure called distance to default, which scales the log distance between the company's asset value and debt principal by the asset return volatility. We perform a simple implementation of the Merton model as in Bai and Wu (2016) by taking the total debt (TD) as the debt principal, the market capitalization (MC) as the equity value, and the estimate of 1-year stock return historical volatility as the equity return volatility (σ_E) while assuming zero interest rate and a 10-year debt maturity ($T=10$). With these assumptions, we solve the asset value (A) and asset return volatility (σ_A) at each date from the following two equations:

$$\text{(17)} \qquad \text{MC} = A \times \text{N}(\text{DD} + \sigma_A\sqrt{T}) - \text{TD} \times \text{N}(\text{DD}),$$

$$\text{(18)} \qquad \sigma_E = \text{N}(\text{DD} + \sigma_A\sqrt{T})\sigma_A(A/\text{MC}),$$

where $\text{N}(\cdot)$ denotes the cumulative standard normal distribution, and the standardized variable DD, often referred to as the distance-to-default measure, is as follows:

$$\text{(19)} \qquad \text{DD} = \frac{\ln(A/\text{TD}) - \frac{1}{2}\sigma_A^2 T}{\sigma_A\sqrt{T}}.$$

We use the estimated distance to default as a standardized financial leverage measure $A_3 = \text{DD}$ and report its summary statistics in Panel C of Table 1. Both A_2 and A_3 are constructed using book value of total debt and market capitalization, except the A_3 measure is scaled by asset return volatility. Comparing their summary statistics, we observe that the sample averages for A_2 and A_3 share the same cross-sectional rank. However, due to the scaling by asset return volatility, the standard deviation estimates for A_3 are close to 1 for all companies.

C. Summary Statistics of Option-Implied Volatilities

Listed options on individual stocks have fixed strike prices and fixed expiry. Their time to maturity and moneyness vary over time, making it difficult to perform time-series analysis. OptionMetrics addresses this issue by constructing time series of implied volatilities at fixed time to maturities and fixed options delta via nonparametric smoothing across nearby contracts. From these smoothed data, we take the average of the 3-month 50-delta call and 50-delta put series as a proxy for the implied volatility level, and we report its summary statistics for the 5 selected companies and the S&P 500 index in Panel A of Table 2. Among the 5 companies, the average implied volatility level is the highest at 32.72% for the bank holding company JPM, which has the highest financial leverage and shortest distance to default. The time-series variation of the implied volatility is also the highest for JPM, with a standard deviation estimate of 13.80%. The utility company DUK

TABLE 2
Summary Statistics of Option-Implied Volatilities

Table 2 reports summary statistics of option-implied volatilities for the 5 selected companies and the Standard & Poor's (S&P) 500 index. The statistics are computed on standardized implied volatility series from OptionMetrics for the sample period Jan. 1997–Oct. 2014. The implied volatility level in Panel A is proxied by the average of the standardized 3-month 50-delta call and 3-month 50-delta put implied volatility series. The implied volatility skew in Panel B is proxied by the difference between the 3-month 25-delta put and 3-month 25-delta call implied volatility series, in percentage of the 50-delta implied volatility level and scaled by 1.349 as an approximate measure of the standardized strike distance. The last row of each panel reports the correlation estimates between the weekly changes in each series and the corresponding weekly returns on the stock or stock index.

Statistics	GE	WMT	JPM	DUK	XOM	SPX
Panel A. 3-Month Implied Volatility Level						
Mean	27.70	24.72	32.72	22.46	23.05	19.73
Median	25.68	21.75	30.80	19.36	22.40	19.06
Standard deviation	12.00	9.62	13.80	9.59	6.64	6.67
Minimum	12.38	11.11	14.56	10.66	12.53	10.03
Maximum	108.43	54.78	94.13	71.02	67.07	57.06
Correlation	−0.73	−0.55	−0.76	−0.56	−0.65	−0.82
Panel B. 3-Month Implied Volatility Percentage Skew						
Mean	14.97	12.89	15.68	14.82	14.34	23.44
Median	14.83	12.89	15.94	14.91	13.44	23.58
Standard deviation	6.52	6.27	5.99	8.53	6.47	5.06
Minimum	−5.70	−6.33	0.40	−14.41	−2.77	5.15
Maximum	37.54	31.94	35.32	38.40	33.99	35.52
Correlation	0.02	0.22	0.08	0.14	0.22	−0.10

has the lowest average implied volatility level, due to a combination of low risk for the regulated business and a moderate level of financial leverage. Although the energy company XOM has very little financial leverage, its average implied volatility level is not the lowest, potentially due to its high business risk; nevertheless, its time-series standard deviation is the lowest at 6.64%. Finally, because of the diversification effect, the S&P 500 index has the lowest average implied volatility level at 19.73%.

The last row of Panel A in Table 2 reports correlation estimates between weekly stock return and weekly changes in the implied volatility level. The correlation estimates are all strongly negative. Among the 5 individual companies, the most negative correlation estimate comes from the bank holding company JPM at −0.76, which has the highest financial leverage. Conversely, even with very little financial leverage, XOM also shows a strongly negative correlation estimate at −0.65, highlighting the contribution of the volatility feedback effect even in the absence of financial leverage. Finally, at −0.82, the return volatility correlation for the equity index is more negative than for any of the selected companies, despite the fact that the index has an "average" level of financial leverage. Under our model, this stronger negative correlation for the equity index can come from two sources. First, the volatility feedback effect is stronger for the equity index because shocks to the index portfolio are mostly systematic due to diversification and thus generate the strongest impact on the cost of capital and, accordingly, market valuation. Second, the self-exciting jump behavior can also be stronger for the index due to aggregation of cross-company propagation of large negative shocks through structurally connected or related businesses.

Negative return volatility correlations, among other things, generate negative skewness for the stock return distribution, which can show up in options as a negative implied volatility skew when the implied volatility at the same maturity is plotted against some measure of moneyness. We construct an option-implied volatility skew measure by taking the difference between the 3-month 25-delta put and 25-delta call implied volatilities, in percentages of the 3-month 50-delta call and put average implied volatility level, and scaling the percentage difference with the standardized strike distance between the two series, approximated based on the delta as $N^{-1}(0.75) - N^{-1}(0.25) = 1.349$. This skewness measure is more positive when the risk-neutral return distribution is more negatively skewed. Panel B of Table 2 reports the summary statistics on this skewness measure. Among the single names, JPM not only has the highest average implied volatility level but also shows the strongest negative skewness, highlighting the contribution from its large financial leverage. Conversely, the stock index shows much higher negative skewness than any of the single names, potentially due to its stronger volatility feedback effect and self-exciting jump behavior. Furthermore, the skew estimates for the index stay negative for the whole sample period and show the smallest intertemporal variation. The skew estimates for the individual stocks, however, have larger standard deviation estimates and can switch signs over time, except for JPM.

We also measure the correlation between weekly stock return and weekly changes in the implied volatility skew and report the statistics in the last row of Panel B of Table 2. The correlation estimate for the equity index is negative at -0.10, suggesting that a downturn in the stock market is not only associated with heightened option-implied volatility level but can also be associated with a more negatively skewed risk-neutral return distribution. Under our model, the self-exciting jumps can contribute to this behavior: A negative jump in the market increases the intensity of more negative jumps to come in the future, thus raising the negative skewness of the return distribution. By contrast, the correlation estimates for the single names are all positive, suggesting that the self-exciting effect is weaker for individual companies.

