PETER CARR'S

HALL OF MIRRORS

Put-call symmetry may have been the start but, for Peter Carr, the importance of invariance extends into surprising realms of possibility. Dan Tudball traces Carr's kaleidoscopic journey…

It's the mid-1990s, the Wall Street of the Masters of the Universe is partying like it's 1999, LTCM and the dotcom bubble are yet to spoil the good times. Peter Carr and Dilip Madan have developed a robust hedge for variance swaps using vanilla options such that static positions in the options combine with dynamic trading in the underlying. The amazing thing is that the hedge works in a big class of models.

Carr, then at Morgan Stanley, goes to a guy in marketing and says: "Wow! We should be selling these variance swaps; they're simple contracts, people should want them, and they can be hedged in a big class of models!"

The marketing guy says: "No one's interested in variance. If you can create a volatility swap, *that's* what we can sell." And probably turns his attention back to his mid-morning glass of Petrus.

An appreciation of symmetry, in all its forms, is essential to understanding Peter Carr

To a marketer, the difference between a variance and a volatility is just a detail in a contract. Carr, however, could not see past the square root which had just punched him in the face. The square root you have to take to go from a variance to a volatility, to someone who is trying to hedge in a model-free way, is a huge problem because it's a nonlinear function. "Trust me," Carr says, "It's not a minor detail."

But Carr's overwhelming sense of diligence won't allow him to drop it and he asks himself: "Is there an analog to this variance swap, where you can take the square root or more generally any nonlinear function? If you do a linear function, it's obvious that it would work, but with a nonlinear function could you still do it?"

Carr "struggled and struggled and struggled," but couldn't make it work robustly: "You could do it all in theory but not in the absence of strong assumptions, let's say." The struggle would result in the piece of work of which Carr is most proud – even though it has never been published.

With the help of Roger Lee, the realization came that you could do it, if you were willing to make one more assumption compared to the standard theory. That one additional assumption is called 'uncorrelatedness,' a technical assumption. "It's an assumption that feels wrong in many contexts, and that's what held me back from publishing it," Carr says, but he "… was happy to make this progress because at least it was only one more assumption and not a lot more."

Five years after the comment that sent him off on his quest, Carr then bumps into a different marketing type and says: "Hey! I can do a vol swap now instead of a variance swap, semi-robustly!"

"No one's interested in vol swaps now," the marketer replies." Everybody's doing variance swaps."

An appreciation of symmetry, in all its forms, is essential to understanding Peter Carr.

Reflections

Prior to chatting with Peter Carr, I had taken the opportunity to ask colleagues and friends for any reflections and anecdotes they had on the affable quant finance luminary, who was then in the midst of making the switch back to academia after 20 years on the industry side. His long-time collaborator, Dilip Madan, was particularly generous with his insights.

A day or two before our scheduled call, Peter informed me that the time had to be shifted a little later owing to a prior engagement, and Madan's notes proved useful as I speculated as to why this might have occurred.

Based on Madan's reflections, I mused that perhaps the shift in time was due to Carr's heroic devotion to financial mathematics: while attending a conference Down Under, Madan and Carr found themselves on a boat trip to marvel at the spectacle of the Great Barrier Reef with other speakers and delegates. "Everyone was either taking a dive or going out on a snorkel," Madan recalls. "But Peter was on the boat interviewing conference attendees about results in financial mathematics and taking notes on his laptop."

Or perhaps the delay was due to the logistics of shifting to NYU? More specifically, shifting his books. "Peter's library of books is huge," Madan shares. "It occupied significant parts of a whole floor at Morgan Stanley. Probably needs a huge office at NYU Tandon."

As it turned out, Madan had indeed provided a clue to the delay, via the observation that Carr has an insatiable appetite for new technology; acquiring the latest gadgets as soon as they come out, introducing such novelties as Google Glass to a perhaps somewhat bemused Madan at the earliest opportunity.

