A Simple Robust Link Between American Puts and Credit Protection

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We develop a simple robust link between deep out-of-the-money American put options on a company's stock and a credit insurance contract on the company's bond. We assume that the stock price stays above a barrier B before default but drops below a lower barrier A after default, thus generating a default corridor $[A, B]$ that the stock price can never enter. Given the presence of this default corridor, a spread between two co-terminal American put options struck within the corridor replicates a pure credit contract, paying off when and only when default occurs prior to the option expiry. (*JEL* C13, C51, G12, G13)

In a classic paper, Merton (1974) links a firm's equity and its debt through their common status as contingent claims on the assets of the firm. In his model, the firm has a simple capital structure, consisting of a single zero-coupon bond and equity. The firm's shareholders default at the debt's maturity date if the firm's value is below the debt principal at that time. Under this structural model, the credit spread on the bond becomes a function of the firm's financial leverage and its asset volatility. The financial leverage links equity to debt and relates firm volatility to equity volatility. Various modifications and extensions on the debt structure, default triggering mechanism, firm value dynamics, and

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implementation procedures have been proposed in the literature.[1] Empirically, many studies have also linked corporate credit spreads to the firm's financial leverage, stock return realized volatility, stock option implied volatility, and stock option implied volatility skews across different strike prices.[2]

The possibility that a company might default on its borrowing has led to the rapid expansion of a market for credit insurance. Credit derivatives such as credit default swaps (CDS) represent a natural step in the evolution of derivatives technology, which began its modern era with the introduction of listed equity calls in 1973 and listed equity puts in 1977. Since the birth of modern financial theory with the publication of the Modigliani and Miller (1958) theorem, much attention has been devoted to the interplay between debt and equity values. Now that financial derivatives co-exist on both corporate debt in the form of CDS and corporate equity in the form of equity options, it seems natural to further examine the interactions between these highly liquid derivative securities.

It has been known for a long time that the possibility of default has relevance for the pricing of equity options. The first explicit recognition of this relation seems to be in another classic paper by Merton (1976). While Merton could have captured these linkages though the structural models that he pioneered, he instead chose to directly model the impact of corporate default on the stock price process by assuming that the stock price jumps to zero and stays there upon the random arrival of a default event. Extensions and estimations of such jump-to-default models include Carr and Wu (2007, 2010), Carr and Linetsky (2006), and Le (2007). In these reduced-form models, the inputs needed to value the option extend beyond the usual inputs such as the risk-free rate, the stock price, and the stock volatility. In particular, one needs an estimate of the risk-neutral arrival rate of default, which can be obtained from corporate bond spreads or CDS. When the option payoff can be replicated, the dynamic trading strategy will in general use risk-free bonds, the underlying shares, and instruments sensitive to credit risk such as corporate bonds or CDS. The particular mix of these instruments depends on the moneyness and maturity of the option and whether it is a call or a put. Focusing on put options, the more

1 Prominent examples include Black and Cox (1976), Geske (1977), Ho and Singer (1982), Ronn and Verma (1986), Titman and Torous (1989), Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), Leland (1994, 1998), Anderson and Sundaresan (1996), Anderson, Sundaresan, and Tychon (1996), Leland and Toft (1996), Briys and de Varenne (1997), Mella-Barral and Perraudin (1997), Garbade (1999), Fan and Sundaresan (2000), Duffie and Lando (2001), Goldstein, Ju, and Leland (2001), Zhou (2001), Acharya and Carpenter (2002), Huang and Huang (2003), Hull, Nelken, and White (2004), Bhamra, Kuehn, and Strebulaev (2010), Buraschi, Trojani, and Vedolin (2007), Gray, Merton, and Bodie (2007), Chen, Collin-Dufresne, and Goldstein (2009), Cremers, Driessen, and Maenhout (2008), and Gray, Merton, and Bodie (2009).

2 Examples include Carey (1998), Pedrosa and Roll (1998), Bevan and Garzarelli (2000), Frye (2000), Collin-Dufresne, Goldstein, and Martin (2001), Delianedis and Geske (2001), Elton et al. (2001), Aunon-Nerin et al. (2002), Bangia et al. (2002), Campbell and Taksler (2003), Cremers et al. (2008), Ericsson, Jacobs, and Oviedo (2009), Hilscher (2004), Consigli (2004), Vassalou and Xing (2004). Altman et al. (2005), Bakshi, Madan, and Zhang (2006), Zhang, Zhou, and Zhu (2009), Berndt and Ostrovnaya (2007), Cao, Yu, and Zhong (2010), and Wu and Zhang (2008).

out-of-the-money the put, the greater the reliance on credit-sensitive securities relative to the underlying stock.

In this article, we explore the theoretical possibility that for put options struck deepest out-of-the-money on the stock, pricing is *entirely* driven by the default possibility, rendering the precise behavior of the underlying stock price process irrelevant for the pricing of these puts. The existence of such an approach would bypass the difficult task of specifying and estimating the volatility process of the stock, as all of the relevant information for pricing such equity puts would actually be found in the markets for credit insurance.

The main contribution of this article is to propose a new, simple, and robust link between deep out-of-the-money (DOOM) American-style equity put options and a standardized credit insurance contract linked to default on the corporate debt of the company. This link is established under a general class of stock price dynamics. The key sufficient condition enabling our result is that the stock price is bounded below by a strictly positive barrier $B > 0$ before default, but drops below a lower barrier $A < B$ at default, and stays below A thereafter. The interval $[A, B]$ defines a *default corridor* that the stock price can never enter by assumption. Given the existence of this default corridor, we show that a spread between any two co-terminal American put options struck within the default corridor replicates a pure credit insurance contract that pays off when and *only* when the company defaults prior to the option expiry.

Structural models of default typically assume continuous dynamics for the firm's asset value and that the firm defaults when this asset value touches the debt value or some other floor. Under such assumptions, the equity would be worth zero right before default and stay at zero afterward. As a result, there would be no default corridor. On the other hand, when a firm's asset value can jump, the firm's equity can have strictly positive value just before a default occurs, and a much lower value afterward. These implications accord with certain well-known defaults, e.g., Lehmann Brothers, and with the idea that CDS acts as insurance, providing protection against rare sudden events. Furthermore, recent studies recognize the strategic nature of the default event and find that debt holders have incentives to induce or force bankruptcy well before the equity value completely vanishes. Theoretical work on strategic default includes Leland (1994), Leland and Toft (1996), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Fan and Sundaresan (2000), Goldstein, Ju, and Leland (2001), Broadie, Chernov, and Sundaresan (2007), and Hackbarth, Hennessy, and Leland (2007). Carey and Gordy (2007) develop a model and find empirical support for active strategic behavior from private debt holders in setting an asset value threshold below which corporations declare bankruptcy. In particular, they find that private debt holders often find it optimal to force bankruptcy well before the equity value vanishes. Our specification of the strictly positive barrier B is in line with such evidence and the strategic default literature. Our assumption that default induces a sudden drop in equity value from above B to below A can be justified in several ways, including loss of optionality and direct costs such as legal fees and liquidation costs associated with the bankruptcy process.

We assume both the presence of the default corridor $[A, B]$ and the availability of two American put options of the same maturity T with distinct strikes $K_1 \in [A, B)$ and $K_2 \in (K_1, B]$. With $A \leq K_1 < K_2 \leq B$, a vertical spread of the two American put options, scaled by the strike distance, $U^p(t, T) \equiv (P_t(K_2, T) - P_t(K_1, T))/(K_2 - K_1)$, replicates a standardized credit insurance contract that pays one dollar at default whenever the company defaults prior to the option expiry and pays zero otherwise. If the company does not default before the option expiry, the stock price stays above K_2 by our assumption and hence neither put option will be exercised, as they both have zero intrinsic value. If default does occur at some time prior to the option expiry, the stock price falls below K_1 and stays below afterward by assumption. As a result, it becomes optimal to exercise both options at the default time, and the scaled American put spread nets a payoff of one dollar at the default time. As long as the default corridor exists and there are two traded American put options struck within it, this simple spreading strategy replicates the target standardized credit contract robustly, irrespective of the details of the stock price dynamics before and after default, the interest rate dynamics, and default risk fluctuations.

We henceforth refer to the target standardized credit contract as a *unit recovery claim* or URC. This fundamental claim is simply a fixed-life Arrow and Debreu (1954) security paying off one dollar at the default time if and only if default occurs before expiry. In 2007, the Chicago Board Options Exchange (CBOE) launched the URC under the name Credit Event Binary Options. The subsequent failure of this contract is consistent with our hypothesis that the contract is redundant in the presence of listed DOOM puts. After coming across our article, the CBOE launched a website (http://www.cboe.com/Institutional/DOOM.aspx) showing how DOOM puts can be related to CDS.

Our simple and robust strategy for replicating the URC suggests that the value of these fundamental claims can be extracted from market quotes of American put options, written on single shares. These equity options were first listed by the CBOE in 1977 and now trade actively in the United States on several options exchanges. A practically important special case of our framework arises when the lower bound of the default corridor vanishes ($A = 0$), so that the stock price falls to zero at the default time. In this case, we can set $K_1 = 0$ and use a scaled position in a single American put option to replicate the URC, $U^p(t, T) = P_t(K_2, T)/K_2$.

When a single put is used to replicate a URC, the intuition behind our approach can be simply stated. The price of the stock underlying a DOOM put either spends time below the strike price or it does not. Similarly, a URC either pays off one dollar because default occurred or it expires worthless. While there are 2x2 = 4 logical partitions of the state space, two of these 4 partitions are fairly unlikely. While a default could occur with the stock price above the DOOM put's strike price both before and after default, such a scenario is highly implausible. Conversely, the stock price could fall below the DOOM put's strike price without triggering a default. While this scenario is also unlikely, it does occur when the company is deemed to be too big to fail. Our simple model assumes that both of these unlikely scenarios cannot occur. To the extent that one has reason to believe that the company is too big to fail, our model should not be applied.