Different from the listed options, the OTC SPX option-implied volatility quotes that we use for the equity index model estimation are in fixed time to maturities and relative strikes in the percentage of the spot index level. The quotes also cover a much wider range of maturities from 1 month to 5 years, thus allowing us to gain a better understanding of the index option's term structure behavior. To capture the term structure of the implied volatility level, we take the implied volatility level at 100% strike for each maturity and report the summary statistics in Panel A of Table 3. The average implied volatility shows an upward-sloping term structure pattern, starting from 19.19% at 1-month maturity to 23.52% at 5-year maturity. The upward-sloping mean term structure is often regarded as evidence of the presence of the variance risk premium. The standard deviation of the volatility series declines with the option maturity, a sign of mean reversion in the risk-neutral volatility dynamics; however, the decline is very slow, as the implied volatility series shows a significant amount of time-series variation even at 5-year maturity. The last row reports the correlation between the weekly index return and the weekly changes in each series. The correlation estimates are all strongly negative, more so at short maturities than at long maturities. Comparing the

TABLE 3
Term Structure of SPX Option-Implied Volatility Levels and Skews

Table 3 reports summary statistics on the Standard & Poor's 500 index (SPX) over-the-counter (OTC) option-implied volatility quotes at different maturities. Panel A reports the statistics on the at-the-money implied volatility quotes where strike prices are at 100% of the spot index level. Panel B reports the statistics on the implied volatility skew, defined as the difference between 80% and 120% strike implied volatilities, divided by the absolute difference in the corresponding standardized moneyness measure $d \equiv \ln(K/S)/(\mathrm{IV}\sqrt{\tau})$, where IV is the implied volatility at the relative strike K/S and time to maturity τ. The last row in each panel reports the correlation between the weekly index return and the corresponding weekly changes in the implied volatility and skew series.

	Maturity (months)							
Statistics	1	3	6	12	24	36	48	60
Panel A. Implied Volatility Level								
Mean	19.19	19.87	20.37	20.90	21.63	22.29	22.92	23.52
Median	18.31	19.26	19.83	20.66	21.39	22.05	22.64	23.28
Standard deviation	7.74	6.76	6.03	5.36	4.85	4.62	4.48	4.37
Minimum	8.46	10.43	11.09	11.88	13.04	13.85	14.54	14.90
Maximum	69.06	59.00	52.55	46.71	42.47	40.97	40.52	40.38
Correlation	−0.81	−0.83	−0.82	−0.79	−0.73	−0.69	−0.65	−0.60
Panel B. Implied Volatility Percentage Skew								
Mean	16.30	19.28	21.49	23.59	25.16	25.46	25.32	25.06
Median	16.32	19.17	21.27	23.29	24.86	25.53	25.38	24.98
Standard deviation	2.78	3.12	3.66	4.17	4.67	4.88	5.17	5.58
Minimum	7.85	10.69	11.32	12.82	13.96	12.88	14.40	12.34
Maximum	27.08	29.29	32.41	33.85	35.70	36.25	36.81	37.36
Correlation	−0.11	−0.34	−0.33	−0.21	−0.09	0.00	0.04	0.04

summary statistics on the 3-month OTC series with the corresponding statistics in Panel A of Table 2 computed from listed SPX options, we observe very similar behaviors.

From the OTC quotes at fixed relative strikes (K/S), we also compute an implied volatility skew measure by taking the difference between the 80% strike and 120% strike implied volatility, in percentage of the 100% strike implied volatility, and scaling the difference by the absolute distance in the standardized moneyness measure $d = \ln(K/S)/(\mathrm{IV}\sqrt{\tau})$, where IV denotes the implied volatility quote at the relative strike (K/S) and time to maturity τ. Panel B of Table 3 reports the summary statistics of the implied volatility skew at different maturities. The average skewness increases with the option maturity from 16.30% at 1-month maturity to 25.06% at 5-year maturity, suggesting that the risk-neutral return distribution becomes increasingly more negatively skewed at longer conditioning horizons. The minimum skew estimates are positive across all option horizons, showing that the risk-neutral index return distribution remains negatively skewed across all conditioning horizons over the whole sample period. The last row reports the correlation between weekly index return and weekly changes in the implied volatility skew. Potentially due to cleaner OTC quotes and hence better skew measurements, the correlation estimate at 3-month maturity is more negative at −0.34 than the estimate from the listed options reported in Panel B of Table 2. As with the return correlation with the volatility level, the correlation estimates peak at 3-month maturity and decline as the option maturity increases. The correlation becomes virtually 0 after 3 years. If this negative correlation is induced by the self-exciting behavior, the term structure pattern suggests that its impact on options is mainly at intermediate maturities.

D. Model Estimation and State Identification

The objective of the estimation is to identify the values of the structural parameters that govern the financial leverage and risky-asset dynamics and to extract the levels of the three state variables $(\widetilde{X}_t, v_t^Z, v_t^J)$ at different time periods. The estimates for the structural parameters help our understanding of the dynamics and interactions between the different risk sources, whereas the levels of the three state variables at different sample periods shed light on the relative contribution of each risk source at different historical times.

To estimate the model with the option observations, we cast the model into a state-space form by treating the three state variables as hidden states and the option observations as measurements with errors. Let $V_t \equiv \left[\widetilde{X}_t, v_t^Z, v_t^J\right]^\top$ denote the state vector at time t. We specify the state propagation equation based on a Euler approximation of their statistical dynamics:

$$V_t = f(V_{t-1}; \Theta) + \sqrt{Q_{t-1}}\,\varepsilon_t, \tag{20}$$

where ε_t denotes the standardized forecasting error vector; $f(V_{t-1}; \Theta)$ denotes the conditional forecasts as a function of state vector V_{t-1} and the parameter set Θ, given by

$$f(V_{t-1}; \Theta) = \begin{bmatrix} \widetilde{X}_t + \widetilde{X}_t^{1-p}(\widetilde{a}_X - \kappa_L^\top V_{t-1})\Delta t \\ \kappa_Z \theta_Z \Delta t + (1 - \kappa_Z^{\mathbb{P}} \Delta t) v_{t-1}^Z \\ \kappa_J \theta_J \Delta t + (1 - \kappa_J^{\mathbb{P}} \Delta t) v_{t-1}^J \end{bmatrix}, \tag{21}$$

with $\Delta t = 7/365$ denoting the weekly frequency of the data, $\kappa_L = [\widetilde{\kappa}_{XX}, \widetilde{\kappa}_{XZ}, \widetilde{\kappa}_{XJ}]^\top$; and Q_{t-1} denotes the forecasting error covariance matrix, which is a diagonal matrix with the three diagonal elements given by

$$Q_{t-1} = \begin{bmatrix} \widetilde{X}_{t-1}^{2-2p} \Delta t & & \\ & \sigma_Z^2 v_{t-1}^Z \Delta t & \\ & & \sigma_J^2 (v_{J-}^{\mathbb{P}})^2 v_{t-1}^J \Delta t \end{bmatrix}. \tag{22}$$

The measurement equations are specified on the option observations, with additive, normally distributed measurement errors:

$$y_t = h(V_t; \Theta) + \sqrt{R}\,e_t, \tag{23}$$

where y_t denotes the time t forward value of the out-of-the-money options computed from the implied volatility, scaled by the Black–Scholes vega of the option,[4] and $h(V_t; \Theta)$ denotes the corresponding model value as a function of the state vector V_t and the parameter set Θ. We assume that the pricing errors on the scaled option prices are independent and identically distributed (IID) normal with zero mean and constant variance.

Estimating the model on the OTC index options data involves 40 measurement equations built on the 40 implied volatility series across 5 relative strikes at

[4] See, for example, Bakshi et al. (2008) for a detailed discussion on the rationale for the option pricing transformation and scaling for model estimation.

each maturity and 8 time to maturities. When we estimate the model on the listed options for the 5 selected companies, the dimension of the measurement equation varies over time as the number of option observations, as well as their relative strikes and time to maturities, varies over time.

