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Theory and evidence on the dynamic interactions between sovereign credit default swaps and currency options

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Abstract

Using sovereign CDS spreads and currency option data for Mexico and Brazil, we document that CDS spreads covary with both the currency option implied volatility and the slope of the implied volatility curve in moneyness. We propose a joint valuation framework, in which currency return variance and sovereign default intensity follow a bivariate diffusion with contemporaneous correlation. Estimation shows that default intensity is much more persistent than currency return variance. The market price estimates on the two risk factors also explain the well-documented evidence that historical average default probabilities are lower than those implied from credit spreads.

JEL classification: F34; G12; G13

Keywords: Currency options; Sovereign credit default swaps; Default arrival rate; Return variance dynamics; Term structure of credit spread; Option pricing; Time-changed Lévy processes

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1. Introduction

Economic or political instability in a country worsens its credit quality and often leads to depreciation and heightened volatility in its currency. Such instability generates positive co-movements between sovereign creditworthiness, currency depreciation rates, and currency return volatility. When financial markets are operating efficiently, anticipated changes in the credit quality of a sovereign borrower should be reflected in the prices of credit-sensitive instruments such as sovereign credit default swaps (CDS). Likewise, anticipated changes in the realized volatility of currency returns are reflected in the prices of volatility-sensitive instruments such as currency options. It follows that when instability is anticipated, one byproduct is positive covariation between observable quantities such as CDS spreads and currency option implied volatilities. Using the factor model perspective from modern portfolio theory, one can take the view that sovereign CDS spreads and currency option implied volatilities each have a positive exposure to a country specific risk factor. As a result, they covary positively whenever this factor moves.

This positive covariation also has its support from a microeconomic perspective. Under conditions expounded in Modigliani and Miller (1958), corporate debt and equity sum to the present value of future earnings. Sims (1999) and Cochrane (2001, 2003) propose an analogy between this corporate valuation equation and the budget constraint for an economy, which requires that foreign and domestic debt sum to the present value of the future budget surplus. In this analogy, domestic debt and fiat money act like equity in a corporation. Furthermore, the currency price, e.g., the US dollar price of the local currency, is analogous to the stock price. When instability is anticipated, the currency devalues, its volatility rises (due to the leverage effect), and credit quality deteriorates. Assuming efficient markets, this analogy suggests a positive linkage between sovereign credit spreads and currency return volatilities, analogous to the linkage identified between corporate credit spreads and stock return volatilities.[2]

Motivated by these considerations, this paper examines the relation between sovereign credit spreads and currency return volatilities using data on sovereign CDS spreads and currency options. A sovereign CDS is an over-the-counter contract that provides insurance against sovereign default. The protection buyer pays a fixed fee, called the CDS spread, to the seller until the earlier of maturity or the occurrence of a certain pre-specified credit event. If the credit event occurs prior to maturity, the protection seller pays compensation to the protection buyer. Sovereign CDS contracts have been traded actively on major emerging market economies.

Currency options are natural instruments for trading on currency return volatilities. Furthermore, when a sovereign country defaults on its foreign debt, the country's currency inevitably drops in value by a sizeable amount. As a result, the possibility of default on the sovereign debt generates negative skewness in the conditional distribution of currency returns when the currency price is quoted as the US dollar price per unit of the local currency.[3] This negative skewness shows up in the relative pricing of currency options across

[2] See for example, Bevan and Garzarelli (2000), Frye (2000), Pedrosa and Roll (1998), Collin-Dufresne et al. (2001), Aunon-Nerin et al. (2002), Bangia et al. (2002), Altman et al. (2005), Bakshi et al. (2006), Ericsson et al. (2004), Carr and Wu (2005) and Wu and Zhang (2005).

[3] In this article, we adopt the viewpoint of a US investor without loss of generality. Investors based in other countries can substitute their currency for the dollar.

different strikes. When the Black and Scholes (1973) implied volatility is plotted against some measure of moneyness at a fixed maturity, the average slope of the plot is positively related to the risk-neutral skewness of the currency return distribution. In the over-the-counter currency options market, this slope is directly captured by risk reversal quotes, which measure the implied volatility difference between an out-of-the-money call option and an out-of-the-money put option at the same (absolute) delta. Recent empirical studies, e.g., Carr and Wu (2005) and Cremers et al. (2004), show that corporate CDS spreads covary with both stock option implied volatilities and the slopes of the implied volatility plots against moneyness. Analogously, we conjecture that sovereign CDS spreads covary with both currency option implied volatilities and risk reversals.