As listed puts on single shares are American style, one has to deal with the possibility of early exercise. In the option pricing literature, the fact that American put options are rationally exercised early has been a tremendous source of difficulty. The problem of finding an exact analytical solution relating the price of an American put option to the price of its underlying stock is notoriously difficult, even in the benchmark Black and Scholes (1973) model. As is well known, the difficulty lies in analytically characterizing the early exercise region. In contrast, our dynamic assumptions lead to a simple characterization of this region, making it straightforward to value American put options struck within the default corridor in closed form, under standard simplifying assumptions such as a constant interest rate and a constant default arrival rate. Furthermore, the American feature embedded in the options is consistent with the payout timing in the URC contract. Using two analogous European put options would create a contract that pays one dollar at the option expiry, instead of at default, when a default event occurs prior to the option expiry.

The most actively traded credit insurance contracts in the over-the-counter market are CDS written on corporate bonds. A CDS contract provides protection against credit risk. The protection buyer pays a fixed premium, called the CDS spread, to the seller periodically over time. If a certain pre-specified credit event occurs, the protection buyer stops the premium payments and the protection seller pays the par value in return for the corporate bond. Assuming a fixed and known bond recovery rate, we show that the value of the protection leg of the CDS contract is proportional to the value of the URC, with the proportionality coefficient being the loss given default. Assuming deterministic interest rates, we can also represent the value of the premium leg as a function of the URC term structure. By assuming both a fixed recovery rate and deterministic interest rates, we can obtain the URC term structure from the entire term structure of CDS spreads. No specification of the mechanism triggering default is required for this purpose. Unfortunately, it is not possible at this time to directly observe the entire term structure of CDS contracts. Fortunately, if one is willing to assume a constant interest rate and a constant default arrival rate, we can analytically infer the value of a URC from a single CDS quote.

To test the empirical validity of the theoretical linkage, we collect data on both American put options on stocks and CDS spreads on corporate bonds. Over a sample of 121 companies and 186 weeks from January 2005 to August 2008, we construct 5,276 pairs of URC estimates. For each pair, one value is computed from the price of a deep out-of-the-money American put on the company's stock, and the other is computed from the five-year CDS spread on the company's corporate bond, with the assumption of fixed and known bond recovery rate and constant interest rate and default arrival rate. A comparative analysis shows that the two sets of URC estimates share similar magnitudes and statistical behaviors. When we estimate a linear relation between the two sets of estimates, we obtain a slope estimate that is not significantly different from the null hypothesis of one. When the URC estimates from the two markets deviate from each other, the deviations cannot be fully explained by contemporaneous variables commonly used for explaining variations in stock option values and credit spreads. However, the cross-market deviations can predict future movements in both the American put prices and the CDS spreads, reflecting two-way information flow between the two markets.

Our article offers several new insights. First, many structural models with strategic default and/or discontinuous firm value dynamics imply the existence of a default corridor, but the simple robust linkage that we identify in the presence of the corridor is new. Second, compared to the many linkages identified in the literature through parametric (structural or reduced-form) model specifications, our identified linkage between equity American put options and credit insurance is much simpler, as it does not require computational methods such as Monte Carlo, Fourier transforms, or lattices. Third, our linkage is also more robust, as it does not depend on any particular parameterizations of pre- and post-default stock price dynamics, interest rate variations, and default risk fluctuations. Fourth, our proposed theoretical linkage enjoys strong empirical support: The URC estimates constructed from American puts and default swaps show similar magnitudes. When they deviate from each other, the deviations predict future market movements. Finally, we show that the key underlying assumption on the existence of a default corridor can be readily and reasonably accommodated in both reduced-form and structural models.

The rest of the article is structured as follows. The next section presents the theoretical framework under which we build the linkage between equity American put options and credit insurance. Section 2 describes the data selection procedure and the URC construction process from both American puts written on the stock and CDS contracts written on the bond of the same company. Section 3 compares the two sets of URC value estimates and presents supporting evidence for our proposed theoretical linkage. Section 4 provides further theoretical justification for the key assumption underlying our proposed linkage by exploring both reduced-form and structural models that are consistent with the existence of a default corridor. Section 5 offers concluding remarks.

1. Linking DOOM Puts to Credit Protection

The key assumption underlying our proposed linkage is the existence of a default corridor $[A, B]$ that the stock price can never enter. Specifically, we assume that the stock price is bounded below by a strictly positive barrier

$B > 0$ before default, but drops below a lower barrier $A < B$ at default, and is bounded above by A afterward. The existence of such a default corridor is only needed up to the expiry of the American stock options under consideration. An important special case is when the equity value drops to zero upon default, $A = 0$.

Consider an American equity put option with an expiry date T and a strike price K falling within the default corridor, $A \leq K \leq B$. Prior to default, the stock price evolves randomly above B, which is above the strike price of the put option. Therefore, the option will never be exercised conditional on no default. On the other hand, if default occurs at some default time $\tau \leq T$, the stock price jumps at τ to some random recovery level $R_\tau \leq A$ and stays below A afterward. Since A is below the strike price, the American put option will always stay in the money conditioning on the occurrence of default.

Let $\tau_x \in [\tau, T]$ be the exercise time chosen by the put option holder. Since $S_{\tau_x} \leq A \leq K$ by assumption, the continuation value of this American put is just the value of the corresponding forward contract with the same strike, $\mathbb{E}_\tau^{\mathbb{Q}}\left[e^{-r(\tau_x-\tau)}(K - S_{\tau_x})\right]$, where r denotes the continuously compounded interest rate, which we assume is constant for notational clarity, and $\mathbb{E}_\tau^{\mathbb{Q}}[\cdot]$ denotes the expectation operator under the risk-neutral measure $\mathbb{Q}$ and conditional on the time-τ filtration $\mathcal{F}_\tau$. Reasonably assuming that the company suspends any dividends after defaulting, this continuation value becomes $e^{-r(\tau_x-\tau)}$ $K - S_\tau$. Under positive interest rates, this value is maximized by setting $\tau_x = \tau$. In words, it is optimal to exercise all American put options struck within the default corridor at the default time τ.

Now consider two American equity put options with a common expiry date T and two distinct strike prices both falling within the default corridor, $A \leq K_1 < K_2 \leq B$. Let $\triangle K \equiv K_2 - K_1$ denote the strike difference and $\triangle P_t(T) \equiv P_t(K_2, T) - P_t(K_1, T)$ denote the value spread, where $P_t(K_1, T)$ and $P_t(K_2, T)$ are the two observable put option prices at time t. Suppose that an investor buys $\frac{1}{\triangle K}$ units of the K_2 put and writes an equal number of the K_1 puts. The time-t cost of this normalized American put spread is $\triangle P_t(T)/\triangle K$. If no default occurs prior to expiry ($\tau > T$), the put spread expires worthless. If default occurs before or at expiry ($\tau \leq T$), the American put spread pays out one dollar at the time of default so long as both parties behave optimally. Furthermore, so long as the American put prices are consistent with optimal behavior, these prices can be used to value credit derivatives.

To illustrate this point, consider a URC contract, which pays one dollar at τ if $\tau \leq T$ and zero otherwise. Let $U(t, T)$ denote the time-t value of this claim. Assuming constant interest rates (r) and default arrival rates (λ), the value of this URC is

$$U(t, T) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r\tau} 1(\tau < T)\right] = \int_t^T \lambda e^{-(r+\lambda)s} ds = \lambda \frac{1 - e^{-(r+\lambda)(T-t)}}{r + \lambda}. \tag{1}$$

For comparison, we can write the risk-neutral default probability over the same horizon as

$$\mathbb{D}(t, T) = \mathbb{E}_t^{\mathbb{Q}}[1(\tau < T)] = 1 - e^{-\lambda(T-t)}, \tag{2}$$

which is the forward price of a claim paying one dollar at expiry if there is a prior default. Comparing the two expressions, we obtain the following inequality:

$$U(t, T) \leq \mathbb{D}(t, T) \leq e^{r(T-t)} U(t, T), \quad r \geq 0. \tag{3}$$

The risk-neutral default probability is higher than the present value of the URC, but lower than the forward price of the URC given the payment timing difference.

Since the URC and the put spread have exactly the same payoff, no-arbitrage dictates that they should have the same price. Thus, if the market prices of two American puts $P_t(K_2, T)$ and $P_t(K_1, T)$ are available, we can infer the value of the URC from them:

$$U^p(t, T) = \frac{P_t(K_2, T) - P_t(K_1, T)}{K_2 - K_1}, \tag{4}$$

where the superscript p denotes the information source as American put options on the underlying stock.

Conversely, if the interest rate (r), the risk-neutral default arrival rate (λ), and the equity recovery level R_τ for a company are known, one can price an American-style put option on the company's stock that is struck within the default corridor. In particular, assuming that the interest rate and default arrival rate are constant and that the stock price recovers just to the present value of A, i.e., $R_\tau = Ae^{-r(T-\tau)}$, we can derive the American put option value analytically. Since the American put option will be exercised only upon default, we have

$$\begin{aligned} P_t(K, T) &= \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r\tau}[K - R_\tau]1(\tau \leq T)\right] \\ &= \int_t^T \lambda e^{-\lambda s} e^{-rs}[K - Ae^{-r(T-s)}]ds \\ &= K\left[\lambda \frac{1 - e^{-(r+\lambda)(T-t)}}{r + \lambda}\right] - Ae^{-rT}\left[1 - e^{-\lambda(T-t)}\right]. \end{aligned} \tag{5}$$

Equation (5) shows that the value of an American put struck within the default corridor depends only on the default risk of the company, but not on the pre-default stock price dynamics. In particular, conditional on a fixed default arrival rate λ, the American put value does not depend on the stock price level and hence exhibits zero delta. Similarly, the American put value does not depend on the pre-default stock return volatility and in this sense has zero vega.