When the state propagation and the measurement equation are Gaussian linear, the Kalman (1960) filter provides efficient forecasts and updates on the mean and covariance of the state vector and observations. Our state-propagation equations and measurement equations do not satisfy the Gaussian and linear conditions. We use an extended version of the Kalman filter, the unscented Kalman filter, to handle the deviations. The Internet Appendix provides the technical details on the procedure.

Although we assume IID measurement errors for model estimation, the actual pricing errors from the estimated model tend to show strong serial persistence. Persistence in pricing errors makes economic sense. If the pricing errors are caused by temporary supply–demand shocks, their dissipation takes time. In general, systematic market movements tend to be more persistent than supply–demand shocks. Thus, lower persistence for the pricing errors is an indication of better performance for the model in separating systematic market movements from idiosyncratic supply–demand shocks. Furthermore, because supply–demand shocks in option contracts mainly dissipate via hedging with nearby contracts (Wu and Zhu (2016)), the pricing errors of nearby contracts tend to show a positive correlation, but the correlations tend to decline as the contract terms grow further apart. We confirm these behaviors for the pricing errors from our estimated model in the Internet Appendix.

Despite observations on pricing error persistence and cross correlation, we maintain an IID measurement error structure in equation (23) for model estimation. Bakshi and Wu (2010) and Bates (2000), among others, propose the use of more general measurement error structures to accommodate these serial and contemporaneous correlations. Our experience suggests that imposing a diagonal measurement error variance structure for model estimation brings more numerical stability to the estimation procedure and the extracted states. The intuition is similar in spirit to the idea of ridge regression. The Internet Appendix provides more details on the rationale behind this choice.

Given the forecasted option prices $\overline{y}_t$ and their conditional covariance matrix $\overline{\Sigma}_{yy,t}$ obtained from the unscented Kalman filtering, we compute the quasi-log likelihood value for each week's observation on the option prices assuming normally distributed forecasting errors:

$$(24) \qquad l_t(\Theta) \quad = \quad -\frac{1}{2}\log\left|\overline{\Sigma}_{yy,t}\right| - \frac{1}{2}\left(\left(y_t - \overline{y}_t\right)^\top \left(\overline{\Sigma}_{yy,t}\right)^{-1}\left(y_t - \overline{y}_t\right)\right).$$

We estimate the model parameters by numerically maximizing the sum of the conditional log likelihood value on each date:

$$(25) \quad \Theta \quad \equiv \quad \arg\max_{\Theta} \mathcal{L}(\Theta, \{y_t\}_{t=1}^N), \quad \text{with} \quad \mathcal{L}(\Theta, \{y_t\}_{t=1}^N) \quad = \quad \sum_{t=1}^N l_t(\Theta),$$

where N denotes the number of weeks in the sample.

The model has nine parameters $(p,\kappa_Z,\theta_Z,\sigma_Z,\rho,\kappa_J,\theta_J,\sigma_J,v_J)$ and three state variables (X_t,v_t^Z,v_t^J) to price the equity and equity index options. The model parameters are estimated to match the average shape of the option-implied volatility surfaces via the measurement in equation (23), with the three states capturing the time variation of the volatility surface. In addition, the model has six parameters $(\widetilde{a},\widetilde{\kappa}_{XX},\widetilde{\kappa}_{XZ},\widetilde{\kappa}_{XJ},\kappa_v^{\mathbb{P}},\kappa_J^{\mathbb{P}})$ to control the statistical dynamics, which dictate the state propagation equation in (20) and are hence identified by the time-series behavior of the option-implied volatility series. The differences between $(\kappa_v^{\mathbb{P}},\kappa_J^{\mathbb{P}})$ and (κ_v,κ_J) determine the market prices of the diffusion and jump variance risk (γ^v,γ^J), respectively.

IV. Economic Channels of Equity and Equity Index Variation

With the model estimated on both the index options and individual stock options for the 5 selected companies, we first analyze the dynamic behaviors of the three economic channels at the aggregate level and then examine how they show up differently in different types of companies.

A. Aggregate Market Behaviors Extracted from Equity Index Options

Table 4 reports the model parameter estimates and their standard errors (in parentheses) for the S&P 500 index. For ease of discussion, we group the parameters into three panels, with each describing the dynamics of one source of volatility risk. Given the parsimony of the specification and the large amount of data used for the model estimation, all parameters are estimated with strong statistical significance.

1. Disentangling Different Sources of Stochastic Volatility and Skew

Our model allows three distinct channels of volatility variation: i) the variation of the standardized ratio of equity to risky assets, ii) the variation of diffusion risk in the risky-asset portfolio, and iii) the variation of discontinuous risk in the

TABLE 4

Maximum Likelihood Estimates of Model Parameters for the S&P 500 Index

Table 4 reports the maximum likelihood estimates of the model parameters and their standard errors (in parentheses) on the Standard & Poor's (S&P) 500 index. The parameters are grouped into three panels, each describing the dynamics of one source of volatility risk.

Panel A. Leverage Effect

Estimates	p	κ_{XX}	κ_{XZ}	κ_{XJ}	a_X
Coefficient	3.4922	0.0000	17.4087	–0.0743	0.0009
Standard error	(0.0084)	(0.0000)	(0.0537)	(0.0000)	(0.0000)

Panel B. Volatility Feedback

Estimates	ρ	κ_Z	θ_Z	σ_Z	γ^v
Coefficient	–0.8879	3.0125	0.0251	0.5979	–17.8608
Standard error	(0.0023)	(0.0089)	(0.0000)	(0.0019)	(0.0731)

Panel C. Self-Exciting Market Disruptions

Estimates	v_J	κ_J	$\kappa_j\theta_J$	σ_J	γ^J
Coefficient	0.1951	0.0010	0.1155	5.6460	0.4513
Standard error	(0.0000)	(0.0000)	(0.0000)	(0.0107)	(0.0006)

aggregate market. The instantaneous return variance contributions from the three sources are $\widetilde{X}_t^{-2p}$, v_t^Z, and $v_J^2 v_t^J$, respectively. Based on the sample averages of the filtered state values and the parameter estimates on (p, v_J), we compute the average return variance from each source at 0.0115, 0.0257, and 0.0168, respectively, or at 10.73%, 16.04%, and 12.96% in volatility terms, respectively. Therefore, all three sources contribute to a significant portion of the index return variance.

The three sources of stochastic volatility interact dynamically with the index return and contribute to the implied volatility skew at different maturity ranges. The financial leverage effect is captured by the power coefficient p. A coefficient of 0 would imply zero dependence of return variance on the financial leverage. The larger the estimate, the stronger the leverage effect. The estimate for the power coefficient is 3.4922, suggesting a strong dependence of the index return variance on the financial leverage level.

The volatility feedback effect is captured through the instantaneous correlation (ρ) between the diffusion movement in the asset value and its variance rate. The estimate for the correlation is highly negative at -0.8879, suggesting that the volatility feedback effect is very strong at the market level. This feedback effect can generate a negative implied volatility skew at intermediate maturities. The variance rate v_t^Z is modeled as a mean-reverting square-root process, with κ_Z measuring the risk-neutral mean-reverting speed, θ_Z the risk-neutral long-run mean, and σ_Z the volatility of the volatility coefficient. The estimate of $\kappa_Z = 3.0125$ indicates that this variance-rate process is highly mean-reverting under the risk-neutral measure. As a result, shocks on the variance rate v_t^Z dissipate quickly as the option maturity increases. The mean estimate of $\theta_Z = 0.0251$ implies a return volatility contribution of $\sqrt{\theta_Z} = 15.83\%$ under the risk-neutral measure from this particular variance rate. The estimate for the volatility of the volatility coefficient is at $\sigma_Z = 0.5979$, which contributes to the curvature of the implied volatility smile at intermediate maturities.