We take Mexico and Brazil as two example countries and collect data on their sovereign CDS spreads and over-the-counter currency option quotes. The quotes for CDS spreads are available at three fixed maturities of one, three, and five years. The quotes for currency options are available in terms of delta-neutral straddle implied volatilities, 25-delta risk reversals, and 25-delta butterfly spreads at five fixed maturities at one, two, three, six, and 12 months. We find that the CDS spreads show strong contemporaneous correlations with both the delta-neutral straddle implied volatilities and the risk reversals.

Guided by this evidence, we propose a joint valuation framework for sovereign CDS and currency options on the same reference country. We model sovereign default by a Poisson process with stochastic arrival rate. Upon arrival, the dollar price of the local currency falls by a random amount. The pre-default currency price follows a continuous process with stochastic volatility. We specify the default arrival rate and the diffusion variance rate via a joint bivariate process with dynamic interactions that capture the empirical evidence on sovereign CDS spreads and FX options. We label this model as the CDFX model. Under this joint specification, we derive tractable pricing solutions for currency options and CDS spreads.

We estimate the model jointly on the three sovereign CDS spread series and the 15 currency option series for both Mexico and Brazil. From the estimation, we extract the time series of the diffusion variance rate and the default arrival rate, and learn about the dynamic interactions between the two processes. Our estimates show that the default arrival rate is much more persistent than the diffusion variance rate under both the statistical measure and the risk-neutral measure. The statistical persistence difference suggests that the default arrival rate is more difficult to predict than the diffusion variance rate. The risk-neutral persistence difference implies that the default arrival rate dynamics have larger impacts on longer-term contracts. Furthermore, the market price estimates on the default risk factor are significantly negative in all cases. We show that the negative market price estimates explain the well-documented evidence that historical average default probabilities are lower than those implied from credit spreads.

In related literature, Pan and Singleton (2005) perform specification analysis on default and recovery dynamics using the term structure of sovereign CDS. Zhang (2003) analyzes the market expectations of the Argentina default using sovereign CDS data. Andrizky (2003) analyzes default and recovery dynamics using Argentine Eurobonds. In common with these studies, we study sovereign default dynamics. In contrast to them, we link sovereign default dynamics to currency option pricing, both theoretically and empirically. The identified cross-market linkage and the proposed joint modeling framework provide a basis for cross-market trading and hedging and for economic policy analysis.

The remainder of this paper is organized as follows. The next section describes the data set and documents the covariation between currency option quotes and CDS spreads. Section 3 proposes a joint valuation framework for sovereign CDS and currency options. Section 4 describes the joint estimation procedure. Section 5 presents the results and discusses their implications. Section 6 develops and estimates an extended version of the model with stochastic central tendency on the default arrival rate. Section 7 concludes.

2. Co-movements between sovereign CDS spreads and currency options

We use Mexico and Brazil as two examples and collect data from Bloomberg on sovereign CDS spreads on the two countries and options quotes on the two underlying currencies against the US dollar. The industry quotation convention for the two currencies is the number of Mexican pesos per US dollar and the number of Brazilian reals per US dollar, respectively.

The data are from January 2, 2002 to March 2, 2005, sampled weekly on every Wednesday. For both countries, CDS spread quotes are available at three fixed maturities of one, three, and five years. For over-the-counter currency options, the industry convention is to quote them in the form of delta-neutral straddle implied volatilities, 25-delta risk reversals, and 25-delta butterfly spreads at each maturity.[4] We obtain these quotes at five fixed maturities of one, two, three, six, and 12 months. A straddle is a portfolio of a call option and a put option on the same underlying currency with the same strike and time to maturity. By market convention, the strike price (K) neutralizes the Black–Scholes delta:

$$d_1 = \frac{\ln(F_t/K)}{\mathrm{IV}\sqrt{\tau}} + \frac{1}{2}\mathrm{IV}\sqrt{\tau} = 0, \tag{1}$$

where F_t denotes the forward currency price, τ is the time-to-maturity, and IV is the implied volatility quote that is used with the Black–Scholes formula to generate the invoice price for the underlying option. This implied volatility quote is often referred to as the at-the-money implied volatility (ATMV).