The American put value does depend on the equity recovery level R_τ; however, the value of a vertical spread of two American puts both struck within the default corridor does not. This value is purely proportional to the strike price difference. Given the validity of our assumptions, the proportionality coefficient represents the value of the URC.

Exchange-traded individual stock options in the U.S. are all American-style, making them perfect candidates for inferring the value of URCs on the company. Had the put options been European-style, the normalized European-style put spread would pay one dollar at the option expiry if and only if default occurs before or at the option expiry. The forward value of this European-style put spread would just be the risk-neutral default probability over the horizon of the option maturity, $\mathbb{D}(t, T)$.

On the other hand, the most actively traded credit contract takes the form of a credit default swap (CDS) written on corporate bonds. A CDS contract provides protection against credit risk. The protection buyer pays a fixed premium, termed the CDS spread, to the seller for a period of time. If a certain pre-specified credit event occurs, the protection buyer stops making the premium payment and the protection seller pays the par value in return for the corporate bond. The CDS spread is set such that the values of the premium leg and the protection leg are equal at the inception of the contract.[3] Assuming fixed and known bond recovery rate R^b, constant interest rate, and constant default arrival rate, it is well known that the CDS spread k has a flat term structure and is proportional to the constant default arrival rate, $k = \lambda(1 - R^b)$. Thus, we can also compute the URC value from a single CDS spread as

$$U^c(t, T) = \xi k \frac{1 - e^{-(r+\xi k)(T-t)}}{r(t, T) + \xi k}, \quad \xi = 1/(1 - R^b), \tag{6}$$

where the superscript c on the URC value $U(t, T)$ reflects that the information is from the CDS market. Assuming a constant interest rate and default arrival rate, a CDS spread of any maturity can be used as the CDS term structure is flat. The Appendix discusses the relation between CDS contracts and URCs under more general conditions.

2. Sample Selection and Data Construction

To gauge the empirical validity of the simple theoretical linkage between deep out-of-the-money American puts and credit protection, we estimate the URC values from both American puts on a company's stock and CDS spreads on the

3 Currently, the over-the-counter CDS market is undergoing structural and contractual reforms. To reduce counterparty risk, the market is moving toward central clearing. To facilitate netting, the market is also moving toward contract specifications with upfront payments and fixed coupons of either 100 or 500 basis points.

same company's corporate bonds. Our analysis covers three and a half years from January 2005 to August 2008. The options data are from OptionMetrics. The CDS spreads are from Bloomberg.

First, we construct a reference date list on every Wednesday from February 2, 2005, to August 27, 2008. When the Wednesday is a holiday, we choose the previous business day of that week. On each chosen date, we look through the options data to select a list of companies with put options that satisfy the following criteria: (1) The bid price is greater than zero; (2) The open interest is greater than zero; (3) The time-to-maturity is greater than 360 days; (4) The strike price is \$5 or less; and (5) The absolute value of the put's delta is not larger than 15%. For companies with multiple put options that satisfy the above criteria, we choose the put option with the highest open interest.

The requirements on non-zero bid price and non-zero open interest are used to ensure that the put price is valid and that there is genuine interest in the option contract. The maturity requirement is to minimize the term mismatch with the corresponding CDS contract. The combined requirements of a low strike price and a low delta are used to identify strikes within the default corridor.

Our model assumes the existence of a default corridor $[A, B]$, which the stock price can never enter. We do not know the location of this corridor ex ante. If we could observe American put prices across a continuum of strikes at the same maturity, this corridor would reveal itself because American put prices are linear in the strike price within the default corridor, as shown in Equation (5). The slope of this linear relation is equal to the value of the URC. Outside the default corridor, the American put price is usually considered to be a strictly convex function of the strike price.

In reality, options are only listed at a finite number of strikes. Detecting the default corridor requires additional assumptions. To help identify the corridor, we assume in our empirical analysis that the stock price drops to zero upon default, i.e., $A = 0$. We also set the lower of the two strikes in the put spread to zero so that we only need a single put to create the desired payoff. To locate the strike of this put option, we require both a low strike (\$5 or less) and a low delta to ensure that the chosen strike is below the upper barrier B.

Once the put contract is chosen, we take the mid quote of the American put option $P_t(K, T)$ and divide the mid quote by its strike price K to arrive at the URC value, $U^p(t, T) = P_t(K, T)/K$.

The above procedure selects a total of 452 companies over 187 reference weeks. For each company of these companies, we retrieve its Bloomberg ticker for the five-year CDS. Out of the 452 companies, 152 companies have a valid five-year CDS ticker. Some of the 152 companies have the CDS ticker, but do not have valid CDS quotes during the relevant sample period. For companies with a valid five-year CDS quote k_t at the required sample date, we estimate the value of the URC at the corresponding put option maturity T by assuming a fixed bond recovery rate of 40% ($R^b = 40\%$) and constant interest and default rates,

$$U^c(t,T) = \xi k_t \frac{1 - e^{-(r(t,T)+\xi k_t)(T-t)}}{r(t,T) + \xi k_t}, \tag{7}$$

where $\xi = 1/(1 - R^b)$ and $r(t,T)$ denotes the time-t continuously compounded spot interest rate of maturity T. We obtain U.S. dollar LIBOR and swap rates from Bloomberg and strip the continuously compounded spot interest rate $r(t,T)$ based on a piecewise constant forward rate assumption. At each reference date and for each chosen company for that reference date, we compute the two sets of URC values $U^p(t,T)$ and $U^c(t,T)$ *daily* for a 60-trading-day window centered on the reference date. We use the 30 days of data before the reference date for contemporaneous regressions on control variables, and we use the 30 days of data after the reference date for a forecasting exercise. Cross-market comparisons of the two sets of URC values are performed on the reference dates.

The exchange-listed American stock options are at fixed expiry dates, but the over-the-counter CDS quotes are at fixed time to maturities. In earlier versions of this article, we retrieved CDS quotes at one-, two-, and three-year maturities, and linearly interpolated the CDS spreads across maturities to obtain a CDS spread at the maturity matching the option expiry date, with which we compute the URC value according to Equation (7). By matching the maturities between the options and the CDS, we tried to reduce the potential bias due to maturity mismatch. However, CDS quotes are most readily available and most reliable at five years to maturity. By requiring companies to have reliable CDS quotes at one-, two-, and three-year terms, we ended up with a very small universe of companies. To obtain a larger universe of companies with reliable CDS quotes, we have decided to use five-year CDS spreads in the current analysis.

For companies with both American put quotes and five-year CDS quotes in the required sample period, we further filter the data and require that the URC values computed from the CDS market be no less than 3%. A 3% URC value corresponds to a \$0.15 mid price for a \$5-strike American put. For companies with even lower default probabilities, our non-zero bid requirement on the American put selection and the discreteness of the tick size would artificially overestimate the default probabilities from the options.[4] The intention of our 3% minimum is to mitigate this bias. We also require that the URC value computed from the put option be less than one. By no-arbitrage, the American put price should always be lower than its strike price and hence the URC value computed from the put should always be less than one. We use this criterion as a filter for data errors, which happen in a few cases.

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[4] The tick size is five cents for options less than three dollars and ten cents for options above three dollars. Deep out-of-the-money options can have two to four ticks as the bid-ask spread. Most recently, a pilot program was started in quoting options on certain stocks in penny increments.

After all the filtering, our final sample includes 121 companies at 186 reference weeks.[5] The number of companies at each week ranges from 10 to 61, with an average of 28 companies. At long maturities and for deep out-of-the-money put options, open interest concentrates on two strikes at $2.5 and $5. Our selected sample includes 5,276 option contracts, with 1,622 struck at $2.5 and 3,635 struck at $5. The remaining 9 contracts are struck at $4. The maturities of the chosen option contracts at the reference date range from 360 to 955 days, with an average of 568 days. Panel A in Figure 1 plots the number of selected companies at each reference date of our sample period. The number of companies increased markedly since mid-2007, coinciding with the start of the financial crisis. Panel B of Figure 1 plots the number of selected put options contracts across different times to maturity.

3. Comparative Analysis

At each reference date, we generate a list of companies that have viable quotes for both deep out-of-the-money put options on their stocks and CDS spread quotes on their corporate bonds. From the two data sources, we generate two estimates on the value of the same URC contract. If our proposed theoretical linkage is valid and the assumptions underlying our empirical implementations are reasonable, we should expect that (i) the two sets of URC value estimates are close to each other in magnitude; (ii) their time-series and cross-sectional variations show strong co-movements; and (iii) their deviations are temporal rather than permanent and hence predict future movements in the put options and the CDS.

3.1 General characteristics of the URC value estimates

The circles in Figure 2 represent the 5,276 pairs of URC value estimates for 121 different companies and at 186 reference dates. The 45-degree dash-dotted line represents our null hypothesis that the two sources of estimates should be the same. The data points scatter around the 45-degree line. The scattering deviations from the 45-degree line reveal potential data noise and/or violations of our theoretical hypothesis and implementation assumptions.

Table 1 reports the summary statistics of the URC values estimated from the two markets. The statistics show that the two sources of estimates are similar in average magnitudes and other statistical behaviors. The estimates from the put options have a slightly smaller sample mean and median, but a slightly larger standard deviation than the estimates from the CDS. The two sets of estimates have a cross-correlation of 70.34%.