Parameter estimates for the self-exciting downside jump dynamics are summarized in Panel C of Table 4. The average size of the downside jump under the risk-neutral measure is governed by v_J, which is estimated to be at 19.51%. The negative jump directly induces a negative skewness in the risk-neutral return distribution and, accordingly, a negative implied volatility skew at short maturities. The self-exciting behavior of the downside jump extends its impact to longer option maturities. The jump intensity v_t^J shows little mean reversion under the risk-neutral measure because the mean-reversion speed estimate is very small at $\kappa_J = 0.001$. The slow mean reversion helps sustain the jump effect to long option maturities. The degree of self-excitement is measured by σ_J, which measures the magnitude of the intensity of the upside jump per downside jump in the asset return. The large estimate of $\sigma_J = 5.6460$ indicates that a downside jump in the asset return evokes a very large upside jump in the jump intensity itself, highlighting the significance of the self-exciting behavior.

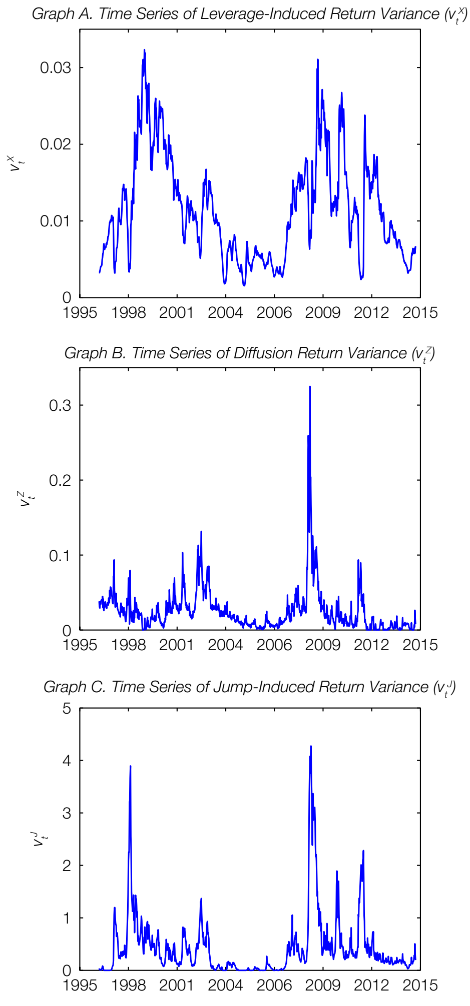
2. Dissecting the Sample Variation of Different Sources of Volatility Risks

Figure 2 plots the time series of the three state variables extracted from the equity index options data. The standardized financial leverage ($\widetilde{X}_t$) contributes to the return variance through the transformed variance rate $v_t^X = \widetilde{X}_t^{-2p}$, which is

FIGURE 2

Time Series of the Three Sources of S&P 500 Index Return Variance

In Figure 2, the time series of the three variance rates (v_t^X, v_t^Z, v_t^J) are extracted from the observed Standard & Poor's (S&P) 500 index option-implied volatility quotes using the unscented Kalman filter under the estimated model parameters. Each graph is for a single variance rate.



plotted in Graph A. The time series for the diffusion variance rate v_t^Z and the jump intensity v_t^J are plotted in Graphs B and C, respectively.

Inspecting the time-series plots, we can trace various historical events to variations in the three variance rates. For example, the 1997 Asian crises were associated with a moderate spike in the diffusion variance rate (v_t^Z) and a small spike in the jump arrival rate v_t^J, followed by a downward drop in the financial-leverage-induced variance rate v_t^X. These variations suggest that the Asian crises increased perceived business risks in the United States in terms of both diffusion-type fluctuations and downside jump risks. The heightened business risks were followed by an immediate deleveraging process that sharply lowered the level of financial leverage and its associated volatility contribution.

The hedge fund crisis in late 1998 was also associated with a moderate spike in the diffusion variance rate, but the associated spike in the jump intensity v_t^J was much larger, surpassed only by the spike during the 2008 financial meltdown, suggesting that market participants were highly concerned with the potential impact of the hedge fund crisis on the U.S. financial system. The 1998 crisis was followed by a deleveraging process that resulted in another sharp drop in the leverage-induced variance rate v_t^X. After this, the leverage-induced variance rate increased steadily and reached its maximum during mid-1999. This period of increasing leverage was associated with declining perceived business risks of both types (v_t^Z and v_t^J).

The burst of the NASDAQ bubble in early 2000 started a protracted deleveraging process as v_t^X started a slow downward trend. This trend was reverted in mid-2007. The 2003 economic recession induced heightened levels for the diffusion variance rate, but the market showed only moderate concern for downside jump risk. The jump-risk concern stayed low during the extended period from mid-2003 to 2007.

The 2008–2009 financial meltdown drew the largest spikes for both the diffusion variance rate v_t^Z and the jump intensity v_t^J. The equity market went down so much that despite market-wide deleveraging efforts, the aggregate financial leverage stayed high. The market experienced jitters in 2010 and 2011, amid European debt crisis, before it finally started to calm down at the end of the sample period.

The time-series analysis shows that volatility variations at different sample periods come from different risk sources. The 2003 recession is mostly associated with an increase in diffusion-type economic uncertainty, whereas the 1998 hedge fund crisis is perceived as having the potential of rocking the stability of the financial system, as shown by the large spike in the jump intensity. The 2008–2009 financial meltdown, however, induces the largest spikes in both types of variations. Furthermore, market-wide deleveraging often follows when the perceived downside jump risk increases.

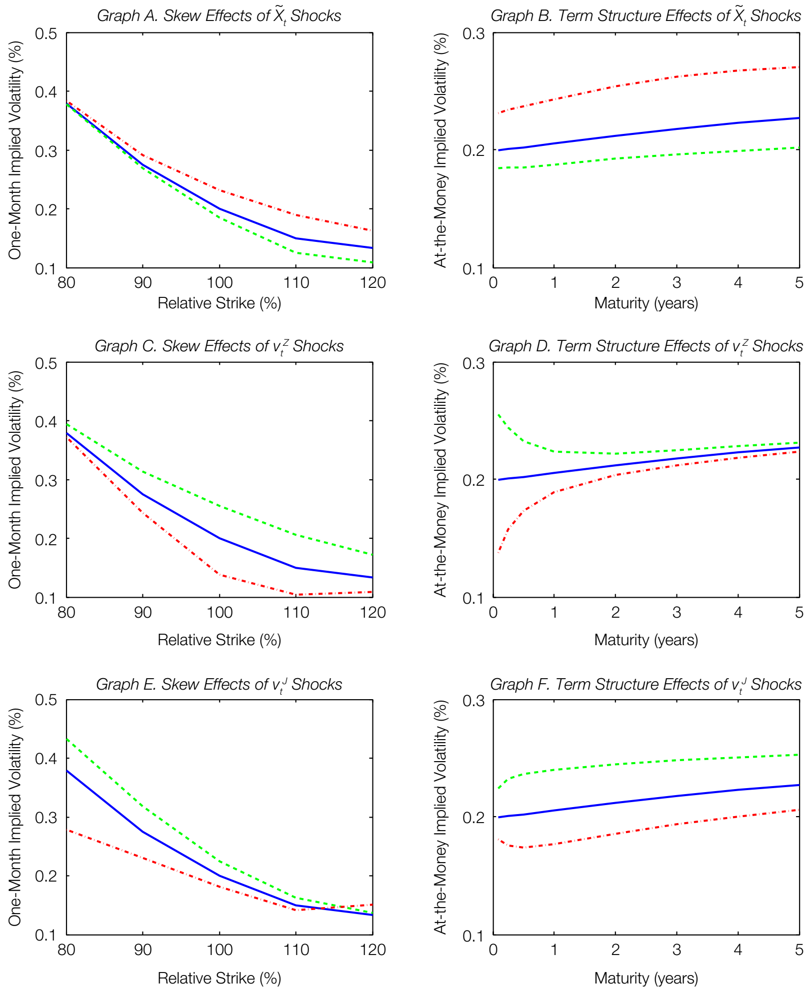
3. Differentiating Volatility Responses to Different Types of Economic Shocks