The 25-delta risk reversal (RR) quote measures the difference in Black–Scholes implied volatilities between a 25-delta call option and a 25-delta put option

$$\mathrm{RR} = \mathrm{IV}(25c) - \mathrm{IV}(25p), \tag{2}$$

where $25c$ and $25p$ denote a 25-delta call and put, respectively. Hence, the risk reversal is a direct measure of the slope of the implied volatility plot against moneyness. The 25-delta butterfly spread (BF) measures the difference between the average implied volatility of the two 25-delta options and the delta-neutral straddle implied volatility

$$\mathrm{BF} = (\mathrm{IV}(25c) + \mathrm{IV}(25p))/2 - \mathrm{ATMV}. \tag{3}$$

Hence, the butterfly spread measures the average curvature of the implied volatility plot against moneyness.

For model estimation, we first convert the risk reversal and butterfly spread quotes into implied volatilities at the two deltas. Then, we use the Black–Scholes formula to convert the implied volatilities into out-of-the-money option prices. The conversion requires

[4] See Carr and Wu (forthcoming) and Malz (1996, 1997) for discussions on over-the-counter currency option quoting and trading conventions.

information on domestic and foreign interest rates from one to 12 months. For US dollar interest rates, we download the US dollar LIBOR rates from Bloomberg with maturities from one to 12 months. We directly convert them into continuously compounded interest rates. For interest rates on the two local currencies, we download from Bloomberg the spot and forward exchange rates on the peso and the real against the dollar for the corresponding maturities. We then derive the interest rates on the local currency using covered interest rate parity. To value the CDS spreads, we download from Bloomberg the Eurodollar swap rates and strip the spot rate curve assuming piece-wise constant forward rates.

Table 1 reports the summary statistics on the CDS spreads and currency option quotes. For Mexico, the mean CDS spreads are 0.55% at the one-year maturity, 1.29% at the three-year maturity, and 1.81% at the five-year maturity, generating a steep upward-sloping mean term structure. The standard deviation estimates also show an upward-sloping term structure. The weekly autocorrelation estimates for the three series are all around 0.98, manifesting the high persistence of the spreads. The CDS spreads are positively skewed, but show little excess kurtosis.

Table 1
Summary statistics on sovereign CDS spreads and currency option implied volatilities

Country	Mexico					Brazil				
Statistics	Mean	Std	Auto	Skew	Kurt	Mean	Std	Auto	Skew	Kurt
CDS spreads										
1y	0.55	0.32	0.98	1.06	0.01	8.48	12.04	0.99	1.71	1.41
3y	1.29	0.71	0.98	1.08	0.31	10.86	10.26	0.99	1.51	1.04
5y	1.81	0.82	0.98	0.95	0.06	11.25	8.94	0.99	1.48	1.04
Delta-neutral straddles										
1m	9.56	1.79	0.93	0.08	−0.86	17.96	9.48	0.96	1.41	1.53
2m	9.81	1.66	0.94	0.11	−0.87	18.56	9.11	0.96	1.40	1.47
3m	10.05	1.57	0.94	0.23	−0.57	19.09	8.82	0.97	1.39	1.33
6m	10.52	1.34	0.94	0.33	−0.44	19.99	8.13	0.97	1.36	1.06
12m	10.98	1.09	0.93	0.64	−0.11	21.17	7.44	0.98	1.29	0.63
Risk reversals										
1m	1.57	0.81	0.91	0.28	−0.59	3.18	1.89	0.96	1.15	0.34
2m	1.82	0.83	0.93	−0.04	−0.86	3.73	1.91	0.96	1.22	0.54
3m	2.06	0.97	0.95	−0.12	−0.91	4.15	1.95	0.96	1.30	0.85
6m	2.50	1.01	0.96	−0.32	−1.05	4.91	1.96	0.96	1.25	0.66
12m	3.02	0.88	0.96	−0.31	−0.99	5.73	1.98	0.96	1.24	0.89
Butterfly spreads										
1m	0.54	0.17	0.52	1.18	0.82	0.97	0.58	0.90	1.06	0.60
2m	0.55	0.16	0.57	1.25	0.94	1.10	0.66	0.91	1.07	0.66
3m	0.63	0.21	0.72	0.65	−0.96	1.22	0.72	0.92	1.15	0.92
6m	0.55	0.13	0.54	2.59	11.05	1.43	0.85	0.93	0.92	−0.07
12m	0.63	0.14	0.52	3.22	16.52	1.58	0.93	0.95	1.14	0.48

Entries report the sample estimates of the mean, standard deviation (Std), weekly autocorrelation (Auto), skewness (Skew), and excess kurtosis (Kurt) of sovereign CDS spreads and currency option implied volatilities for Mexico and Brazil, all in percentage points. The currency options are quoted in terms of delta-neutral straddles, 25-delta risk reversals, and 25-delta butterfly spreads for peso and real prices per one dollar. The statistics are based on weekly sampled data (every Wednesday) from January 2, 2002 to March 2, 2005 (166 observations for each series).