Peter Carr on the line: "Thanks a lot for being flexible, Dan. I was doing a test drive of a Tesla and they ran late, so I really couldn't talk…" He was in the midst of moving into a new home, and he went on to tell me that it was, in fact, his birthday that day; he, his wife, and daughter were on their way out for a steak dinner to celebrate. Taken aback at the subject's generosity with his time, I offered to reschedule but he assured me that it was fine, and I *was* assured – within a minute of speaking to Peter Carr you are put immediately at ease: despite the formidable resume and list of achievements. Twenty years in industry at Morgan Stanley, Bloomberg, BOA. Seminal work on variance and volatility. Awards aplenty. Carr doesn't have to be that nice, and yet he is.

So, being his birthday, it seemed fitting to go back to the very beginning – which served to provide an interesting contrast with quite how far he's come; a useful prolog to a discussion of the fascination with symmetry in all its forms that defines precisely the intriguing Professor Carr.

Angle of incidence

Peter's father had grown up in relative poverty in Malta. The island, just 122 square miles, had been bombed beyond recognition during World War II, infrastructure was non-existent, and making a living was near impossible – particularly for a young man who had been forced to drop out of school at the equivalent of grade 6 in order to help out with the family business – driving buses. Carr shares the reflection in Lindsey and Schachter's *How I Became a Quant* that his father was very smart, and managed to pull himself out of that tragic history to achieve what is now a comfortable life in Southern California.

Between 1948 and 1967, 13,000 Maltese emigrated to Canada under an assisted-passage program that effectively amounted to people being paid to leave Malta. Among these émigrés were Peter Carr's parents, who made their way separately to Toronto, Ontario in the early- to mid-1950s. There, they met, as part of the Maltese church community. The couple married in 1957 and Peter arrived on December 22, 1958, the first of three sons. The family settled in two suburbs of Toronto; the first called York, the second called Etobicoke.

It was in Etobicoke that his paperboy route first sparked Peter's interest in math and its financial implications – looking at the changes in the USD/CAD rates that were published daily in the papers he delivered. Carr the Elder, an electrician by trade, had observed that if the US dollar was at 1.01, then the Canadian dollar was at 0.99 and, taking an arithmetical approach, assumed that if the USD rose to 1.02, then the CAD would fall to 0.98.

Carr the Younger was eager to explain to his father that the relationship between the two numbers was geometric: "I was trying to explain this in my way to him but I couldn't use words like 'arithmetic' and 'geometric,' so I just said, what if instead of the USD being at 1.01 to the CAD it was actually 2? Then, he would, using his arithmetic scheme, get zero, when in fact it's just ½, which is very different from zero! So, I was trying to exaggerate to make a point. He got irritated because zero is obviously the wrong answer, like you can't have an exchange rate of 'zero,' right? Well, it happened in Zimbabwe but it wasn't going to happen between Canada and the United States. I guess that he got irritated because I had sort of shown him up but, you know, I don't think he knew what the right answer was to that question. He knew it couldn't be zero but he didn't know it would be a half."

Peter was banished to his room for his impudence, where he found himself reading up on Pythagoras, which ironically led to a reconciliation between father and son, when Peter realized that the theorem could be used to work out the length of wire needed to run diagonally across a room. To put this in context, the reader needs to know that Peter's father was an electrician.

Transformations

Peter Carr possesses a self-deprecating chuckle that seems to simultaneously communicate that the insight he's sharing was achieved with not inconsiderable effort and that in retrospect he can't believe it was such hard work. The laugh surfaces at numerous points throughout our conversations – for example, when talking about his first exposure to finance via a compulsory class taught as part of his Bachelor of Commerce degree at the University of Toronto.

"I had a fantastic teacher, Dan Thornton, and he took a mathematical approach to the subject which really attracted me. I was mostly taking accounting at that time, which, you know, was kind of, I don't know, not so exciting," Carr chuckles. "Anyway, when I saw that people were doing really sophisticated things in finance, which, when I look back on it now, seems like really elementary stuff, like when you compute the variance of a portfolio. How you do *x* times omega *x*, where *x* is a vector and omega is a covariance matrix; I thought that was really sophisticated at the time!"

Peter did well in the class, and the professor asked him to act as a teaching assistant when Thornton taught next. "It was good to TA it because when you're reading people's answers to questions, it gets you thinking," Carr recalls. "I didn't know what I was doing when I was younger, so when I finished my undergrad degree I went into auditing, and I kind of hated it. I decided to go back and get into finance…" Having the Bachelor of Commerce degree from the University of Toronto allowed Peter to return to the school and complete the MBA course in one year rather than the usual two. Classes taught by Laurence Booth were particularly inspiring and led Peter to decide on pursuing a PhD.