To understand how the volatility surface responds differently to shocks from the three risk sources, we shock each of the three state variables $(\widetilde{X}_t, v_t^Z, v_t^J)$ from its mean level to the 10th percentile and 90th percentile, respectively, while holding the other two at their respective mean levels. Figure 3 plots the responses of the volatility skew and term structure to shocks from the three risk sources, with one source in each graph. In each graph, the left panel plots the response of the 1-month implied volatility skew, and the right panel plots the response of the at-the-money (100% relative strike) implied volatility term structure. The solid lines represent the model-generated implied volatility values when evaluated at the sample averages of the state variables, the dashed lines represent the model-generated values when evaluated at the 90th percentile of the state variable, and the dash-dotted lines represent responses to a shift to the 10th percentile for the state variable.

FIGURE 3

Shocks and Implied Volatility Responses on S&P 500 Index Options

The solid lines in Figure 3 represent implied volatilities generated from the estimated model on the Standard & Poor's (S&P) 500 index evaluated at the sample average of the state variables. The dashed lines are obtained by setting one state variable to its 90th percentile while holding the other two to their averages. The dashed-dotted lines are obtained by setting one state variable to its 10th percentile while holding the other two to their averages.



When we shock the standardized equity-to-asset ratio $\widetilde{X}_t$ from its sample average to its 90th-percentile value, financial leverage is reduced, and its return variance contribution (X_t^{-2p}) is reduced accordingly. Thus, the dashed lines in Graph A of Figure 3 are below the corresponding solid lines. When the equity-to-asset ratio is reduced to its 10th-percentile value, financial leverage increases, and implied volatilities (dash-dotted lines) increase, moving above the solid line. The left panel shows that shocks to $\widetilde{X}_t$ affect the implied volatilities more at-the-money than out-of-the-money. As a result, a positive shock to financial leverage increases the at-the-money implied volatility level while reducing the implied volatility skew. The right panel shows that the responses of the at-the-money implied volatilities to financial leverage variation are relatively uniform across maturities. A shock in $\widetilde{X}_t$ induces a near-parallel shift in the at-the-money implied volatility term structure.

When the diffusion variance rate v_t^Z experiences a positive shock, its variance contribution increases. In response, option-implied volatilities move up, but the implied volatility skew declines due to the increase of the diffusion component relative to the jump component. This effect shows up clearly in the left panel of Graph B in Figure 3, where the dashed line is above the solid line but becomes slightly less skewed. A negative shock, conversely, reduces the volatility level but increases the steepness of the skew. The term structure plot on the right side highlights the transient nature of v_t^Z due to its high risk-neutral mean-reversion speed ($\kappa_Z = 3.0125$). Shocks to v_t^Z induce large responses at short maturities, but the responses decline quickly as the option maturity increases.

When the arrival rate of the downside jump v_t^J experiences a positive shock, return variance increases from the contribution of the jump component. Furthermore, a positive shock to v_t^J also increases the negative skewness of the instantaneous return distribution. Thus, as shown in the left panel of Graph C in Figure 3, a positive shock to v_t^J not only raises the implied volatility level but also steepens the negative skew by raising the low-strike implied volatility more than it raises the high-strike implied volatility. The right panel in Graph C shows a hump-shaped term structure effect, where the response is the largest at 3- to 6-month maturities.

The response analysis shows that all three sources of variation contribute to the return variance, but their contributions are quite different across different strikes and maturities. Along the strike dimension, financial leverage ($\widetilde{X}_t$) variation affects at-the-money volatility more than out-of-the-money volatility, and diffusion variance (v_t^Z) variation affects volatility at high strikes more than at low strikes, but the jump-intensity (v_t^J) variation affects volatility at low strikes more than at high strikes. Across maturities, shocks from the diffusion variance rate are the most transient, and thus their impacts are mostly on short-term options; by contrast, both the self-exciting downside jumps and the leverage effect have long-lasting impacts at both short and long maturities. These different response patterns allow us to disentangle the three sources of risk by using options across a wide span of strikes and maturities.

Linking the shock and response plots in Figure 3 to the index option summary behaviors in Table 3, we note that all three sources of variation contribute to the negative correlation between the index return and changes in the implied volatility level. The fact that the correlation declines with increasing option maturity highlights the contribution of the transient volatility feedback effect. In contrast, the self-exciting jump behavior contributes to the negative correlation between the index return and changes in the implied volatility skew. In particular, the hump-shaped correlation term structure pattern is consistent with the similar term structure pattern for the response function to the self-exciting risk factor v_t^J.

4. Decomposing Different Sources of Equity Index Variance Risk Premium

Several studies (e.g., Bakshi and Kapadia (2003a), (2003b), Carr and Wu (2009)) have documented strongly negative variance risk premiums on stock indexes. Several ensuing studies propose explanations for the negative variance risk premium (e.g., Bakshi and Madan (2006), Drechsler and Yaron (2011)) or explore the variance risk premium as a predictor for other financial behaviors (e.g., Bollerslev et al. (2011), Bollerslev et al. (2009), and Zhou (2010)). Missing from these studies is the realization that variance risk and hence the variance risk premium can come from several distinct sources. Our model decomposes the index return into three risk sources (W_t, Z_t, J_t), all contributing to the stochastic return variance and thus the return variance risk premium. Based on the parameter estimates and the extracted state variables, we calculate the average contribution of each risk source to the variance risk premium.

The standardized financial leverage factor $\widetilde{X}_t$ generates an instantaneous index return variance of $v_t^X = \widetilde{X}_t^{-2p}$. The instantaneous risk-neutral drift of v_t^X is $\mu(v_t^X)^{\mathbb{Q}} = p(2p+1)(v_t^X)^2$, and the corresponding statistical drift is $\mu(v_t^X)^{\mathbb{P}} = p(2p+1)(v_t^X)^2 - 2p(v_t^X)^{3/2}(\widetilde{a}_X - \widetilde{\kappa}_{XX}X_t - \widetilde{\kappa}_{XZ}v_t^Z - \widetilde{\kappa}_{XJ}v_t^J)$. The sample averages of the two drifts are 0.0050 and 0.0089, respectively, generating a slightly positive average risk premium of 0.0039. Thus, the observed negative variance risk premium does not come from the variance risk induced by financial leverage variations.

The diffusive component of the asset return contributes to the instantaneous index return variance by v_t^Z, which follows a square-root diffusion dynamics, with the instantaneous risk premium on v_t^Z given by $\gamma^v \sigma_Z v_t^Z$. With a negative estimate on the market price $\gamma^v = -17.8608$, the instantaneous risk premium averages at -0.2746.