To see how the two sets of estimates relate to each other, we perform various regression analyses. The results are summarized in Table 2. Panel A reports the ordinary least squares regression results. When we regress U^p on U^c,

[5] We fail to obtain any data on May 31, 2006.

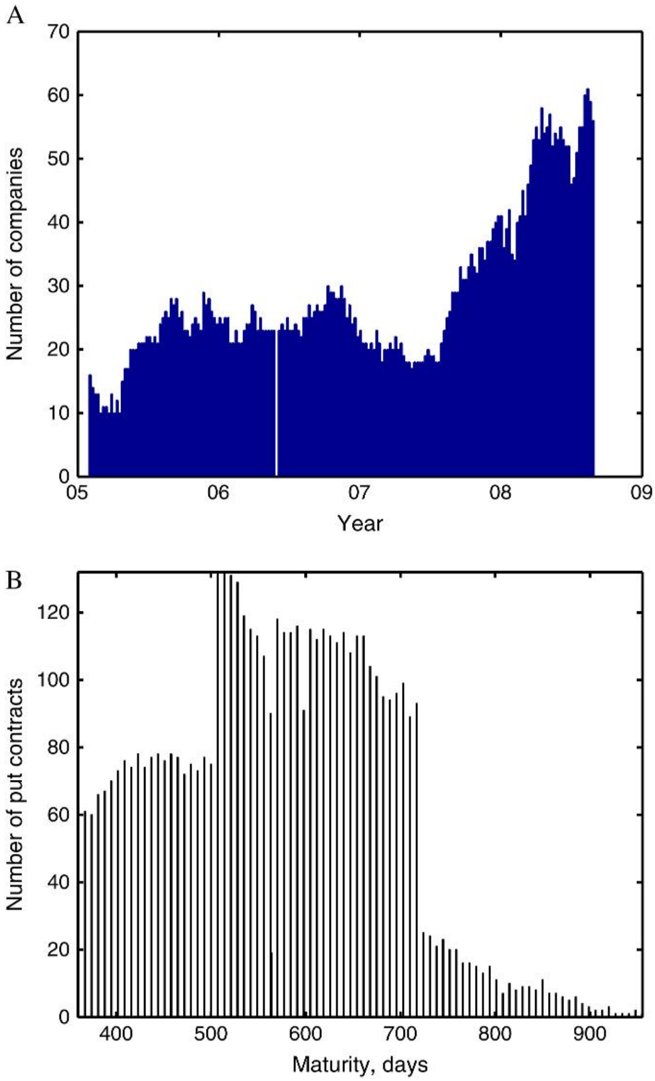

Figure 1
Sample characteristics
Panel A plots the number of companies chosen at each reference date of our sample period. Panel B plots the number of chosen put options across different times to maturity.

we obtain a slope coefficient of $\beta_{pc} = 0.727$. Reversely, when we regress U^c on U^p, the slope estimate is $\beta_{cp} = 0.681$. The fact that both slope estimates are lower than one suggests that both of the two sets of URC values contain measurement errors. Measurement errors in the regressor induce a downward bias in the slope estimate.

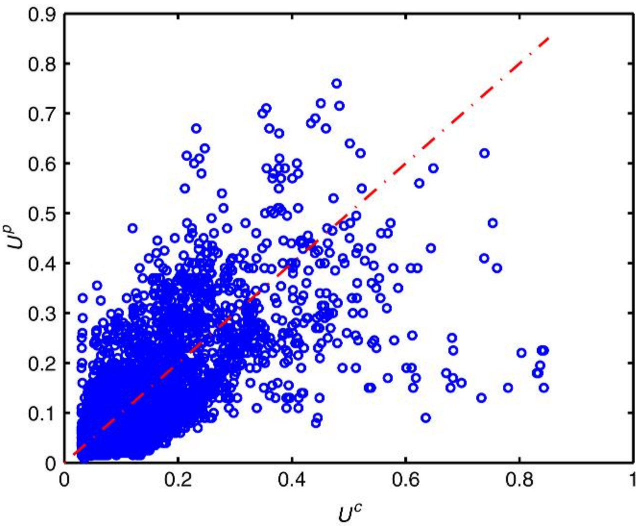

Figure 2
Scatter plots of the two sets of URC value estimates
Circles denote the two sets of URC value estimates over 186 weeks and for 121 different companies. The dash-dotted line reflects the null hypothesis that the two sets of estimates should be identical.

Table 1
Summary statistics of URC estimates from DOOM puts and CDS

	Mean	Median	Min	Max	Std
U^p	0.111	0.080	0.009	0.760	0.101
U^c	0.127	0.099	0.030	0.843	0.098

Entries report the sample average (Mean), median, minimum (Min), maximum (Max), and standard deviation (Std) of the two sets of URC estimates on 5,276 contracts over a sample of 121 different companies at 186 reference dates. U^p denotes the value estimated from deep out-of-the-money American puts on the company's stock. U^c denotes the value estimated from the five-year CDS spread on the company's bond. The two sets of estimates have a cross-correlation estimate of 70.34%.

To correct for the errors-in-variables issue, we also perform the Deming (1943) regression, or the total least squares regression, under which both the dependent variable (y) and the independent variable (x) are assumed to be estimated with error:

$$y = y^* + \varepsilon, \quad x = x^* + \eta, \tag{8}$$

where (y^*, x^*) denote the underlying true values and (ε, η) denote the measurement errors. Assume that the underlying true values have a linear relation: $y^* = \alpha + \beta x^*$, the total least squares regression, estimates the linear relation

Table 2
Ordinary and total least squares regressions relating the two sets of URC estimates

Relation	Intercept		Slope		R^2(%)
	Panel A. Ordinary least squares				
$U^p = \alpha_{pc} + \beta_{pc} U^c$	0.019	(0.003)	0.727	(0.026)	49.47
$U^c = \alpha_{cp} + \beta_{cp} U^p$	0.051	(0.001)	0.681	(0.015)	49.47
	Panel B. Total least squares				
$U^p = \alpha_{pc} + \beta_{pc} U^c$	−0.022	(0.004)	1.047	(0.036)	86.31
$U^c = \alpha_{cp} + \beta_{cp} U^p$	0.021	(0.003)	0.955	(0.033)	83.99

Entries report the ordinary least squares (in Panel A) and total least squares (in Panel B) estimation results of a linear relation between the two sets of URC estimates. The total least squares estimation assumes equal measurement error variance in the two sets of URC value estimates. The standard errors (in parentheses) for the parameter estimates are obtained from bootstrapping. The R-squares (R^2) for the total least squares are defined as one minus the ratio of the variance of the measurement error to the variance of the original series for the dependent variable.

by minimizing the weighted sum of the squared measurement errors from both sources,

$$\min_{\alpha,\beta,x^*} \sum_{i=1}^{n} \left(\frac{(\varepsilon_i)^2}{\sigma_\varepsilon^2} + \frac{(\eta_i)^2}{\sigma_\eta^2} \right), \tag{9}$$

where n denotes the number of observations and $(\sigma_\varepsilon^2, \sigma_\eta^2)$ denote the measurement error variance. If the variance ratio $\delta = \sigma_\varepsilon^2/\sigma_\eta^2$ is known, one can compute the linear relation coefficients as

$$\beta = \frac{s_{yy} - \delta s_{xx} + \sqrt{(s_{yy} - \delta s_{xx})^2 + 4\delta s_{xy}^2}}{2 s_{xy}}, \quad \alpha = \overline{y} - \beta \overline{x}, \tag{10}$$

where $(\overline{x}, \overline{y})$ denote the sample mean and s_{xx}, s_{xy}, s_{yy} denote the sample variance and covariance estimators of x and y:

$$s_{xx} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2, \quad s_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}),$$

$$s_{yy} = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \overline{y})^2. \tag{11}$$

The filtered values of x and y are given by

$$x^* = x + \frac{\beta}{\beta^2 + \delta}(y - \alpha - \beta x), \quad y^* = \alpha + \beta x^*. \tag{12}$$

Panel B of Table 2 reports the slope estimates under the assumption of equal measurement error variance from both sets of URC value estimates ($\delta = 1$).

With the correction for measurement errors, the slope coefficients are no longer significantly different from the null value of one. Under the total least squares regression, we define the R-squared as one minus the ratio of measurement error variance to the variance of the original series, $1-\sigma_\varepsilon^2/\sigma_y^2$. By using information from both markets, one can explain a higher percentage of variation in the two markets.

The intercept estimates from the total least squares regressions remain significantly different from the null hypothesis of zero. In particular, the intercept on U^p is significantly lower than zero ($\alpha_{pc} = -2.2\%$) and the intercept on U^c is significantly greater than zero ($\alpha_{cp} = 2.1\%$). Over the whole sample period, the difference between the two sets of URC values ($U^p - U^c$) has a sample average of -1.56%, a median of -2%, and a standard deviation of 7.66%.

To see how the bias varies over time, we also compute the cross-market differences at different reference dates. Figure 3 plots the median difference (the solid line) and the 25th and 75th percentiles (dash-dotted lines) at each reference date. Historically, the URC values estimated from the American puts are lower than those from the CDS market, in line with the general perception of practitioners: Selling credit insurance through the CDS market and buying deep out-of-the-money puts to hedge the credit risk have been regarded as a profitable trading strategy. However, this general bias seems to have disappeared since the start of the financial crisis in mid-2007.