For Brazil, the mean CDS spreads are 8.48%, 10.86%, and 11.25% at the three maturities, respectively. The much higher mean spreads suggest that the market expects a higher probability of default during our sample period for Brazil than for Mexico. The mean term structure is also upward sloping, albeit not as steep as for Mexico. Furthermore, the term structure of the standard deviation is downward sloping, implying that long-term CDS spreads vary less. The spreads on Brazil show both positive skewness and positive kurtosis, suggesting that the spreads have experienced large positive movements during our sample period.

The second panel in Table 1 reports the summary statistics of the delta-neutral straddle implied volatility quotes. The mean implied volatilities of the real are more than twice as much as those of the peso, suggesting that returns on the real have been more volatile during our sample period. The mean term structure of implied volatilities are upward sloping for both currencies, but the term structures of standard deviation are both downward sloping. The autocorrelation estimates suggest that the implied volatilities are also persistent, but not as much as the CDS spreads. For the implied volatility of the peso, the skewness estimates are positive but small, and the kurtosis estimates are small and even negative. In contrast, the estimates for both skewness and kurtosis are positive and relatively large for the Brazilian real.

The third panel in Table 1 reports the summary statistics on the 25-delta risk reversals. The mean risk reversals are positive for both currencies, increasingly so as the option maturity increases. The positive risk reversal indicates that the risk-neutral distributions for peso and real returns on investing in US dollars are positively skewed. Conversely, the dollar returns from investing in pesos and reals are negatively skewed. The risk reversals on the real are about twice as much as those on the peso. The risk reversals are about as persistent as the delta-neutral straddle implied volatilities. The last panel in Table 1 reports summary statistics on butterfly spreads. The average butterfly spreads are about half a percentage point for the Mexican peso and about one to two percentage points for the Brazilian real.

To measure the covariation between the sovereign CDS market and the currency options market, we report in Table 2 the cross-correlation estimates between the three CDS spreads series on the one hand and the 15 currency option quotes on the other, both in levels and in weekly differences. The CDS spreads show strong positive correlations with the at-the-money implied volatilities. The estimates between levels range from 0.31 to 0.8 for Mexico and 0.93 to 0.96 for Brazil. The estimates between weekly differences range from 0.29 to 0.4 for Mexico and from 0.34 to 0.5 for Brazil. Strong positive correlations are also observed between levels of CDS spreads and risk reversals, but the estimates on weekly differences are smaller than those between CDS spreads and implied volatilities. The cross-correlation between weekly changes of butterfly spreads and CDS spreads are close to zero, indicating that the variation of butterfly spreads is not very informative.

3. Joint valuation of sovereign CDS and currency options: The CDFX model

We propose an internally consistent framework to value both sovereign CDS spreads and currency options written on the same economy. The model framework captures the observed link between the CDS market and the FX market. We christen our framework as the CDFX model. Under this framework, we model the sovereign default on its foreign debt as a compound Poisson process with a stochastic arrival rate $\lambda(t)$. Upon default, we

Table 2
Correlations between sovereign CDS spreads and currency option implied volatilities

Country	Mexico						Brazil					
CDS	1y	3y	5y	1y	3y	5y	1y	3y	5y	1y	3y	5y
	Levels			Weekly differences			Levels			Weekly differences		
Delta-neutral straddles												
1m	0.31	0.49	0.56	0.34	0.29	0.33	0.93	0.95	0.95	0.47	0.39	0.37
2m	0.39	0.57	0.63	0.39	0.36	0.39	0.93	0.95	0.96	0.47	0.38	0.36
3m	0.44	0.62	0.68	0.38	0.33	0.40	0.93	0.96	0.96	0.48	0.39	0.36
6m	0.52	0.69	0.75	0.28	0.29	0.37	0.93	0.96	0.96	0.50	0.40	0.36
12m	0.60	0.76	0.80	0.30	0.34	0.40	0.93	0.95	0.96	0.50	0.39	0.34
Risk reversals												
1m	0.65	0.74	0.77	0.17	0.19	0.15	0.80	0.83	0.84	0.13	0.12	0.09
2m	0.65	0.73	0.77	0.14	0.18	0.17	0.78	0.82	0.83	0.15	0.13	0.11
3m	0.64	0.72	0.76	0.17	0.21	0.22	0.78	0.82	0.83	0.16	0.13	0.11
6m	0.66	0.74	0.78	0.09	0.15	0.15	0.78	0.81	0.83	0.15	0.14	0.11
12m	0.59	0.68	0.73	0.05	0.11	0.08	0.74	0.77	0.78	0.16	0.14	0.11
Butterfly spreads												
1m	0.05	0.08	0.11	0.01	0.09	0.06	0.44	0.49	0.52	0.03	−0.01	−0.02
2m	0.06	0.10	0.14	−0.00	0.06	0.03	0.48	0.54	0.57	0.02	−0.02	−0.02
3m	−0.24	−0.22	−0.18	−0.01	0.07	0.02	0.51	0.58	0.60	0.03	−0.02	−0.02
6m	0.06	0.15	0.20	−0.02	−0.02	−0.08	0.50	0.58	0.60	0.01	−0.03	−0.02
12m	0.10	0.14	0.16	−0.03	0.02	−0.08	0.53	0.59	0.62	0.02	−0.03	−0.03