As Peter tells it, weather seems to have been a major factor in making his decision as to which schools to apply to for the PhD because he only applied to institutions in California: "You know I'm from Toronto, so I really wanted to get out of the cold…"

Thornton and Booth wrote letters of recommendation on Carr's behalf and he secured offers from Berkeley and UCLA. "UCLA had a really good finance faculty, according to the people I trusted," Carr explains. "So, I went there and, well, it was really tough at the beginning. They were really tough on doctoral students. When I arrived, they had actually failed everybody who had taken this comprehensive exam for the past five years and so it was kind of a shock when you get there and you see that they are not letting anybody out!"

The first months were filled with trepidation at the prospect of being ground through the academic mill, only to be spat out at the end with nothing to show for the effort – the rules were that you could only sit the paper once a year, and if you failed it twice you were out on Sunset Boulevard on your rear.

"The summer I arrived, they gave this comprehensive exam and they had finally passed some people – I think it was four out of seven – so, I thought, great – they are finally passing people!" But hopes were quickly dashed when the next cohort sat the exam at the end of Peter's first year. "They had seven people take it, and they failed them all, including someone who was in my year who had come really prepared and decided to take it early – even though they had this record of failing people!" Carr recalls that there was one student among the cohort who just looked set to succeed. "He was such an impressive guy, I couldn't believe that he had failed this exam. They don't give the marked papers back, they just tell you sorry you didn't pass. So, he was shocked because he thought he had done well. He went to Dick Roll, who was a big name in the finance department, and said: 'I understand that I failed – can I see my exam, so I can at least see where I'm going wrong for the next time?'"

Roll, now Professor Emeritus at UCLA Anderson School of Management, took some sympathy on the student and agreed to pull out a copy of the exam. "So," Carr continues, "they met at his office to look over the photocopy of the exam and it turned out that Dick Roll had only been given one side of each page that the student had written – he had written on both sides of each page and whoever had collected the exam papers and distributed them to the faculty to be graded had only photocopied one side of each page!"

Once again, faint hope raised its head across the school as speculation grew that perhaps all the other failures were due to only half the submitted answers being marked. Alas, not true

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– other candidates had not been as lengthy in their answers as the determined student, who passed, once his submission was reassessed. "They were basically back to their ways of failing everybody because they only passed one out of seven in the end," Carr says ruefully. "Then, my second year comes and I have to take it – there were four of us altogether – three others and me – and we knew they were so tough on people. We said, you know what, let's study together. That way, we walk in knowing the exact same things, and maybe they'll either fail us all or they'll pass us all; if we know the same things, they can't really distinguish between us, right? So, that's what we did; we studied together for a whole year, the four of us, and then took the exam and, believe it or not, they passed us all! So, it kind of worked." Chuckle.

"We used to call UCLA 'University of Chicago at Los Angeles' because it was so tough," Carr recalls in a tone of awe and disbelief, even after all these years. The University of Chicago had a longstanding reputation for intellectual standards that might best be described as excoriating; a large number of the faculty at UCLA had come from Chicago and were happy to beat their acolytes with the same yardstick.

"You had to switch gears from taking courses and passing exams on the one side, to doing original research on the other." Carr confesses that he found the transition arduous. "I kind of floundered actually. You can be good at understanding what people are telling you and then parroting it back but that is a completely different set of skills from coming up with something entirely new. Anyway," he sighs, "I kind of floundered for a year; I couldn't think of something good to work on. Then, luckily enough, these two professors came from the University of British Columbia; one was Michael Brennan, the other was Eduardo Schwartz, and they had actually worked together a lot. They were like Carr and Madan – you've probably heard of them?" A relieved chortle. "They switched from UBC to UCLA and they sort of saved my life. So, I took a Schwartz doctoral seminar and I learned a lot of useful things; Michael Brennan suggested a topic I could work on for my thesis, which I did..."