Finally, the jump component of the asset return contributes to the instantaneous index return variance by $(v_J)^2 v_t^J$ under the risk-neutral measure and by $(v_J^{\mathbb{P}})^2 v_t^J$ under the statistical measure. The difference is induced by the market price of jump risk γ^J, which is estimated at $\gamma^J = 0.4513$. The sample averages of the two variance series are 0.0168 (12.96% in volatility term) and 0.0142 (11.91% in volatility term), respectively, thus generating a negative variance risk premium of -0.0026. Furthermore, the market price of the jump risk also induces a difference between the statistical and risk-neutral drifts of the jump-intensity process v_t^J at $\sigma_J(v_J^{\mathbb{P}} - v_J)v_t^J$, which averages at -0.0393.

With the results taken together, our model estimation confirms the existence of a negative variance risk premium on the equity index, but it also highlights its complex composition. The instantaneous variance risk premium is a combined result of four different types of risk premiums. When one investigates the variance risk premium over different investment horizons and using different variance instruments (e.g., 1-month investment in 1-month variance swap as in Carr and Wu (2009) or 2-month investment in 2- to 12-month variance swaps as in Egloff, Leippold, and Wu (2010)), these four sources of instantaneous variance risk premium further interact with the corresponding variance dynamics to determine the expected excess returns. Because the diffusion variance rate v_t^Z is more transient than the jump arrival rate v_t^J, we expect the risk premium from the diffusion variance rate to dominate investments on short-term variance swap contracts. By contrast, excess returns on long-term variance swap contracts can be dominated by risk premiums from the persistent jump intensity v_t^J and the mean-repelling financial leverage variation $\widetilde{X}_t$. This decomposition is consistent with the empirical findings of Egloff et al. (2010) that short-term variance swap contracts generate more negative risk premiums than do long-term variance swap contracts.

5. Detecting Aggregate Capital Structure Behaviors from Equity Index Options

The model uses four parameters to capture how the standardized equity-to-asset ratio $\widetilde{X}_t$ responds to the levels of financial leverage and the two types of business risks, the diffusion risk v_t^Z and the downside jump risk v_t^J. Parameter $\widetilde{\kappa}_{XX}$ measures the dependence of the capital structure decision on the current leverage level. A large positive estimate $\widetilde{\kappa}_{XX}$ would suggest mean-reverting behavior in capital structure decisions, as proposed by, for example, Collin-Dufresne and Goldstein (2001). The maximum likelihood estimate for $\widetilde{\kappa}_{XX}$ is virtually 0, suggesting that for the aggregate market, changes in the standardized financial leverage are largely independent of its current level.

Parameter $\widetilde{\kappa}_{XZ}$ captures how the capital structure target responds to diffusion business risk v_t^Z. Interestingly, the estimate is strongly positive at $\widetilde{\kappa}_{XZ} = 17.4087$, suggesting that when the diffusion-type fluctuations increase, $\widetilde{X}_t$ declines, and hence the financial leverage actually increases. Conventional wisdom holds that one wants to reduce financial leverage to mitigate risk when business risk increases; however, our estimation suggests that for the aggregate market, financial leverage does not decrease but rather increases when diffusion-type business risk increases.

On the other hand, the capital structure response to jump business risk is estimated to be negative at $\widetilde{\kappa}_{XJ} = -0.0743$. The market reduces financial leverage when the expected downside jump risk increases but increases financial leverage when the diffusion risk increases. Therefore, the main concern with using financial leverage is not normal daily business fluctuations but instead unexpected, large downside jumps that can lead to a self-exciting spiral.

Traditional corporate finance often links financial leverage targets to the riskiness of the underlying business. Our model decomposes business risk into two types and allows the financial leverage decision to respond differently to the two types of business risk. Model estimation shows that the aggregate financial leverage in the U.S. market does not always decline with increased business risk. Instead, financial leverage can actually increase with increasing business risk if the risk is driven by small, diffusive market movements. Only when the perceived risk of self-exciting market disruptions increases does the market become truly concerned and start the deleveraging process.

B. Cross-Sectional Variation in Individual Stock Volatility Dynamics

Table 5 reports the model parameter estimates and standard errors when the model is applied to match individual stock options on the 5 selected companies. The cross-sectional variation in the parameter estimates highlight how the three economic channels show up differently for different types of companies.

TABLE 5
Maximum Likelihood Estimates of Model Parameters for Selected Companies

Table 5 reports the maximum likelihood estimates of the model parameters and their standard errors (in parentheses) for selected companies, using options data on each company's stock.

Coef.	GE	WMT	JPM	DUK	XOM
Panel A. Leverage Effect					
p	3.8216	2.6433	1.6368	1.1315	0.5777
	(0.0011)	(0.0099)	(0.0008)	(0.0029)	(0.0008)
κ_{XX}	0.0002	0.0617	0.7465	0.0000	0.0038
	(0.0028)	(0.0011)	(0.0118)	(0.0000)	(0.0029)
κ_{XZ}	18.5264	−2.1296	15.6550	20.2717	4.5964
	(0.0331)	(0.2682)	(0.0059)	(2.5409)	(1.2206)
κ_{XJ}	−0.0901	−0.0976	−0.1386	−0.0917	−0.0983
	(0.0000)	(0.0319)	(0.0420)	(0.2292)	(0.0634)
a_X	0.0003	0.0000	0.0000	0.0000	0.0026
	(0.0013)	(0.0000)	(0.0000)	(0.0000)	(0.0001)
Panel B. Volatility Feedback					
ρ	−0.2333	−0.4619	−0.4542	−0.3610	−0.9637
	(0.0029)	(0.0027)	(0.0012)	(0.0044)	(0.0010)
κ_Z	3.6640	0.2143	1.4070	7.2309	6.6415
	(0.0147)	(0.0131)	(0.0023)	(0.0402)	(0.0198)
θ_Z	0.0286	0.0793	0.0485	0.0174	0.0091
	(0.0001)	(0.0002)	(0.0001)	(0.0002)	(0.0000)
σ_Z	0.6285	0.2204	0.5541	1.0610	0.9138
	(0.0033)	(0.0029)	(0.0013)	(0.0084)	(0.0011)
γ_V	−15.2698	−15.4399	−6.9716	−0.0159	−11.8360
	(0.2768)	(0.4942)	(0.1438)	(0.2696)	(0.0495)
Panel C. Self-Exciting Market Disruptions					
v_J	0.1728	0.2581	0.5043	0.3885	0.2856
	(0.0003)	(0.0003)	(0.0000)	(0.0000)	(0.0000)
κ_J	0.0008	0.0077	0.0004	0.0000	0.0023
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$\kappa_J\theta_J$	0.1055	0.1926	0.2797	0.2149	0.4811
	(0.0002)	(0.0028)	(0.0008)	(0.0025)	(0.0007)
σ_J	5.0776	29.2038	18.9226	12.3199	19.8787
	(0.0290)	(0.0412)	(0.0016)	(0.0024)	(0.0002)
γ_J	0.4565	0.1073	0.7829	2.3948	0.3964
	(0.0022)	(0.0100)	(0.0030)	(0.1229)	(0.0034)

1. Cross-Sectional Variation of Capital Structure Decisions and the Leverage Effect

From the estimates on the leverage effect coefficient p in Panel A of Table 5, we observe that the financial leverage effect is the strongest for the manufacturing company GE, with $p = 3.8216$. The leverage effect remains strong for the retail company WMT at $p = 2.6433$ but is markedly weaker for the bank holding company JPM at $p = 1.6368$. Among the three companies, JPM has the highest financial leverage but the lowest leverage effect. Manufacturing companies tend to have the same debt for a long period of time, without active rebalancing on their capital structures. As a result, their financial leverage tends to vary passively with the stock price fluctuation, generating the classic leverage effect as described by Black (1976). By contrast, financial firms tend to actively manage their capital structures according to changes in market conditions. Bank holding companies also need to maintain a certain level of financial leverage to satisfy regulatory requirements.