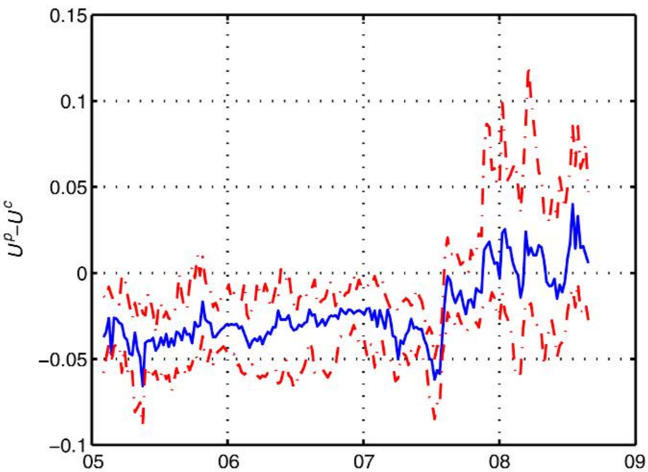

Figure 3
Cross-market deviations in URC values over different reference dates
The solid line plots the median difference at each reference date between the URC values estimated from the put options (U^p) and that from the CDS quote (U^c). The two dash-dotted lines represent the 25th and 75th percentiles.

3.2 Characterizing cross-market deviations in URC values

To understand whether and how the cross-market deviations in URC values are related to the characteristics of the chosen put option contract and the company, we regress the deviations on a list of put option contract and company characteristics. The regressions are performed on the pooled data of 5,276 pairs of URC value estimates over 186 reference dates and 121 companies. Each regression includes one characteristic and a calendar-date dummy variable to control for the calendar-day effect. The dependent variable is the cross-market deviation measured in either level differences ($U^p - U^c$) or log relatives ($\ln U^p - \ln U^c$). Table 3 reports the slope estimates, with standard errors in parentheses, and the R-squared from each regression.

First, we analyze whether the cross-market deviation is related to the URC level of the company. We use the average of the two estimates $(U^p + U^c)/2$ as a proxy for the URC level. The slope estimates change signs as we alter specifications, suggesting that the deviation does not depend on the URC level in any clear manner.

To analyze whether the deviation is related to the moneyness level of the put option, we consider two moneyness measures: the Black-Scholes delta (in absolute magnitude), |Delta|, and the log strike price (K) deviation from the spot level (S) of the stock, $\ln(K/S)$. For both measures, the lower the magnitude,

Table 3
Cross-sectional characteristics of cross-market deviations in the URC estimates

y	$U^p - U^c$			$\ln U^p - \ln U^c$		
X	Slope		R^2	Slope		R^2
$(U^p + U^c)/2$	−0.082	(0.012)	18.1	0.519	(0.519)	27.6
|Delta|	0.540	(0.024)	24.9	6.557	(6.557)	47.3
$\ln(K/S)$	0.041	(0.002)	23.5	0.410	(0.410)	38.5
IV_p	0.018	(0.004)	17.7	0.171	(0.171)	27.5
$ATMV$	0.076	(0.006)	20.2	0.734	(0.734)	31.9
RV^{30}	0.038	(0.003)	21.0	0.256	(0.256)	31.2
RV^{360}	0.054	(0.005)	19.5	0.457	(0.457)	30.9
TD/BE	−0.003	(0.001)	18.6	−0.028	(−0.028)	29.9
TD/MC	0.050	(0.011)	18.2	0.508	(0.508)	28.5
DF	0.080	(0.009)	18.8	0.807	(0.807)	29.8
OI	−0.297	(0.033)	18.7	−0.841	(−0.841)	27.3

Entries report the slope estimates (with standard errors in parentheses) and the R-squared from regressing the cross-market deviations in the URC value estimates on each of the listed characteristics (X) of the put option contracts and the companies. The cross-market deviations are defined in level differences ($U^p - U^c$) on the left-hand side and log differences ($\ln U^p - \ln U^c$) on the right-hand side. Each regression contains one of the characteristics (X) and a calendar-day dummy variable. Among the chosen characteristics, $(U^p + U^c)/2$ acts as a proxy for the average level of the URC value, |Delta| denotes the Black-Scholes delta (in absolute magnitude) of the put option, $\ln(K/S)$ measures how far the strike price (K) is away from the current spot level of the stock (S), IV_p denotes the Black-Scholes implied volatility of the put option contract, *AMTV* is the interpolated one-year 50-delta put option implied volatility, RV^{30} and RV^{360} are 30- and 360-business-day realized variance estimators, *TD* denotes the total book value of debt, *BE* denotes the total book value of common equity, and *MC* denotes the market capitalization. *DF* denotes a default-probability measure estimated based on the structural model of Merton (1974) using total debt, one-year at-the-money option implied volatility, and market capitalization. *OI* is the open interest of the put option contract.

the further away the put option strike price from the spot level. Regressing the cross-market deviations on the two moneyness measures generates significantly positive slope estimates in all specifications, suggesting that, given fixed URC value from the CDS market (U^c), the put option value (U^p) increases as the spot price moves closer to the chosen strike price. Since our chosen strike prices are largely fixed at either \$2.5 or \$5, the moneyness variation mainly reflects the variation of the stock price level. If our theoretical assumption is correct and the chosen strike price is within the default corridor, the American put value should not depend on the stock price once the default risk is fully accounted for. Thus, the positive dependence of the cross-market deviation on the delta measures suggests either or both of two possible scenarios: (1) Our assumptions are wrong and the chosen put options have an explicit stock price dependence in addition to its dependence on default risk; or (2) the stock price movements reveal credit risk information not fully captured by the CDS market. In the latter case, a falling stock price may be a sign of increasing default risk. If this increasing default risk is not fully captured by the CDS spread, we would identify a positive relation between the cross-market deviation between the two URC value estimates and the falling stock price.

We also analyze whether the cross-market deviations depend on the volatility of the stock. We choose four different volatility measures: the Black-Scholes implied volatility of the chosen put option contract (IV_p), the one-year 50-delta put interpolated Black-Scholes implied volatility ($ATMV$), and the 30- and 360-business-day realized variance (RV^{30} and RV^{360}). The option implied volatilities are available from OptionMetrics. The realized variance estimators are available from Bloomberg. All four volatility measures yield positive slope estimates, suggesting that the put value increases with stock volatility. Most options have strictly positive volatility dependence under normal circumstances, yet under our model, the American put options struck within the default corridor do not have explicit dependence on stock return volatility once the default risk of the company is accounted for. Thus, once again, the positive slope estimates suggest either or both of two possibilities: (1) Our model assumptions are wrong and the chosen put option has explicit volatility dependence in addition to its dependence on default risk; or (2) increasing volatility reflects increasing default risk that is not fully captured by the corresponding CDS spread.

When we regress the deviation on financial leverage measures (the ratio of total book debt to total common equity), the slope is negative when the book value of equity (BE) is used, but positive when market value of common equity (MC) is used. We also estimate a default probability (DF) based on the Merton (1974) structural model using the total debt (TD), the market capitalization (MC), and the one-year 50-delta put implied volatility (ATMV) as input, and use the option maturity as the target debt maturity. Unconditionally, the structural model default probability estimate has a correlation of 89.8% with the URC value computed from the American put (U^p) and a lower correlation of 61.9% with the URC value computed from the CDS spread (U^c). When we regress the cross-market deviation on this default probability estimate, the slope coefficients are all positive, suggesting that the American put may contain extra credit-risk information in addition to that contained in the CDS spread.

Finally, we also observe that the cross-market deviation becomes more negative when the open interest of the put option is high. Potentially, the open interest is due to hedge demand from credit insurance sellers. The hedge demand becomes higher when the put options are regarded as cheap compared to the corresponding CDS contract.

Since we have chosen several similar variables for each category of the characteristics, putting all the chosen variables in one multivariate regression is not a viable choice. On the other hand, results from the different univariate regressions show how the cross-market deviations depend on different aspects of the option and firm characteristics.

According to our model, if the put strike is within the default corridor, the put value depends only on the default risk and the equity recovery level upon default. Once the credit risk is controlled for, the put premium has no additional dependence on the option delta or pre-default stock return volatility. The regression results show that the difference between the put- and CDS-implied URC values reacts positively to the absolute delta, volatility, and default probabilities estimated from Merton's structural model, and negatively to the options open interest. Such dependence can come from violations of our model assumption on the existence of the default corridor and assumptions made in the empirical implementation. For example, if the stock price can diffuse to the chosen strike of the put option either because there is no default corridor or because the chosen strike is above the corridor, both diffusion volatility and the delta of the option will affect the put value.

Furthermore, we assume that the stock price drops to zero upon default, $R_\tau = 0$. If the stock recovery value at default R_τ is strictly above zero, the scaled American put option $P_t(K, T)/K$ will pay off $(K - R_\tau)/K$ at default when the default time is at or prior to the option expiry date. This payoff is lower than the one dollar from the URC contract. Accordingly, the scaled American put value under estimates the URC value. The lower the strike price for the chosen put option (and hence the lower the absolute put option delta), the larger this downward bias becomes.

When we estimate the URC values from the CDS spreads, we also make a series of simplifying assumptions. First, we assume that the bond recovery rate is known and fixed at 40%. Different recovery rates assumed by the market participants would bias our URC value estimate from the CDS spread. For example, during the most recent financial crisis, the prospect of government bailout of certain financial firms' debt during a bankruptcy can significantly lower the CDS spreads for these firms even if their default probabilities and hence the American put price on their stocks remain high. Second, we use the five-year CDS spread to compute the URC value at the put option maturity based on a constant default arrival rate assumption. If the CDS term structure is not flat, the maturity mismatch between the put option and the five-year CDS would induce another bias. Third, the CDS contract also protects against credit events other than bankruptcy, such as distressed exchanges and missed payments. In these cases, the stock price might not jump through the default corridor and the proposed linkage might not hold. Finally, since the CDS contracts are over-the-counter products, counterparty risk can potentially bias the valuation and thus generates another deviation from the American put market.[6] The current efforts toward central clearing of over-the-counter CDS contracts can potentially reduce the counterparty risk and remove this particular bias.