General covariance

When Carr was teaching the first time round in the early 1990s, and had yet to take the leap with both feet into Wall Street, he was consulting for firms in New York. "I was kind of intrigued that people were using these theories to make real money on Wall Street. So, I transitioned in '96 and went to Morgan Stanley, and that was really a fantastic experience, going to a place like that, I remember; this is a true story – I don't know how this is going to play out!" He chuckles.

"I remember my pay tripled when I went from academia to Morgan Stanley, and when I got there the guy who hired me said he was really grateful that I wasn't too demanding when it came to salary, so it made it a lot easier for him to hire me – so, I said yeah, well, no problem. That's what it was like back then, you know..."

Carr reflects on the ground covered in the conversation. "The biggest difference between industry and academia, by the way, is that in academia you get to choose your problem; in industry, they force the problem on you and you really can't change the problem."

So, how do you deal with that when you're on the commercial side of things? "What you'll see industry people doing, and I was one of them for 20 years, is change the concept of 'solution.' If you can't change the problem, you can at least relax the notion of solution – you can say: 'I don't want an exact solution, I just want to get close.'

Carr feels that practitioners are much more sympathetic to approximating, by dint of the fact that the problem is set in stone

Carr feels that practitioners are much more sympathetic to approximating, by dint of the fact that the problem is set in stone. Academics will simply change the problem until they can find exact solutions. "It's interesting to think about the sociological differences. I feel I have a bit of a unique perspective because I was an academic and then a practitioner and am now an academic again, so I see what's going on...

"Yeah, I mean, I think each side does contribute something to the overall quest for understanding but also, they differ in terms of what the payoff is, let's say. For an academic, it is understanding; for the practitioner – the function of the firm is to generate profit, so – it'll lead you to forget about understanding as a secondary objective, which can in the end hurt profit. But let's say, in the short term, you discover some black box and they see money, you'll tend to work with it, right?"

Perhaps his habit of developing a toy model rather than going to the general has always been a sort of insulation against the context – approaching the problem by establishing a problem that you're interested in. Aligning personal goals and preferences with the institution or task at hand and hoping that they align. Would it be safe to assume that where they don't align, Carr was prepared to move on?

"It's a difficult balance," Carr allows, "there's following the leader and then there's being a leader intellectually. You are given a task, it's a difficult balance; you can solve that task or you can think, is this the right task? I am probably more the latter, and say, here are the results – perhaps the original task doesn't get solved but, instead of getting a passable solution to the problem at hand, the hope is that you can think outside the box and open up whole new vistas."

Carr feels that the opening of new vistas happened many times.

"There's been a lot of change over the time I've been in industry as a result. People thought, at first, they had this turnkey solution, then things like the crisis made them realize it wasn't so turnkey.

"It's had a whole lot of repercussions; it's created huge departments surrounding model risk and things like that and, well, I feel lucky to have lived long enough, let's say, where my original pessimism or skepticism about these turnkey solutions were proved right.

"With Dilip, he was definitely in the vanguard of jump models," Carr reflects. "At first, back in the dark ages, in the stone age before the crisis, no one paid attention to them; then, the crisis came and people started to pay attention. They are relatively intractable compared to the nonrealistic continuous models but that's the price you have to pay.

"If you want to be germane to the problem at hand, you just have to deal with the nonlocality and deal with the fact that you are going to have to use numerical methods. That's a very pragmatic philosophy that he espouses and I also share."

About the axis

Espen Haug testifies to the impact of Carr's result in put-call symmetry, describing it as a very robust hedge that he and other uber-traders had put into practice, with excellent results. Haug's observation provided access to Carr's hall of mirrors.

"I'm a big believer in what's called symmetry – or it's also called invariance," Carr enthuses. "These are mathematical principles that seem like a curiosity when you first encounter them, and then it turns out that people, especially applied mathematicians and physicists, eventually realized that it was at the core of understanding the universe and math.

"I feel like, in finance, there's a few people who get it but most people don't, and so it's a kind of a passion for me to just understand where the invariances are, where the symmetries are in any problem.

