Option Pricing and Stochastic Calculus FRE6233, Fall 2018
New York University Tandon School of Engineering,
Department of Finance and Risk Engineering
Monty Essid, essid@cims.nyu.edu
Tuesdays 6:00-8:30pm, Rogers Hall Room 201

Prerequisites : Quantitative Methods in Finance FRE-GY 6083 or equivalent

Course Description : This course provides the mathematical foundations of Option Pricing models. The techniques covered include arithmetic and geometric Brownian motion, first passage time, the reflection principle, the stochastic Ito integral, Ito differential Calculus, change of probability measure, martingales, Stochastic Differential Equations and Partial Differential Equations. Some of the pricing models considered are the European, Barrier, Asian and American options. These problems are either solved analytically by the martingale approach or numerically, by applying approximation and simulation methods. Since the same techniques allow the treatment of more complex financial products, examples of credit derivatives will be also presented.

Course Objectives :

- After taking this course, the students will be able to price any common derivative security.
- This introductory course in the Computational Finance track is a building block for more advanced quantitative courses such as FRE 6311, or some of the specialized Topics courses.
- The material taught is part of the common knowledge shared by quants in the Financial industry and taking this course will enable the students to apply for quantitative positions.

Website : Lecture notes, assignments and slides will be posted on the NYU classes website.

Teaching assistant/grader : Senpei Hou, sh4646@nyu.edu

Office hours : Mon 10am-12pm

I will be available by appointment either at the beginning of the class. We can also meet at my adjunct office; Office 518, Courant Institute of Mathematical Sciences, 251 Mercer Street, 10012 New York, NY (you need an NYU ID to have access to the building).

Readings : The required texts for the course are:

- S. Shreve, *Stochastic Calculus for Finance II: continuous-time models*, 2nd edition, 2004, Springer (**Reference we will use the most for the class. Note that I couldn't select this book on the NYU website, but this will be the most useful book to buy**)
- Tomas Bjork, *Arbitrage Theory in continuous time*, Oxford University press, 3rd edition, 2009 (**Very good book, with some overlapping material with Shreve, will be mostly used for the american options part**)

Additional reading (optional) :

- Probability Essentials, Jean Jacod and Philip Protter, Universitext, Second Printing, Second Edition, 2004, Springer (covering Basic Probability)
- Financial Modelling With Jump Processes, Rama Cont and Peter Tankov, 2004, Chapman and Hall/CRC
- Stochastic Differential Equations: An introduction with Applications, Bernt Oksendal, Universitext, Third Printing, Sixth Edition, 2009, Springer (Excellent reference, somewhat slightly more advanced than this class)
- Brownian Motion and Stochastic Calculus (Graduate Texts in Mathematics) (Volume 113) 2nd Edition, Ioannis Karatzas , Steven Shreve (Also one of the Bible of Stochastic Calculus, more advanced than the class)
- Paul Bourgade's Lecture notes available at : http://www.cims.nyu.edu/ bourgade/SA2010/StochasticAnalysis.pdf (Excellent lecture notes, PhD level class)
- Heard on the street: Quantitative Questions from Wall Street Job Interviews, Timothy Falcon Crack, revised 15th Edition, 2014 (very good interview book)
- A Practical Guide To Quantitative Finance Interviews, Xinfeng Zhou (another very good interview book)

Coding : Some assignments will include some coding. The recommended programming languages

- Matlab : https://www.mathworks.com (not free)
- Python : https://www.python.org/. A very good free alternative to Matlab is to use Python with the Scipy,Numpy libraries available at https://www.scipy.org/. An alternative way to install all of these hassle free is to use the Anaconda distribution https://www.continuum.io/downloads
- R available at http://www.r-project.org

Course Requirements : There will be weekly homework sets, an in-class midterm exam on Tue. 10/16 , and a final exam on Tue. 12/18 (in the normal class time and place). The semester grade will be based on the HW (1/3), the midterm (1/3), and the final exam (1/3).

Collaboration on homework is encouraged (homeworks are not exams) but registered students must write up and turn in their solutions individually.
If you work with another student, please name him or her on your solution sheet. HW may not be turned in late; **no extension is allowed**.
I understand that people might punctually get overwhelmed by other classes/events, so I will drop the worst grade out of all HW. This should be used as your 'joker' and it is advised to keep it only in case you can't turn in your HW in time.
Doing the HW is important, not only because it counts as part of the grade, but also because if you don't do the HW, you probably won't do well on the exams.
Copying solutions from other students (or past solution sheets) is not permitted, and not a good idea: you won't have the intended experience, and won't learn the material well.
Homework are due at the beginning of the class (Tuesdays 6:00pm) to be uploaded on NYU classes.

The midterm and final exams will be closed-book, but you may bring one handwritten sheet of notes (8.5" $\times$ 11", both sides) to the midterm, and to the final.

Tentative Schedule :
Part I: Ito Stochastic Calculus

- Week 1 (09/04) : The Brownian motion and filtrations
- Week 2 (09/11) : The Ito integral and Ito's lemma in several dimensions
- Week 3 (09/18) : Application of stochastic calculus to the Black-Scholes model

Part II: The no arbitrage theory in continuous-time

- Week 4 (09/25) : The martingale approach
- Week 5 (10/02) : The Partial Differential Equations approach
- **Week 6 (10/09) : No Class (classes will meet according to a Monday schedule)**
- **Week 7 (10/16) : Midterm Exam**
- Week 8 (10/23) : The Asian Option model
- Week 9 (10/30) : The American option
- Week 10 (11/06) : Finite Difference scheme for the American option
- Week 11 (11/13) : Change of numeraire and application to global finance
- Week 12 (11/20) : T-forward measure and introduction to fixed-income derivatives

Part III: Jump-diffusion processes and incomplete market models

- Week 13 (11/27) : Stochastic Calculus for jump-diffusion processes
- Week 14 (12/04) : Option pricing under jump-diffusion models
- Week 15 (12/11) : Problem solving session
- **Week 15 (12/18) : Final Exam**