The cross-market deviations in the URC values can also come from temporary default risk information delays in either market. Potentially, high stock volatility, high leverage, and low stock price can all indicate heightened credit risk. If such information is not fully reflected in the CDS market, it can show up as a deviation between the URC values estimated from the put and the CDS markets. On the other hand, if the default risk revealed in the CDS market has not been fully captured by the stock and stock options market, the American put value and the stock option volatility can be temporarily lower, and the stock price temporarily higher, than the corresponding fully informed values should be.

3.3 Information flow between the American put and the CDS markets

If the cross-market deviations between the two URC value estimates are purely due to violations of our model and implementation assumptions, the deviations shall be fully explained by current variables and shall not have any forecasting power on future movements in either the American put premium or the CDS spread. On the other hand, if the deviations are due to temporary information delays in either market or data noise, current deviations will become predictive of future price movements in the corresponding put and CDS contract. Furthermore, if one of the URC values fully reflects the default risk information, the deviation will be driven by mispricing (or lagged information flow) on the other contract. In this case, the deviation will predict the future price of movement of the mispriced contract, but will not predict the price movement of the fully informed contract. Hence, analyzing relations between the cross-market deviation and future price movements in the American put and the CDS contract provides further evidence on the overall validity of our assumptions and on the information flow between the two markets.

To analyze the cross-market information flow, we define the cross-market deviations as $D_t = U_t^p - U_t^c$, and we use this deviation to predict future

6 We thank the referee for pointing out several of the cases that can induce bias between the two markets.

price changes in the URCs in the two markets: $\Delta U^j_{t+\Delta t} = U^j_{t+\Delta t} - U^j_t$, with $j = p, c$ denoting the put and the CDS market, respectively, and Δt denoting the forecasting horizon. Under our model assumption, the calculated URC values reflect purely default risk. To account for potential violations of the assumption, we also regress the deviation D_t on various option and company characteristics (X_t),

$$D_t = a + bX_t + \mathcal{D}_t, \tag{13}$$

and use the regression residual $\mathcal{D}_t$ to predict future URC value movements.

At each reference date and for each chosen company for that reference date, we perform the contemporaneous regression in Equation (13) using daily data over the past 30 trading days to obtain the residual $\mathcal{D}_t$. For each control variable X_t, 5,276 daily regressions are performed. Then, the 5,276 regression residuals at the 186 reference dates are pooled for the following forecasting regressions:

$$\Delta U^p_{t+\Delta t} = \alpha^p + \beta^p \mathcal{D}_t + e_{t+\Delta t}, \tag{14}$$

$$\Delta U^c_{t+\Delta t} = \alpha^c + \beta^c \mathcal{D}_t + e_{t+\Delta t}, \tag{15}$$

where t denotes the 186 reference dates and Δt denotes the forecasting horizon. If the cross-market deviation is driven by information delay or data noise in the American puts, we would expect this noise to dissipate in the future and hence a negative slope β^p on the predictive regression in Equation (14). On the other hand, if the American puts contain additional credit risk information not yet fully reflected in the CDS market, we would expect a positive slope β^c on the predictive regression in Equation (15). The specifications in (14) and (15) are in the spirit of the error-correction model of Engle and Granger (1987). We can think of the URC value estimates as containing two components: One is the long-run equilibrium value of the contract, and the other is a short-term noise or market distortion component. Although the fundamental value of the URC is close to be a random walk, the cross-market deviations of the two estimates should be stationary and should converge to the equilibrium value.

Table 4 reports the forecasting regression results. We consider a similar list of control variables (X) as in the previous section (Table 3). When $X = 1$, we simply demean the cross-market deviation by its sample average over the past 30 days to obtain the demeaned residual $\mathcal{D}_t$. For the remaining list of variables, we perform the daily regression in (13) to remove its dependence on that particular variable before we perform the predictive regression.

Although we use the same variables as in Table 3, the regressions are quite different. Table 3 performs pooled (both cross-sectional and time-series) regressions on the 5,276 pairs of put-CDS contracts over 186 reference dates and 121 different companies. By contract, the daily regressions in (13) are performed on each of the 5,276 pairs separately. While the pooled regression

Table 4
Predicting future market movements based on current cross-market deviations in URC estimates

Horizon		$\Delta t = 7$ Days						$\Delta t = 30$ Days					
X	$\overline{R^2}$	β^p		R^2_p	β^c		R^2_c	β^p		R^2_p	β^c		R^2_c
1	0.0	−0.122	(0.013)	1.7	0.138	(0.011)	2.9	−0.194	(0.028)	0.9	0.274	(0.019)	4.0
$(U^p+U^c)/2$	54.0	−0.209	(0.025)	1.4	0.043	(0.022)	0.1	−0.380	(0.054)	1.0	0.097	(0.036)	0.1
|Delta|	51.1	−0.203	(0.017)	2.6	0.202	(0.015)	3.4	−0.320	(0.038)	1.3	0.357	(0.025)	3.7
$\ln(K/S)$	27.1	−0.190	(0.018)	2.2	0.207	(0.015)	3.5	−0.312	(0.039)	1.3	0.353	(0.026)	3.6
IV_p	54.0	−0.037	(0.018)	0.1	0.247	(0.015)	5.1	−0.047	(0.039)	0.0	0.433	(0.025)	5.4
$ATMV$	23.7	−0.116	(0.017)	0.9	0.180	(0.014)	3.0	−0.147	(0.036)	0.3	0.352	(0.024)	4.1
RV^{30}	16.9	−0.137	(0.017)	1.3	0.141	(0.015)	1.8	−0.102	(0.037)	0.2	0.248	(0.025)	2.0
RV^{360}	19.5	−0.156	(0.018)	1.5	0.189	(0.016)	2.9	−0.080	(0.040)	0.1	0.327	(0.026)	3.0
TD/BE	15.6	−0.162	(0.043)	1.2	0.261	(0.034)	5.1	−0.039	(0.088)	0.0	0.535	(0.059)	6.9
TD/MC	25.6	−0.183	(0.018)	2.1	0.215	(0.016)	3.9	−0.271	(0.040)	1.0	0.379	(0.026)	4.3
DF	22.0	−0.124	(0.017)	1.1	0.171	(0.015)	2.8	−0.169	(0.037)	0.4	0.311	(0.025)	3.2
OI	18.5	−0.176	(0.022)	1.8	0.166	(0.020)	1.9	−0.132	(0.048)	0.2	0.341	(0.033)	2.9

First, the cross-market deviations in the URC estimates on each of the 5,276 contracts are regressed against a series of variables (X) on that particular company using daily data over the past 30 days,

$$U_t^p - U_t^c = a + bX_t + \mathcal{D}_t.$$

The second column in the table under $\overline{R^2}$ reports the average value of the R-squares from the daily contemporaneous regressions on each variable X, with $X = 1$ denoting the case where the cross-market deviations is just demeaned by its average value of the past 30 days. From each of the daily regressions, we record the regression residual $\mathcal{D}_t$ at the last date of the regression. Second, the regression residuals are used to predict future changes in the two sets of URC value estimates ($\Delta U^p_{t+\Delta t}$ and $\Delta U^c_{t+\Delta t}$),

$$\Delta U^p_{t+\Delta t} = \alpha^p + \beta^p \mathcal{D}_t + e_{t+\Delta t}, \quad \Delta U^c_{t+\Delta t} = \alpha^c + \beta^c \mathcal{D}_t + e_{t+\Delta t}.$$

The remaining columns of the table reports the slope estimates (with standard errors in parentheses) and the R-squares of the forecasting regressions at forecasting horizons of seven and 30 days.

with a calendar-day dummy variable captures the deviations across different contracts, the daily regression captures the time-series movements of the same contract. The second column under $\overline{R^2}$ in Table 4 reports the average R-squares from the 5,276 daily regressions for each variable. The highest average R-squares come from the average URC value ($(U^p + U^c)/2$), the absolute delta (|Delta|), and the put option implied volatility (IV_p).

For the predictive regressions, we consider two forecasting horizons: $\Delta t =$ 7 days and $\Delta t = 30$ days. Over both forecasting horizons and regardless of the choice of the control variables, the slope estimates β^p for the predictions on the American puts are all strongly negative, and the slope estimates β^c for the predictions on the CDS are all strongly positive. The results support the presence of two-way information flow. The cross-market deviations predict future movements in both markets. Comparing the forecasting results based on different control variables, we observe that controlling for the moneyness of the put option (either delta or log strike over spot) yields high forecasting R-squares for both the American puts and the CDS spread. Comparing the results at the two forecasting horizons, the predictability on the CDS market becomes stronger at the longer forecasting horizon, while that on the American put market becomes weaker. We have also experimented with multivariate regressions on a selected number of control variables. These multivariate regressions increase the R-squares of the contemporaneous regressions, and the regression residuals often show slightly higher forecasting powers on future market movements. The qualitative conclusions, however, are the same.

Taken together, our identified simple theoretical linkage between American puts and the CDS market enjoys strong empirical support. Although there are many possible ways that our model and the implementation assumptions can be violated, we find that the URC values estimated from the two markets share similar magnitudes and other statistical behaviors. When we estimate a linear relation between the two sets of estimates, the slope estimate is not statistically different from the null hypothesis value of one. When the estimates from the two markets deviate from each other, the deviation predicts future movements in both markets to the direction of their future convergence.