"Put-call symmetry is actually due to a guy called David Bates, who did it before me – but let's say that I emphasized it more than he did. I remember getting that idea of hedging a barrier option using the symmetry of vanilla options in Black–Scholes, and it's the weirdest thing – when I had the idea, I said: 'Oh well.' I got the idea in the context of Black–Scholes and when I got it I said, why would we care? We already have a way to replicate a barrier option using dynamic hedging."

Dilip Madan

Peter Carr's long-time collaborator reflects on their work together.

I started by saying to myself that I will ignore commenting on papers coauthored with me, only to realize that I would then have little to talk about.

There have been many substantial papers over the years, and it is clear that Peter is continuously engaged in exciting endeavors of intellectual discovery. He likes to get to the essentials of an argument or exposition, to discover with clarity what is making things work out as they must. I have had many discussions with him on numerous matters and we often disagree and go our separate ways but, equally often, we are intrigued and work to help each other get to the answer.

We both had first degrees in accounting and came to enjoying mathematics later in life, in graduate school. When I came of age, mathematics was making its impressive way into economics, and finance was as yet untouched by mathematics. When Peter came to graduate school, mathematics had begun to shine its light on finance. We met when Peter was an Assistant Professor of Finance at Cornell and I was spending a semester on sabbatical at Cornell. We hit it off as friends with a strong interest in the connections between mathematics and finance. Our lives intertwined considerably thereafter, when Peter came to Morgan Stanley as a derivatives quant and hired me as a consultant in the summer of 1996. We have been in close contact ever since.

Our first success came with trying to find an arbitrage-free model consistent with option prices at a single maturity. We sought a flexible, parametrically parsimonious density capable of being the risk-neutral density at a given maturity. I had been working with the symmetric variance gamma but it lacked the ability to generate a skew. We formulated the asymmetric variance gamma and observed that, yes, it could fit option prices across strike for any maturity and underlier. The model was adopted at Morgan Stanley to mark their option books at market close worldwide.

The marking required computation speed and this led us to develop the application of the fast Fourier transform to address this problem. I remember working with Peter through Thanksgiving Friday on completing this work. It soon became the standard for option calibration for a variety of models.

We were both not satisfied with just explaining the smile and wanted to go across maturity as well. We tried increasing the number of parameters in the variance gamma, to see if this would help, and this led to the CGMY model. But it did not work across maturity.

We then learned that no Lévy process would fit option prices across maturity, no matter how many parameters are added to the Lévy density. This is because, for all such processes, skewness and excess kurtosis fall as the reciprocal of the square root of maturity, and maturity, respectively, and in the data one can see that these moments do not fall with maturity.

We had to go elsewhere and we were at the time collaborating with Helyette Geman and Marc Yor in Paris. I would spend January of each year in Paris on these problems. It was Marc who suggested that we look at the processes developed by Sato as an alternative. This gave us our first parsimonious four-parameter model capable of fitting option prices across both strike and maturity. I use it now to summarize option surfaces and mark-to-market option trading strategies implemented over ten to 15 years.

Peter then moved to Bloomberg and I started consulting there for a little bit, where we implemented work on credit default swaps.

Peter returned to Morgan Stanley in 2010 and we got together on developing hedges for insurance counterparties. This led to our recent work on hedging insurance books to attain levels of risk acceptability. We had worked on acceptable risks earlier in 2001 at an abstract level but this was not implementable at the time. Work with Alexander Cherny in 2009 made all this operational and we brought the methods forward for managing multiple underlier complex risk exposures.

We are both quite consumed by our own agendas and Peter has worked with many others on various issues, but I am not a part of that agenda. I am familiar with the work on variance swaps with Roger Lee and the associated hedge corrections in the context of underlying Lévy processes.

Nevertheless, Carr presented the findings at an industry conference, and the people there – to Carr's surprise – were very excited about the result.

"I soon realized that why they were excited about it was because some people realized it was holding outside of the particular model it was developed in. Espen is one of those people who realizes it."

From that point on, Carr devoted time to trying to find analogs of that result, other robust hedges, and found that often they were based on some kind of symmetry. “The particular kind of symmetry that Espen is referring to is called mirror symmetry – when you look in a mirror, the left side of your face becomes the right side, and so on. But mathematicians use the term in a broader sense, and it’s just that you’ve done one change to one thing but something else didn’t change in response. An even function, one that’s symmetric about the origin, would look the same after [the transformation] – right becomes left and left becomes right. So, that’s mirror symmetry; there’s way more important examples of symmetry that I use.