These active financial leverage management practices mitigate Black's leverage effect.

The leverage effect is even weaker for the utility company DUK at $p=1.1315$. Given the regulated business, utility companies tend to have stable profit streams and stable stock prices. The financial leverage variation is small as a result. The weakest financial leverage effect comes from XOM, which does not have much debt to begin with.

The 5 companies also show distinct behaviors in terms of how they vary their financial leverage according to risk conditions. Among the 5 companies, the bank holding company JPM is the only one that shows strong mean-reverting behavior, with $\kappa_{XX}=0.7465$, consistent with regulatory requirements on capital ratio targets.

Four out of the 5 companies increase their financial leverage when the diffusion risk increases, with large positive estimates on κ_{XZ}. The only exception is the noncyclical company WMT, with $\kappa_{XZ}=-2.2196$. The different estimates suggest that it is possible that companies alter their financial leverage not only based on their own firm-specific situations but also with general market conditions.

Conversely, all 5 companies generate negative estimates for κ_{XJ}, suggesting that they all reduce their financial leverage when the perceived crash risk increases. This response is particularly strong for the bank holding company JPM. Thus, reducing leverage in the presence of increased crash risk is not only a market behavior but is also applicable to the situation of each individual company.

2. Cross-Sectional Variation of Volatility Feedback and Self-Exciting Behaviors

Panel B of Table 5 reports parameter estimates related to the volatility feedback effect and the diffusion variance dynamics. The correlation coefficient ρ captures the volatility feedback effect. The correlation estimates for all 5 companies are less negative than that for the equity index, confirming our argument that the volatility feedback effect is stronger for the equity index than for an average individual company due to the effect of diversification. Only systematic shocks induce changes in the cost of capital and generate the volatility feedback effect. Although shocks to the equity index are mostly systematic, idiosyncratic shocks to individual companies do not induce the feedback effect. The different degrees of the volatility feedback effect from different types of companies highlight their different contributions to the market risk. Among the 5 selected companies, the correlation estimate is the most negative, at -0.9637, for the energy company XOM, suggesting that shocks to energy can have reverberating effects on the aggregate economy.

The diffusion variance dynamics also show cross-sectional variation. Among the 5 companies, the diffusion variance for the noncyclical company WMT shows the most risk-neutral persistence with the smallest κ_Z estimate, at 0.2143, and the smallest instantaneous variation with the smallest σ_Z estimate, at 0.2204.

Panel C of Table 5 reports the parameter estimates that govern the self-exciting market disruptions. Although we regard the self-exciting behavior as an aggregate market behavior, its effect shows up in all 5 selected individual companies. The estimates for the average crash size v_J range from 17.28% for GE to as high as 50.43% for JPM. The estimates for the degree of self-excitement σ_J

are also very large for all 5 companies, ranging from 5.0776 for GE to 29.2038 for WMT. As is the case for the stock index, the estimates for the risk-neutral mean-reversion speed κ_J are close to 0 for all 5 companies. The near-zero mean-reversion estimates help propagate the effect of the self-exciting crash risk to long option maturities.

C. Comparing Option-Implied Financial Leverage to Accounting Measures

The model decomposes equity variation into variations in financial leverage and asset value. With options but without actual financial leverage data, the estimation cannot fully identify the equity-to-asset ratio X_t and can identify only a standardized version of it, which standardizes the equity-to-asset ratio by its volatility scale δ, $\widetilde{X}_t = X_t/\delta^{1/p}$. To examine how much we can infer about the actual financial leverage variation from options data, we compare the $\widetilde{X}_t$ time series extracted from stock and stock index options to alternative financial leverage measures computed from accounting and financial data.

Table 1 computes three alternative measures of financial leverage based on accounting and financial data, with A_1 measuring X using book values of total debt and total asset, A_2 measuring X using book values of total debt and market capitalization, and A_3 proxying a standardized version of X using Merton's (1974) distance to default measure. Table 6 reports the correlation estimates, both in levels (Panel A) and in annual changes (Panel B), between the option-implied $\widetilde{X}$ series and the three alternative accounting measures A_1–A_3 for each of the 5 selected companies and the S&P 500 index. The last column reports the average correlation across the 5 companies and the index.

The average correlation between $\widetilde{X}$ and A_1 is positive but small at 11% both in levels and in annual changes. On levels, the correlation estimates are positive for GE, WMT, and XOM; negative for DUK; and virtually 0 for JPM and the index. On annual changes, the correlation estimates are all positive except for DUK. The low and sometimes negative correlation estimates suggest that

TABLE 6

Linking Option-Implied Financial Leverage to Alternative Accounting Measures

Table 6 reports the correlation between the option-implied standardized equity-to-asset ratio measure $\widetilde{X}$ and three alternative accounting measures, A_1–A_3, both in levels (Panel A) and in annual changes (Panel B). Measure A_1 approximates X using book values of total debt and total assets, A_2 approximates X using book values of total debt and market capitalization, and A_3 proxies the standardized ratio $\widetilde{X}$ with Merton's distance-to-default measure. The last column reports the average of the correlation estimates across the 5 selected companies and the Standard & Poor's (S&P) 500 index.

	Statistics						
Measures	GE	WMT	JPM	DUK	XOM	SPX	Average
Panel A. Correlation between $\widetilde{X}$ and Alternative Accounting Measures							
A_1	0.19	0.47	0.04	−0.28	0.26	−0.02	0.11
A_2	0.34	0.26	0.45	−0.00	0.01	−0.12	0.16
A_3	0.59	0.23	0.78	0.77	0.68	0.65	0.62
Panel B. Correlation between Annual Changes in $\widetilde{X}$ and Alternative Accounting Measures							
A_1	0.07	0.52	0.04	−0.19	0.15	0.07	0.11
A_2	0.19	0.32	0.42	0.14	0.10	0.08	0.21
A_3	0.34	0.33	0.51	0.53	0.61	0.46	0.46

the option-implied $\widetilde{X}$ series do not line up well with this book-value leverage measure.

When we use the book value of total debt but the market value of equity to define the second measure A_2, the average correlation becomes higher at 16% on levels and 21% on annual changes. In particular, the correlation estimates on annual changes become universally positive for all 5 companies and the index, suggesting that option-implied leverage is closer to leverage measures defined with market values of equity.

Finally, in A_3, we use Merton's (1974) distance-to-default measure as a proxy for the standardized leverage measure that is adjusted for volatility. As Table 6 shows, this adjustment makes a big difference in increasing its correlation with the option-implied series. The correlation estimates are positive for all 5 companies and the index, both on levels and on annual changes. The correlation estimates average at 62% on levels and 46% on annual changes. The option-implied series $\widetilde{X}$ is a standardized leverage measure that adjusts for its volatility. When the volatility varies over time, the time variation in the scaling can reduce the correlation between the $\widetilde{X}$ and unadjusted measures such as A_1 and A_2. The much higher correlation estimates with the standardized measure A_3 suggest that the volatility scale adjustment is important in lining up the accounting- and option-implied leverage measures.

Figure 4 compares the time series of the standardized equity-to-asset ratio $\widetilde{X}$ extracted from options (in solid line) to the time series of the three alternative measures A_1 (dotted line), A_2 (dash-dotted line), and A_3 (dashed line), all rescaled to match the average level and scale of $\widetilde{X}$. Each panel plots the time series for one company, with the last panel for the S&P 500 index.

For GE, the option-implied series show some variations in the early part of the sample that are not matched by the alternative measures, but all measures reveal a gradual build-up of financial leverage leading to the 2008 financial crisis and a gradual deleverage process since then.