Entries report the sample estimates of the cross-correlation in both levels and weekly differences between sovereign credit default swap spreads at maturities of one, two, and three years and currency option implied volatilities quoted in delta-neutral straddles, 25-delta risk reversals, and 25-delta butterfly spreads at maturities of one week, and one, two, three, six, and 12 months. The statistics are based on weekly sampled data (every Wednesday) from January 2, 2002 to March 2, 2005 (166 observations for each series).

assume that the return on the US dollar price of the local currency, $\ln(P_t/P_{t-})$, drops by a sizable amount, q, so that the post-default price becomes $P_t = e^{-q}P_{t-}$. Thus, e^{-q} measures the recovery rate on the currency price. The random variable q captures the magnitude of the currency's depreciation upon default.

Formally, let $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{t\geqslant 0}, \mathbb{Q})$ be a complete stochastic basis and let $\mathbb{Q}$ be a risk-neutral probability measure, under which we assume the following currency price dynamics:

$$dP_t/P_t = (r(t) - r^*(t))dt + \sqrt{v(t)}dW_t^s + ((e^{-q} - 1)dJ(\lambda(t)) - \zeta\lambda(t)dt), \tag{4}$$

where the first term represents the interest rate differential between the home (US) and foreign economies, which defines the instantaneous drift of the currency price under the risk-neutral measure $\mathbb{Q}$. The second term uses a Brownian motion W_t^s to capture market movements of the currency price under normal conditions, with $v(t)$ denoting the instantaneous variance rate, which we allow to be stochastic. The last term is a jump martingale that captures the impact of a country's default on the currency price, with $(e^{-q} - 1)dJ(\lambda(t))$ denoting the increments of a Poisson process with stochastic arrival rate $\lambda(t)$ and random jump size $(e^{-q} - 1)$. Conditional on default occurring, we assume that q is normally distributed with mean μ_j and variance v_j under measure $\mathbb{Q}$, with $\zeta = e^{-\mu_j + v_j/2} - 1$ being the mean percentage jump size.

In Eq. (4), we use a continuous process to capture the market risk component of the exchange rate dynamics. In reality, even in the absence of a sovereign default event, exchange rates can move discontinuously (Bates, 1996; Carr and Wu, forthcoming; Daal and Madan, 2005; Bakshi et al., 2005). Thus, it is appropriate to introduce a jump component for the market risk. Although it is straightforward to do so theoretically, our estimation exercise shows that we cannot identify an additional jump component for the market risk. Intuitively, such a jump component has its impact mainly on the tail behavior of the currency return; yet, our currency option quotes are either at-the-money or at 25-delta, and hence do not provide much information beyond the 25 percentile of the distribution. Thus, we choose the more parsimonious specification in (4) for our subsequent analysis.

3.1. Joint dynamics of diffusion variance rate and default arrival rate

We decompose currency movements into two sources of risk: the market risk, which captures the normal market movements through W^s, and the credit risk, the impact of which is captured by a compound Poisson process. The intensities of the two sources of risks, $v(t)$ and $\lambda(t)$, vary randomly over time. We specify the following joint dynamics under the risk-neutral measure $\mathbb{Q}$:

$$\mathrm{d}v(t) = (\theta_v - \kappa_v v(t))\mathrm{d}t + \sigma_v \sqrt{v(t)}\mathrm{d}W_t^v, \tag{5}$$

$$\lambda(t) = \beta v(t) + z(t), \tag{6}$$

$$\mathrm{d}z(t) = (\theta_z - \kappa_z z(t))\mathrm{d}t + \sigma_z \sqrt{z(t)}\mathrm{d}W_t^z, \tag{7}$$

$$\rho^{sv} \equiv \mathbb{E}[\mathrm{d}W^s \mathrm{d}W^v]/\mathrm{d}t, \quad \rho^{sz} \equiv \mathbb{E}[\mathrm{d}W^s \mathrm{d}W^z] = 0, \quad \rho^{zv} \equiv \mathbb{E}[\mathrm{d}W^z \mathrm{d}W^v] = 0. \tag{8}$$