“To me,” Carr reflects, “modeling is the quest for symmetry.

“Once you decide what symmetries work in practice, you’re done, you’ve actually modeled – that’s what modeling is. Hedging and invariance are quite the same thing. Hedging means your wealth is invariant, the value of your positions is invariant to moves in the underlying asset prices, and in math, invariance more generally is that something doesn’t change if something else changes.”

“Once you decide what symmetries work in practice, you’re done, you’ve actually modeled”

Carr entertains me with the anecdote that opened this article – the marketing mind warp that led to the work he is most proud of. “We never published it and people know about it nonetheless. I’m most proud of it because it was a hard problem that I’d worked on for five years, and finally, together with Roger Lee, we solved it. It’s got this Achilles heel that I’ve never been comfortable with, so that’s why I’ve never published it; nonetheless, I feel that it’s a step in the right direction.

“The key to solving that problem of going from variance swaps to nonlinear functions of realized variance, is that you have to think about addition becoming multiplication, so the translation invariance becoming scale invariance. It’s actually all the same thing, so it’s funny that you just have to have the right idea – and it took me five years to have it – but, suddenly, this can all become clear.

“In retrospect, I now realize why I didn’t have that idea; it was because of the notation. It’s actually because I was using these stupid tools, the wrong tools for the actual problem; so, anyway, so that’s, I’ve probably talked for too long – but that paper with Roger Lee is probably my favorite paper.”

Funnily enough, with yet more symmetry at play, Carr is now somewhat obsessed with a further warping of perspective, introducing ratios to derivatives and, specifically, developing a notation for a potentially new branch of derivatives calculus.

Matters of scale

“People don’t see things simply because they don’t have the notation, because the notation hasn’t been developed,” says Carr. He reminds me that Leibniz coinvented ordinary calculus with Newton but the extraordinary contribution he made – the immeasurably valuable contribution – was his notation. The notation made it all obvious what was going on because Leibniz had built his deep understanding of the mechanics of calculus into it.

“A lot of people would say that math is left brain, creativity is right brain. Certainly, in math what is rewarded is creativity, not just being able to follow rules. So, I think that most successful people are able to work from both sides of their brains. I try to be creative, not just follow rules, so I actually spent the last few days developing a notation in math that is related to calculus, and I’m thinking it should be used.”

Carr’s zeal was sparked when he stumbled across an idea, developed by two researchers, that might be easily dismissed as nuts. As a preamble to introducing this idea, Carr starts with the familiar: “So, a standard derivative in calculus is a limiting finite difference. You are taking a ratio of two things and the two things are both getting smaller. So, in the limit, you are coming to 0 over 0, which in math is literally not defined, but as you’re taking a limit with the numerator approaching 0 and the denominator approaching 0, then that limit is well defined and that’s what a derivative is.”

So, the idea is: what if you considered the differences as having ratios?

“At first, when you hear this you think it’s nuts,” laughs Carr, “but I’ve totally bought into this philosophy.”

Galileo asked, is the number 10 closer to 1 or closer to 100? “Somebody who thinks arithmetically, like my father, say, would say, of course 10 is closer to 1 than it is to 100! Arithmetically, that’s true. But if you consider ratios, the ratio of 10:1 is equal to the ratio of 100:10, so, from a ratio perspective, 10 is equidistant, and so these guys are basically saying, well, maybe we could use ratios instead? It’s a choice we could make, and maybe it’s useful in some contexts.”

Authors Robert Katz and Michael Grossman, in their book *Non-Newtonian Calculus*, have invented three new types of derivatives; in the first, there’s still a difference in the numerator, but now you have a ratio in the denominator. The next type does it the other way around and we put the ratio in the numerator position instead. Finally, you could have a third derivative that has a ratio for both numerator and denominator.

Carr thought this was interesting enough to have a go at developing a notation, which he doesn’t really see as a big deal in itself. But this led him to think about lognormal distributions. “Well, you are not going to believe this but I did a calculation and it turns out that if you take that last kind of new derivative, which is a ratio over a ratio, and you apply it twice to this lognormal density, you actually get a constant, which is like kind of shocking!