4. Default-corridor Consistent Dynamics

The key assumption enabling our proposed linkage is the existence of a default corridor $[A, B]$ that the stock price can never enter. In this section, we provide theoretical justification for the existence of such a default corridor. We show that the default corridor can be readily and reasonably accommodated by both reduced-form models and structural models. First, we show that by combining two classic models, i.e., the jump-to-default model of Merton (1976) and the displaced diffusion model of Rubinstein (1983), we can create a reduced-form model that not only matches the observed option price behavior better, but also generates the desired default corridor in the stock price dynamics. Then, we show that the default corridor can be made consistent with a class of firm value dynamics and thus with the structural modeling approach pioneered by Merton (1974).

4.1 The defaultable displaced diffusion stock price dynamics

Black and Scholes (1973) and Merton (1973) propose an option pricing model (henceforth the BMS model) that has since revolutionized the derivative industry. Under the BMS model, the stock price follows a geometric Brownian motion. The risk-neutral stock price dynamics are

$$S_t = S_0 e^{rt+\sigma W_t - \frac{1}{2}\sigma^2 t}, \tag{16}$$

where σ denotes the constant instantaneous return volatility and W_t denotes a standard Brownian motion. We assume zero dividend and constant interest rate r for notational clarity.

Under the BMS model, the stock price S_t can never hit zero if it starts at a strictly positive level $S_0 > 0$; however, for companies with positive default probabilities, the stock price going to zero is a definite possibility. Merton (1976) generalizes the BMS dynamics by allowing the possibility for the stock price to jump to zero and stay there upon default. With a constant arrival rate λ for such a jump, the pre-default risk-neutral stock price dynamics becomes

$$S_{t-} = S_0 e^{(r+\lambda)t+\sigma W_t - \frac{1}{2}\sigma^2 t}, \tag{17}$$

where the subscript $t-$ denotes the pre-jump level at time t. We refer to this model as the Merton-jump-to-default (MJD) model.

In evaluating the validity of any extension of the BMS option pricing model, it has become common practice to use the notion of option implied volatility. The implied volatility of an option is defined as the constant volatility input that one must supply to the BMS option pricing model in order to have the BMS model value agree with a given option price. The given option price can be produced by either a model value or a market price. The model implied volatility of the BMS model is invariant to strike price. In contrast, the model implied volatility of the MJD model is always decreasing in the strike price, generating what is commonly referred to as an implied volatility skew. Carr and Laurence (2006) derive the following short-maturity approximation for the implied volatility function under the MJD model

$$IV_t(d_2, T) \approx \sigma + \frac{N(d_2)}{N'(d_2)}\sqrt{T-t}\lambda,$$
$$d_2 = \frac{\ln(S_t/K) + r(T-t) - \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}, \tag{18}$$

where $N(\cdot)$ denotes the cumulative probability function of a standard normal variate and $N'(\cdot)$ denotes the standard normal probability density function. Under this approximation, the slope of the implied volatility skew against the standardized measure of moneyness d_2 evaluated at $d_2 = 0$ is simply $\sqrt{T-t}\lambda$ and is hence directly determined by the probability of default.

Motivated by the possibility of producing an analytically tractable European option pricing model for which implied volatilities can decrease or increase in strike, Rubinstein (1983) introduces the displaced diffusion option pricing model, henceforth the RDD model. In this model, Rubinstein introduces a new parameter $B \in (-\infty, S_0)$ and suggests that the risk-neutral process for the underlying stock price be constructed by summing Be^{rt} and a geometric Brownian motion,

$$S_t = Be^{rt} + (S_0 - B)e^{rt+\sigma W_t - \frac{1}{2}\sigma^2 t}. \tag{19}$$

As a result, the state space of the underlying stock price process at any time t is (Be^{rt}, ∞) rather than $(0, \infty)$. When $B > 0$, the stock price has a strictly positive lower bound and the option implied volatility for this model is monotonically increasing with the strike price. At short maturities, the implied volatility function can be approximated as

$$IV(K) \approx \frac{\ln(K/S)}{\ln(K-B)/(S-B)}\sigma, \tag{20}$$

where the implied volatility starts at zero when the strike is below the lower bound and increases monotonically as the strike increases. The implied volatility converges to σ as the strike price approaches infinity.

It is interesting to compare the two BMS generalizations: Merton's generalization introduces a scale factor $e^{\lambda t}$ on the allowed sample paths, while Rubinstein's generalization introduces a shift Be^{rt}. Under the MJD model, the underlying stock price has a state space of $[0, \infty)$ and the model implied volatility can only decline in strike price. Under RDD, the underlying stock price has a state space at time t of (Be^{rt}, ∞) and the model implied volatility can only increase with the strike when $B > 0$. Neither model can produce a U-shaped relation between implied volatility and strike price, a shape that is the most commonly observed in individual stock options and is often referred to as the implied volatility smile.

We propose to combine the two classic generalizations to both scale and shift the sample paths. We label this new model as *defaultable displaced diffusion*, or the DDD model. We start with the stock price S following a displaced diffusion as in Rubinstein (1983) over the fixed time horizon $t \in [0, T]$. The stock price process is assumed to be positively displaced, with $B \in (0, S_0]$. Then, we depart from Rubinstein's model by adding the possibility of a down jump in the stock price. If the stock price jumps at some time $\tau \in [0, T]$, we assume that it jumps to a deterministic recovery level $R(\tau) = Ae^{-r(T-\tau)}$. Zero recovery ($A = 0$) is allowed as a special case within our specification. After the jump time τ, the stock price grows deterministically at the risk-free rate r: $S_t = R(t) = R_0 e^{rt}$ for $t \geq \tau$. The risk-neutral arrival rate of the jump in the stock price is assumed to be constant at λ.

To formally model the risk-neutral stock price dynamics in the DDD model, we use G to denote a geometric Brownian motion,

$$G_t = e^{\sigma W_t - \sigma^2 t/2}, \tag{21}$$

starting at $G_0 = 1$, and we let J be another martingale started at one defined by

$$J_t = 1(N_t = 0)e^{\lambda t}, \tag{22}$$

where N_t denotes a standard Poisson process with a constant arrival rate λ. This process drifts up at a constant growth rate of λ and jumps to zero and stays there at a random and exponentially distributed time. We construct the risk-neutral stock price process S by combining the two random processes G and J as

$$S_{t-} = e^{rt}\{R_0 + J_{t-}[B - R_0 + (S_0 - B)G_t]\}. \tag{23}$$

To understand this stock price process, we can rewrite it as the sum of three components,

$$S_{t-} = R_0 e^{rt} + [B - R_0]e^{rt} J_{t-} + (S_0 - B)e^{rt} J_{t-} G_t. \tag{24}$$

Each component has the form of a constant, multiplied by e^{rt}, and then a martingale. Therefore, each component can be interpreted as the time-t value resulting from investing the constant in an asset whose initial price is one. In this sense, we are attributing the equity value of a firm to returns from three types of investments. The first type is a riskless cash reserve defined by the deterministic and non-negative process $R(t) = R_0 e^{rt}$. Zero riskless cash reserve would result in zero equity recovery upon default. The second type is a defaultable risky cash reserve defined by the stochastic and non-negative process $[B - R_0]e^{rt} J_{t-}$, with the martingale J_{t-} capturing the default risk. The third type of investment is a defaultable and market risky asset defined by the stochastic and non-negative process $(S_0 - B)e^{rt} J_{t-} G_t$, with the martingale G_t capturing the market, diffusion-type risk. Under the risk-neutral measure, all three investments generate a risk-neutral expected return of r.

If we set $\lambda = 0$, the DDD model degenerates to the positively displaced diffusion of Rubinstein (1983), for which implied volatility increases with strike. If we set $B = 0$ and assume zero equity recovery, the model degenerates to the MJD model of Merton (1976), for which implied volatility decreases with strike. When λ and B are both positive, the implied volatility displays a smile when graphed against the strike price. This model behavior is broadly consistent with the behavior of implied volatilities obtained from market prices of listed American stock options.

Under the DDD model, $J_t = e^{\lambda t}$ prior to default and $G_t > 0$. Hence, Equation (24) implies that the stock price S_t exceeds $B(t) \equiv R_0 e^{rt} + [B - R_0]e^{(r+\lambda)t} \geq B$. At the default time τ, $J_\tau = 0$, and the stock price jumps down to the riskless cash reserve level $R(\tau)$ and will henceforth evolve risklessly according to $R(t) = R_0 e^{rt}$. Now consider a fixed horizon $T > t$ with $A = R(T)$. Then, the stock price process is random and at least B before default occurs and becomes deterministic and at most A after default, $t \in [\tau, T]$. Thus, this DDD model generates the default corridor $[A, B]$, within which the stock price cannot reside prior to option expiry.

We use the DDD model to show that one can readily construct a simple stock price dynamics that not only matches the stylized behavior of the observed stock options but also exhibits a default corridor. In fact, this default corridor can be retained even if we relax the assumptions of the DDD model substantially. In particular, we can generalize the model along four major dimensions by allowing (i) stochastic interest rates; (ii) stochastic stock recovery after default as long as the price stays below the lower barrier A; (iii) stochastic default arrival; and (iv) stochastic volatilities and jumps in the pre-default stock price dynamics as long as the price stays above the upper barrier B prior to default.

4.2 A motivating structural model that generates the default corridor

The structural modeling approach pioneered by Merton (1974) is widely used by both academics and practitioners as it relates a firm's default probability to its financial leverage and business risk. Unfortunately, Merton's structural model does not support the existence of a default corridor. Under his assumptions on the capital structure and geometric Brownian motion for the firm's value, the equity would be worth zero right before default and stay at zero afterward. In this section, we propose a simple structural model that generates the default corridor in the equity value.