The diffusion variance rate follows a mean-reverting square-root process. The default arrival rate co-varies with the diffusion variance rate through the loading coefficient β. A positive coefficient generates positive contemporaneous correlation between the two sources of risk. In addition, we allow the default arrival rate to have an independent source of variation $z(t)$, which is modeled as another square-root process. Finally, based on the analogy between the currency price for a sovereign country and the stock price for a corporation, we anticipate the correlation ρ^{sv} in Eq. (8) between currency returns and return volatilities to be negative. In contrast, the other pairs of Brownian motions have zero correlation by the independence assumption on $z(t)$.

3.2. Pricing currency options

Taking the dollar as the home currency, we consider the time-t dollar value of a European call option on the dollar price of a local currency with strike price K and expiry date T, $c(P_t, K, T)$. We can write the value as a discounted risk-neutral expectation over the terminal payoff:

$$c(P_t, K, T) = \mathbb{E}_t\left[\exp\left(-\int_t^T r(s)\,\mathrm{d}s\right)(P_T - K)^+\right] = B(t, T)\mathbb{E}_t[(P_T - K)^+],$$

where $\mathbb{E}_t[\cdot]$ denotes the expectation operator under measure $\mathbb{Q}$ conditional on the filtration $\mathcal{F}_t$ and $B(t, T)$ denotes the time-t price of a default-free zero-coupon bond with a face value of one dollar and expiry T. We can solve the expectation by inverting the following generalized Fourier transform on the log currency return

$$\phi(u) \equiv \mathbb{E}_t[\mathrm{e}^{\mathrm{i}u\ln(P_T/P_t)}], \quad u \in \mathcal{D} \subset \mathbb{C}, \tag{9}$$

where $\mathscr{D}$ denotes the subset of the complex plane under which the expectation is well-defined.

Under the dynamics specified in Eqs. (4)–(8), the Fourier transform is exponential affine in the two risk factors $x_t \equiv [v_t, z_t]$ (Duffie et al., 2000):

$$\phi(u) = \exp(\mathrm{i}u(r(t,T) - r^*(t,T))\tau - a(\tau) - b(\tau)^\top x_t), \quad \tau = T - t, \tag{10}$$

where $r(t,T)$ and $r^*(t,T)$ denote the continuously compounded spot interest rate for the domestic and foreign currencies, respectively, at time t and maturity date T, and the time-homogeneous coefficients $[a(\tau), b(\tau)]$ can be solved from the following set of ordinary differential equations:

$$\begin{aligned} a'(\tau) &= b(\tau)^\top \theta, \\ b'(\tau) &= \beta_x - (\kappa^{\mathbb{M}})^\top b(\tau) - \frac{1}{2}\Sigma(b(\tau) \odot b(\tau)), \end{aligned} \tag{11}$$

starting at $a(0) = 0$ and $b(0) = 0$, with $\odot$ denoting the Hadamard product, and

$$\theta = \begin{bmatrix} \theta_v \\ \theta_z \end{bmatrix}, \quad \beta_x = \begin{bmatrix} (\psi(u) + \mathrm{i}u\zeta)\beta + \frac{1}{2}(\mathrm{i}u + u^2) \\ \psi(u) + \mathrm{i}u\zeta \end{bmatrix}, \quad \kappa^{\mathbb{M}} = \begin{bmatrix} \kappa_v - \mathrm{i}u\sigma_v\rho & 0 \\ 0 & \kappa_z \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_z^2 \end{bmatrix}, \tag{12}$$

where $\psi(u) = 1 - \exp(-\mathrm{i}u\mu_j - v_j u^2/2)$ denotes the characteristic exponent of the default-induced currency return jump conditional on $\lambda(t) = 1$. The ordinary differential equations can be solved analytically. Given the Fourier transform $\phi(u)$, option prices can be obtained via fast Fourier inversion (Carr and Wu, 2004).