“I’m basically doing a novel kind of second derivative,” Carr continues. “If you take the usual kind of second derivative, you would not get a constant. If you take this unusual derivative and apply it twice to a lognormal density, you actually get a constant. In this sense, lognormal is like a quadratic function because if you take a standard second derivative and apply it to a quadratic function, you get a constant – so I thought that was cool.

“The lognormal distribution is something that everybody in mathematical finance, like option pricing theories, is intimately familiar with. It’s just like, you talk about it every day and I don’t think that anybody realizes that it has that strange property because nobody ever thought of that strange derivative. I was kind of happy to discover that recently.”

This takes Carr back to symmetry. A standard derivative has translation invariance in both the numerator and the denominator, so, for example, x^2 with respect to x is $2x$ and the first derivative of x^2 plus a constant with respect to x is also $2x$.

These alternative derivatives have scale invariance, meaning that if you were to take some function and then double it, and then compare the derivatives before and after doubling, they'd be the same.

"In some applications, that's really important," Carr stresses. "Getting back to lognormal, that's actually what's key – this Black–Scholes model is all about scale invariance and not about translation invariance.

"It turns out that things are a lot simpler if you use the right tools for the job, and these guys' proposed useful tools when you have scale invariance in a problem like you do in Black–Scholes; and yet, no one is really aware of their work, that's point number one, and point number two is that the few people who are aware of his work would use his notation, which is not great, it's funny."

"You can listen to others but you have to verify it for yourself, so if someone says something is true, well, check it"

Getting back to Leibniz. "The notation for derivatives uses d's and the d's stand for differences. So, I simply said, why aren't these guys who are using ratios everywhere also proposing a new notation, which would be r's, where r stands for ratio?" Carr laughs. "I said that's crazy not to do it, that's what's going on – so that's the simple notation that I started to use."

Transpositions

Peter Jäckel credits Carr's publications, presentations at conferences, and private meetings for many things he has learned in financial mathematics, saying that his work was "… full of detail and academic depth rarely found with other researchers in financial mathematics. He's an excellent academic with vast knowledge and understanding in financial mathematics and is one of the most valuable teachers in financial mathematics."

"I really strongly believe that when it comes to understanding what's going on, never trust anybody," Carr shares. "You can listen to others but you have to verify it for yourself, so if someone says something is true, well, check it, don't take it on face value. Part of it is going to the original source, so I guess, like Peter Jäckel kindly said, I cite people a lot – well, that's because I'm going to the source, so I might as well cite them."

It's one thing to research and develop new ideas, but the ability to actually communicate and describe new ideas and introduce people to new approaches is an entirely different challenge. The best teachers, tired cliché that it is, have the most empathy for their students. That aspect of Carr is obvious from the outset of any conversation, so it was apt to ask whether that aspect of empathy is a conscious thing he works on before and during addressing an audience or is it something achieved unconsciously?

"Probably unconscious," Carr ventures, after pondering the question for a moment. "There's a tendency in finance to keep results to yourself because they potentially have monetary value, and I guess I have never really followed that route. It might have helped that I grew up in Canada, a kind of semi-socialist country, as compared to the United States anyway!"

He reflects on how the sometimes solitary – bordering on hermit-like – pursuit of mathematical understanding can work against the best didactic intentions. "There's this tendency, especially among mathematicians, I'd say, where they understand things at a very general level and have gotten there through struggle. You get to these abstract understandings via examples and then abstracting from the examples, and eventually you get what is going on. There's this natural tendency that when you see what's going on abstractly, you just want to run out and tell people the shortcuts – but most people won't know what the hell you're talking about!

"I have a strong opinion about teaching; in some ways, you can't really just *teach* someone something, which is a strong statement. All you can really do is get them excited about a result and have them learn it for themselves – in some ways, be a guide, which you can't be as a lecturer.