We start by assuming that the debt of the firm has a principal D, a maturity date T, and a continuously paid coupon. The firm can default on either the coupons or principal, but the coupon rate is nonetheless assumed to be equal to the contemporaneous risk-free rate $r_t \geq 0$, which evolves randomly over time. Hence, the company is assumed to have issued floating rate debt with a continuous coupon of size $r_t D$ dollars per unit time. The only other claimants to the firm's cash flows are shareholders who receive dividends continuously at rate $q_t V_t$ dollars per unit time, where we refer to $q_t \geq 0$ as the dividend yield and V_t denotes the time-t value of the firm.

Prior to any default, the coupons to the debt holders and the dividends to the shareholders are financed by any combination of asset sales or additional equity issuance. At any time, the equity holders can choose to stop paying the coupon by declaring bankruptcy. This declaration eliminates all future dividends, causing the share price to hit zero for the first time.

As a result of these assumptions, the firm's equity holders may be viewed as holding a so-termed "cancelable" American call option on the firm value. This call option is issued with strike D and maturity T. It is similar to a standard American call option written on the firm value in that it can be exercised at any time $t \in [0, T]$ for the amount $V_t - D$. Like a standard American call, the cancelable call has the value $(V_T - D)^+$ at expiry. Unlike a standard American call, the cancelable call has a holding cost, given by $r_t D - q_t V$ per unit time. The American call that the shareholders possess is said to be cancelable because the shareholders can at any time cancel it by refusing to pay any more coupons, at the cost of foregoing subsequent dividends and the optionality.

Since the shareholders do receive the cash flows thrown off by the assets prior to any cancellation, it is never optimal for them to exercise their call early for $V_t - D$. In contrast, it can be optimal for them to cancel early and they will optimally cancel when the share price first hits zero from above. The following parity condition links the value $C_t^C(D, T)$ of a cancelable call to the value $P_t^V(D, T)$ of the corresponding standard American put, where both the call and the put are written on the firm value:

$$C_t^C(D, T) = V_t - D + P_t^V(D, T). \tag{25}$$

The first term on the right-hand side capitalizes the dividends paid to shareholders if default was not an option, while the second term capitalizes the coupons under no-default. The final term corrects for the additional value shareholders gain by being allowed to default. The optimal default time for the shareholders is the optimal exercise time for the standard American put. The cancelable call value hits zero for the first time just as the put value hits its exercise value $D - V_t$ for the first time. This parity condition holds for any assumptions on the dynamics of V, r, and q.

As we have no taxes or bankruptcy costs, the value of the floating rate debt, $F_t(D, T)$, is just given by the difference between the firm value and the equity:

$$F_t(D, T) = V_t - C_t^C(D, T). \tag{26}$$

Substituting (25) in (26) implies that the value of the floating rate debt can alternatively be understood as the difference between par and the value of the bankruptcy put:

$$F_t(D, T) = D - P_t^V(D, T). \tag{27}$$

In contrast to the standard paradigm developed by Merton, the bankruptcy put here is American-style, implying that default can occur at any time. Equations (26) and (27) hold for any assumptions on the dynamics of V, r, and q. As a consequence, any result developed in the now voluminous literature on American puts implies a corresponding result for the floating rate debt.

We now further assume that the firm value V follows defaultable displaced dynamics with a default corridor $[A^V, B^V]$, where A^V and B^V sandwich the principal of the debt, i.e., $A^V < D < B^V$. Then, it is never optimal to exercise the American put option on the firm value before the firm defaults, as the exercise value $D - V_t$ is negative. Furthermore, it is optimal to exercise the American put as soon as the company defaults, as the firm value falls below A^V and stays below it afterward. Before default, the firm's equity has a strictly positive value represented by the cancelable call option, $C_t^C(D, T) = V_t - D + P_t^V(D, T)$ for $t < \tau$. In particular, if we let $B^V = D + B$ for $B > 0$, then the equity value stays above $B > 0$, since $V_t - D > B$. In contrast, at the default time τ, the equity value falls to zero since it becomes optimal to exercise the American put immediately and hence $P_\tau^V(D, T) = D - V_\tau$. Thus, assuming defaultable displaced dynamics on the firm value with a corridor $[A^V, D + B]$ leads to defaultable displaced dynamics for the equity value with a corridor $[0, B]$. This motivating example shows that a default corridor in the stock price can also be readily accommodated in a structural model that starts with the firm value dynamics.

5. Concluding Remarks

The literature on strategic default often predicts that a company goes to default strategically well before its firm value falls below its debt. Under these predictions, the stock price stays above a strictly positive barrier prior to default. On the other hand, firm and equity values often experience sudden drops upon default due to deadweight losses such as legal fees and liquidation costs. In this article, we develop a class of models for the stock price that are consistent with these observations. Prior to default, stock price are bounded below by a positive constant B. After default, the stock price drops and stays below another constant $A < B$. We refer to the region $[A, B]$ as the default corridor, within which the stock price can never reside. Such a default corridor on the stock price can also be made consistent with a structural model when the firm value dynamics also exhibit a default corridor.

When the default corridor exists in the stock price, we show that a vertical spread of stock American put options, struck within the default corridor and scaled by the difference in strikes, has the same payoff as a standardized credit claim paying one dollar at default if this event occurs before the options expire, and paying zero otherwise. In the important special case of zero equity recovery ($B = 0$), we can use one American put struck within the default corridor scaled by the strike price to replicate this standardized credit insurance contract, which we label as the unit recovery claim. The replication of this contract is simple and is robust to the details of the stock price dynamics before and after default. Since the two positions pay off the same amount at the same random time, the replication is also robust to the dynamics of interest rates and default arrival rates.

We use the value of the American put spread to infer the value of the unit recovery claim and compare it to the value estimated from the credit default swap market. Collecting data from both markets over 186 weeks and for 121 different companies, we show that the unit recovery claim values estimated from the two markets share similar magnitudes and show strong co-movements. When the estimates from the two markets deviate from each other, the deviations predict future movements in both markets due to the future convergence.

Our identified linkage provides fertile ground for future research. On the theoretical side, research effort can be directed toward specifying the trading strategy that should be enacted when arbitrage arises. On the empirical side, much work is needed in investigating how the put strikes should be chosen and how to deal with maturity mismatches between the two markets.

Appendix: Linking Unit Recovery Claims to CDS Spreads

We analyze how to use URC values to determine the CDS spread in a fairly robust way. Let $V^{prot}(t, T)$ denote the time-t value of the protection leg of a CDS contract with expiry T, and let R^b denote the recovery rate on the corporate bond in case of default. The protection leg pays $1 - R^b$ at the time of default (τ) if $\tau \leq T$ and zero otherwise. The recovery rate on corporate bonds is in general not known ex ante, and it can vary with different bonds and different default situations. Nevertheless, also actively traded nowadays are swaps on the recovery rate of corporate bonds on default. Thus, one can use the recovery swaps to remove the uncertainty in the bond recovery in the credit default swap contract.

Assuming that the recovery rate R^b is known and fixed, we observe that the payoff of the protection leg of the CDS is simply $1 - R^b$ times the payoff of a URC. No arbitrage implies that

$$V^{prot}(t, T) = (1 - R^b)U(t, T). \tag{A1}$$

To value the premium leg of a CDS contract, let $A(t, T)$ denote the value of a defaultable annuity that pays \$1 per year continuously until the earlier of the default time τ and its maturity date T. This annuity value can be written as

$$A(t, T) = \int_t^T \mathcal{S}(t, s)ds, \tag{A2}$$

where $\mathcal{S}(t, s)$ denotes the prevent value of a survival claim that pays one dollar at time s if $\tau > s$ and zero otherwise. Assuming that the risk-free rate is a deterministic function of time $r(t)$, we can write the value of the survival claim in terms of the URC term structure,

$$\mathcal{S}(t, T) = e^{-\int_t^T r(s)ds} - U(t, T) + \int_t^T r(s)e^{-\int_s^T r(u)du}U(t, s)ds. \tag{A3}$$

The equation reflects the fact that the survival claim has the same payoff as the portfolio consisting of (1) long one default-free bond paying \$1 at T and costing $e^{-\int_t^T r(s)ds}$ at time t; (2) short one URC maturing at T and costing $U(t, T)$; and (3) long $r(s)e^{-\int_s^T r(u)du}ds$ units of URC for each maturity $s \in [t, T]$, with each unit costing $U(t, s)$. If no default occurs before T, the default-free bond in the portfolio pays off the desired dollar, while all of the URC contracts expire worthless. If the default occurs before T, the value of the portfolio at the default time $\tau \in [t, T]$ becomes

$$e^{-\int_\tau^T r(s)ds} - 1 + \int_\tau^T r(s)e^{-\int_s^T r(u)du}ds = e^{-\int_\tau^T r(s)ds} - 1 + \int_\tau^T de^{-\int_s^T r(u)du} = 0, \tag{A4}$$

as desired.

Finally, let $k(t, T)$ denote the time-t CDS spread of expiry date T. Assuming continuous premium payments until $\tau \wedge T$, we can represent the CDS spread as

$$k(t, T) = \frac{V^{prot}(t, T)}{A(t, T)}. \tag{A5}$$

Assuming a known bond recovery rate R^b implies that Equation (A1) can be used to relate the numerator to the URC value $U(t, T)$. Assuming deterministic interest rates implies that Equations (A2) and (A3) can be used to relate the denominator to a URC term structure. Making both assumptions allows the CDS spread to be expressed in terms of URC values,

$$k(t, T) = \frac{(1 - R^b)U(t, T)}{\int_t^T \left\{e^{-\int_t^s r(u)du} - U(t, s) + \int_t^s r(u)e^{-\int_u^s r(v)dv}U(t, u)du\right\} ds}. \tag{A6}$$

Reversely, one can numerically infer the URC term structure from the term structure of CDS spreads. Under the assumption of constant interest rates and default arrival rates, we can infer the URC value from one CDS quote as in Equation (6).

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