3.3. Pricing CDS spreads

For a sovereign CDS contract initiated at time t and with maturity date T, let $S(t,T)$ denote the CDS spread, the fixed premium rate paid by the buyer of default protection. Assuming continuous payment and one dollar notional, we can write the present value of the premium leg as

$$\mathsf{Premium}(t,T) = S(t,T)\mathbb{E}_t\left[\int_t^T \exp\left(-\int_t^s (r(u) + \lambda(u))\,\mathrm{d}u\right)\mathrm{d}s\right]. \tag{13}$$

The present value of the protection leg of the contract is

$$\mathsf{Protection}(t,T) = w\mathbb{E}_t\left[\int_t^T \lambda(s)\exp\left(-\int_t^s (r(u) + \lambda(u))\,\mathrm{d}u\right)\mathrm{d}s\right], \tag{14}$$

with $(1 - w)$ denoting the recovery rate, which is set to 40%. By setting the present values of the two legs equal, we can solve for the CDS spread as

$$S(t,T) = w\frac{\mathbb{E}_t\left[\int_t^T \lambda(s)\exp\left(-\int_t^s (r(u) + \lambda(u))\,\mathrm{d}u\right)\mathrm{d}s\right]}{\mathbb{E}_t\left[\int_t^T \exp\left(-\int_t^s (r(u) + \lambda(u))\,\mathrm{d}u\right)\mathrm{d}s\right]}, \tag{15}$$

which can be regarded as a weighted average of the expected default loss.

Our default arrival dynamics specification in Eqs. (5)–(8) satisfy the affine conditions identified in Duffie et al. (2000). We can solve for the present values of the two legs of the contract in analytical form. The value of the premium leg is

$$\mathsf{Premium}(t,T) = S(t,T)\int_t^T B(t,s)\exp(-a_\lambda(s-t) - b_\lambda(s-t)^\top x_t)\,\mathrm{d}s, \tag{16}$$

with

$$\begin{aligned} a'_\lambda(\tau) &= b_\lambda(\tau)^\top\theta, \\ b'_\lambda(\tau) &= \beta_\lambda - \kappa^\top b_\lambda(\tau) - \frac{1}{2}\Sigma(b_\lambda(\tau)\odot b_\lambda(\tau)), \end{aligned} \tag{17}$$

starting at $a_\lambda(0) = 0$ and $b_\lambda(0) = 0$ and with

$$\beta_\lambda = \begin{bmatrix}\beta \\ 1\end{bmatrix}, \quad \kappa = \begin{bmatrix}\kappa_v & 0 \\ 0 & \kappa_z\end{bmatrix}. \tag{18}$$

The present value of the protection leg is,

$$\mathsf{Protection}(t,T) = w\int_t^T B(t,s)(c(s-t) + d(s-t)^\top x_t)\exp(-a_\lambda(s-t) - b_\lambda(s-t)^\top x_t)\,\mathrm{d}s, \tag{19}$$

where the coefficients $[a_\lambda(\tau), b_\lambda(\tau)]$ are the same as in (17), and the coefficients $[c(\tau), d(\tau)]$ can be solved from the following set of ordinary differential equations:

$$\begin{aligned} c'(\tau) &= d(\tau)^\top\theta, \\ d'(\tau) &= -\kappa^\top d(\tau) - \Sigma(b_\lambda(\tau)\odot d(\tau)), \end{aligned} \tag{20}$$

starting at $c(0) = 0$ and $d(0) = \beta_\lambda$. In model estimation, we approximate the integrals in (16) and (19) using summations with quarterly intervals. Combining the solutions for the present values of the two legs solves for the CDS spread $S(t,T)$.

3.4. Market prices and statistical dynamics

Our joint estimation identifies both the statistical dynamics and the risk-neutral dynamics of the bivariate state vector x_t. To derive the statistical dynamics for the bivariate vector x_t under the physical measure $\mathbb{P}$, we assume that the market prices of risks are proportional to the risk level, $\gamma\sqrt{x_t}$, with γ being a diagonal matrix with diagonal elements $[\gamma_v, \gamma_z]$. Under this assumption, the time-series dynamics are

$$\mathrm{d}x_t = (\theta - \kappa^{\mathbb{P}}x_t)\mathrm{d}t + \sqrt{\Sigma x_t}\mathrm{d}W_t^{\mathbb{P}}, \quad \text{with } \kappa^{\mathbb{P}} = \kappa - \sqrt{\Sigma}\gamma. \tag{21}$$