"I believe in just showing them the path via example, rather than the abstract result and then illustrating with examples – that's my teaching philosophy. When I was teaching at Cornell, I really did not try to teach theory but instead just used examples with numbers from the beginning, with no theory. It was kind of amazing and I really learned a lot about how to teach. Luckily, you have a classroom with 60 really smart people in it. When you are doing a numerical example and you haven't dropped an ounce of theory on anybody, and then you ask what amounts to a theoretical question, *somehow* one person will be smart enough to figure it out. It seems like it's amazing that it can happen but it does," Carr marvels. "The wisdom of crowds – you ask the question, somebody will answer it, and everybody else is amazed how the person did it! Then, they are definitely intrigued as to what were the principles that led at least one person to know what was going on."

Warum Buffet?

Never afraid to ask the controversial questions, Wilmott *presses Peter Carr on the question of self-serve catered functions…*

***Wilmott*:** Aaron Brown has noticed that at functions you have a tendency to fill your plate with food, apparently with discrimination, but never eat a bite – is that true?

Carr: (laughing) I didn't notice that; I do end up talking to people a lot and writing things down on napkins instead of eating the food. Espen Haug has noticed that too, so that's probably what it's about. So, let's see – when it comes to eating food, I do have some weird things. I'll eat only one thing at a time, I never mix. At salad bars, I notice that they put the cheap things up front, so that you'll fill your plate with the cheap things; so, I'll start at the opposite end and fill my plate with the less cheap things. Anyway, I mean to eat it but maybe I got sidetracked, it's true!

So, the style was set: example then theory after they are intrigued, and then another example just to nail it.

"It meant that it kind of went slow," Carr admits. "But I'm a big believer in less is more; strange statement, but by trying to relate too much information, you can lose people, so I prefer to just pick what I want them to know and just stick with that. So, anyway, I had fairly poor teaching ratings when I first started out at Cornell because I was adopting the same style I was taught in myself, and then realized how I myself learned, and so switched to that.

"I know you said earlier that teaching is distinct from research, and it is, but I do think that you have to teach yourself in order to do research, and the same kind of tricks that you play on yourself to teach yourself, can be used to teach others.

"With respect to teaching, there's this master teacher at NYU named Aswath Damodaran and he was actually a classmate of mine at UCLA, when we were both there doing PhDs in finance. Damodaran actually won this university-wide teaching award just before he came up for tenure – which was very convenient. I asked him what his secret to success for teaching was; he's a total star, and he said: 'You have to care and you have to show that you care.' Well, it seems so simple; so, anyway, I took that to heart."

Teaching is something that Carr is getting back to at NYU this spring and he's curious to see how it goes, wondering if he should be ambitious and do something completely nonstandard. "That's what some people do. They understand things in a completely different way from everybody else and then they want to share that understanding. I have this friend who did that and he kind of lost everybody, so I'm in two minds as to whether to do it myself. There is some virtue in just teaching the accepted canon of knowledge.

"There are certain things you are expected to know if you are getting a Master's degree, and if you spend the entire limited time doing this alternative view – even if you think it's simpler – they'll get interview questions which are completely classical, and they won't be prepared for them, so, you'll be letting students down. It's a fine balance – teaching is definitely hard."

Carr remarks that nobody has conquered the problem of the inhomogeneity of the students but he suspects that perhaps the best technique to deal with this is the idea of 'flipping' the classroom.

"So, in a flipped classroom model, the lectures are videotaped; the 'flipping' occurs because the students look at the lecture before physically coming to class. When they are physically in class, time is spent on the students doing rather than listening – they have already done the listening outside the classroom and the doing means doing exercises or whatever.

"I haven't done it myself but I'm basically a believer of it at this point – it feels right to me. It feels consistent with my 'induction rather than deduction' philosophy to learning.

"Along those lines, there is a lot of value in writing things down – it's like muscle memory; somehow, things stay in your brain longer if you receive them while writing them rather than just listening – it's definitely more permanent. If that's actually true – say you are alone in your room and writing something down – then that's helping you to remember it; but if you were saying it while you wrote it, then in theory you are hearing it and writing and so it should help with remembering even better! You never see anyone do that!

"I am a big believer in mantras, like when Bill Clinton was running for President he had, 'It's the economy, stupid!' – that was his mantra – and I am a big believer that in teaching and research there should be some guiding principle, and it's worth repeating; eventually you'll get it."

"I am a big believer that in teaching and research there should be some guiding principle"