For WMT, the plot shows that WMT took on additional debt in mid-1999, leading to a sudden drop in both the option-implied measure $\widetilde{X}$ and the three alternative accounting measures. The book-value-based A_1 measure shows the most sudden drop, whereas the distance-to-default measure A_3 shows only a gradual decline due to interactions with volatility. On the other hand, the plateaus around 2006 for both $\widetilde{X}$ and A_3 are potentially driven more by a decline in volatility than by actual raw accounting leverage change.

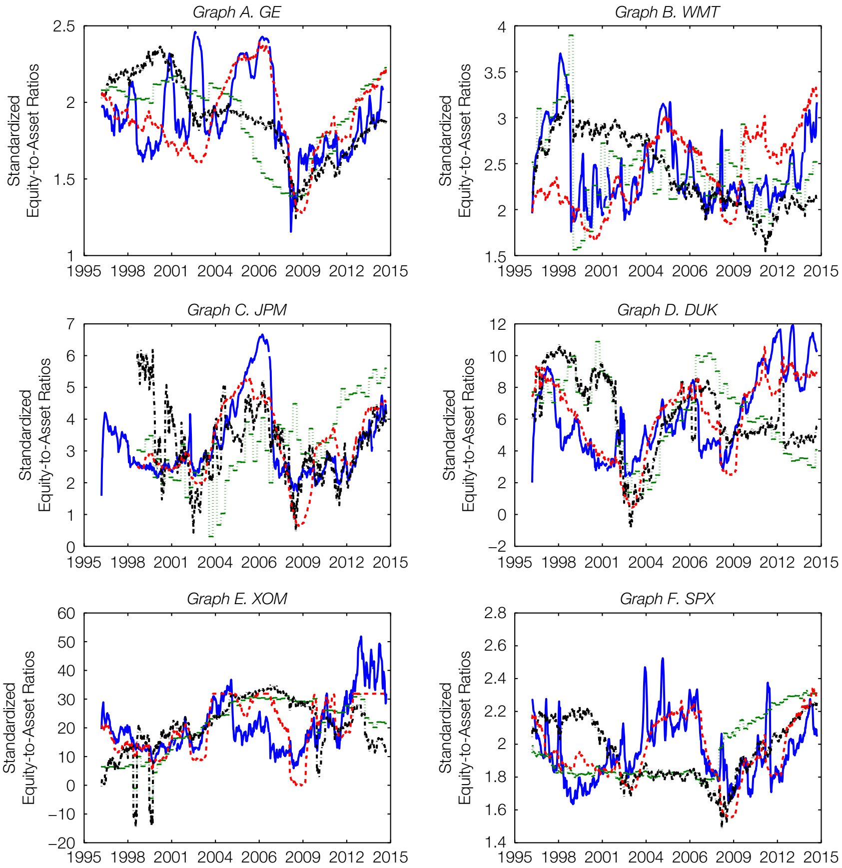
For JPM, the different financial leverage measures share a common cycle of levering up before a recession or crisis and deleveraging right after, highlighting the more active capital structure management behavior of the bank. Among the different lines, the book-valued-based A_1 measure is somewhat lagged behind other market-based measures.

A somewhat similar cycle also shows up for DUK, except that the recent deleveraging process shows up only in the two standardized measures $\widetilde{X}$ and A_3 but not much in A_1 and A_2. For XOM, because the actual leverage is very low, the option-implied standardized equity-to-asset ratio is much higher than that for other companies. Furthermore, because the time-series variations of the two raw

FIGURE 4

Comparing Option-Implied Financial Leverage to Alternative Accounting Measures

Each graph in Figure 4 compares the time series of the standardized equity-to-asset ratio $\widetilde{X}_t$ extracted from options (solid line) to three alternative accounting measures: A_1 (dotted line) using book value of total debt and total assets, A_2 (dash-dotted line) using book value of total debt and market capitalization, and A_3 (dashed line) using Merton's standardized distance-to-default measure.



leverage measures A_1 and A_2 are small, the variations of the two standardized measures $\widetilde{X}$ and A_3 may reflect more variations in volatilities.

For the equity index in the last panel, the two raw leverage measures A_1 and A_2 show little variation in the early 2000s but show a build-up of financial leverage leading to the 2008 financial crisis, and then a prolonged deleveraging process after the crisis. The two standardized measures also show similar variations. In addition, the two standardized measures go up between 2004 and 2007, whereas the two raw measures stay flat, potentially due to the effect of muted volatility during this quiet period.

There are many reasons that the standardized leverage ratio extracted from options can deviate from the various accounting measures. On the one hand, $\widetilde{X}_t$ extracted from options can proxy option movements not related to financial leverage or its volatility. On the other hand, the accounting measures may not truly reflect a company's financial leverage, especially for companies with complex capital structures. Our preliminary analysis, however, suggests that using market values of equity and standardizing the leverage by historical volatility can generate standardized leverage measures that match the option-implied leverage series reasonably well. This observation provides future direction on how accounting leverage measures can be incorporated into option pricing. In particular, the option price behavior does not depend directly on the equity or equity index level, as suggested by the local volatility model of Dupire (1994), nor does it depend directly on the raw debt-to-equity ratio; instead, it reflects the risk induced by financial leverage and its interaction with business risk variation.

V. Concluding Remarks

The variation of equity index volatility and its interaction with the index return can come from three distinct economic channels. First, the index return volatility increases with financial leverage, the variation of which is dictated by the market's aggregate capital structure decisions. Second, positive shocks to systematic risk increase the cost of capital and reduce the valuation of future cash flows, generating a negative correlation between the market's return and its volatility, regardless of the financial leverage level. Finally, large negative market disruptions show self-exciting behaviors. This article develops an equity index dynamic that accommodates all three sources of volatility variation and proposes to disentangle the three sources of variation through the variation in equity index options across a wide range of strikes and maturities. Model estimation shows that the volatility feedback effect reveals itself mainly in the variations of short-term options, the self-exciting behavior affects both short-term and long-term option variations, and the financial leverage variation has its largest impact on long-dated options.

The disentangling of the volatility variation reveals economic insights that one would not be able to obtain from the estimation of a standard reduced-form stochastic volatility model. In particular, the model estimation reveals how the market capital structure responds to different types of risks. Contrary to conventional wisdom, financial leverage does not always decline with increased business risk. Instead, companies respond differently to different types of business risk. The financial leverage can increase with increasing business risk if the risk increase is due to small, diffusive market movements. Only when the self-exciting downside jump risk increases do companies become concerned and start the deleveraging process.

When applied to individual companies, the three economic channels show up differently and to different degrees in companies with different business types and different capital structure behaviors. The leverage effect's contribution to the negative return volatility relation is stronger for companies with passive capital structure behaviors and weaker for bank holding companies that actively manage their capital structures to satisfy regulatory requirements. The volatility feedback effect is stronger for companies that experience mostly systematic shocks or shocks that generate reverberating impacts on the aggregate economy, and it is weaker for companies with mostly idiosyncratic shocks. Finally, the self-exciting behavior is more of an aggregate behavior due to cross-company propagation of negative shocks through structurally connected or related businesses.

Although we estimate the model using only options, the standardized financial leverage series extracted from the options data do show co-movements with various accounting financial leverage measures, especially when we standardize the leverage measures by volatility, as in the distance-to-default measure of Merton (1974). Future research could explore incorporating financial leverage measures in model estimation to achieve better separation of leverage variation and business risk fluctuation. The analysis could also be expanded to the large cross section of individual companies both theoretically and empirically. Theoretically, both systematic and idiosyncratic shocks could be incorporated in the modeling of individual companies. Empirically, the cross-sectional variation of stock option behaviors could be linked to differences in firm characteristics.

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