4. Joint estimation of diffusion variance and default rate dynamics

We estimate the bivariate dynamics jointly using sovereign CDS spreads and currency option quotes. We cast the model into a state-space form and estimate the model parameters using the quasi-maximum likelihood method. In the state-space form, we regard the bivariate risk vector as the unobservable state and specify the state-propagation equation using an Euler approximation of the statistical dynamics in Eq. (21):

$$x_t = \theta\Delta t + \varphi x_{t-1} + \sqrt{\Sigma x_{t-1}\Delta t}\varepsilon_t, \tag{22}$$

where $\varphi = \exp(-\kappa^{\mathbb{P}}\Delta t)$ denotes the autocorrelation coefficient with Δt being the length of the discrete time interval, and ε denotes an iid bivariate standard normal innovation. We sample the data weekly for the estimation, hence $\Delta t = 7/365$.

We construct the measurement equations based on sovereign CDS spreads and currency option quotes, assuming additive, normally distributed measurement errors:

$$y_t = \begin{bmatrix} S(x_t, t+\tau_s; \Theta) \\ O(x_t, t+\tau_O, \delta; \Theta) \end{bmatrix} + e_t, \quad \begin{array}{l} \tau_s = 1, 3, 5 \text{ years}, \\ \tau_O = 1, 2, 3, 6, 12 \text{ months}, \end{array} \tag{23}$$

where y_t denotes the observed series, $S(x_t,t+\tau_s;\Theta)$ denotes the model-implied value of CDS spreads at time t and maturity τ_s as a function of the state vector x_t and model parameters Θ, and $O(x_t,t+\tau_O,\delta;\Theta)$ denotes the model-implied out-of-the-money option value at time t, time-to-maturity τ_O, and delta δ, as a function of the state vector x_t and model parameters Θ. Each maturity has options at three deltas, including a 25-delta call, a 25-delta put, and the delta-neutral straddle. We scale the CDS spreads by their respective sample means for each series. We represent the out-of-the-money option prices in percentages of the underlying spot and scale the price by their Black–Scholes vega. Then, we assume that the three credit default spread series generate iid normal pricing errors with error variance σ_s^2. We also assume that the pricing errors on all 15 option series are iid normal with error variance σ_O^2.

The state-propagation equation is Gaussian-linear, but the measurement equation in (23) is nonlinear. We use an extended version of the Kalman filter to handle the nonlinearity (See Wan and van der Merwe (2001) for details of the filtering technique). Let $\bar{y}_{t+1}$ and $\overline{A}_{t+1}$ denote the time-t ex ante forecasts of time-$(t+1)$ values of the measurement series and the covariance of the measurement series, respectively. We construct the log-likelihood value assuming normally distributed forecasting errors

$$l_{t+1}(\Theta) = -\frac{1}{2}\log|\overline{A}_{t+1}| - \frac{1}{2}((y_{t+1} - \bar{y}_{t+1})^\top (\overline{A}_{t+1})^{-1}(y_{t+1} - \bar{y}_{t+1})). \tag{24}$$

The model parameters are estimated by maximizing the sum of the weekly log likelihood of the data series.

5. Joint dynamics of diffusion variance and default arrival rates

Through the model parameter estimates, we infer the joint dynamics of the diffusion variance and default arrival rates. The estimated risk dynamics highlight the underlying link between the sovereign CDS market and the currency options market.

5.1. Performance analysis

Before we look into the risk dynamics, we first assess the model performance in pricing sovereign CDS spreads and currency options. Table 3 reports the summary statistics of the pricing errors on sovereign CDS spreads and currency option implied volatilities. The pricing errors on the CDS spreads are defined as the difference between data quotes and model-implied values of the spreads, both scaled by the sample mean of the spread series. For currency options, the measurement equations are defined on out-of-the-money option prices scaled by the option vega. To make more economic sense out of the pricing errors, we convert the model-implied option prices into the Black–Scholes implied volatilities. We then define the pricing errors as the difference between the implied volatility quotes and model-implied values in percentage points.

For both Mexico and Brazil, the mean pricing errors on the CDS spreads are negative with one year to maturity but positive at the three and five year maturities. Hence, the model has a systematic bias in overpricing short-term CDS spreads and underpricing long-term spreads. The CDS spreads have a steep mean upward-sloping term structure. The remaining structure in the mean pricing errors suggests that the model-implied term structure is flatter. Hence, the model has difficulties in generating the steep term structure, while at the same time pricing the currency options correctly.

The standard deviations of the pricing errors are relatively flat across maturities, between 0.31 and 0.38 for Mexico and between 0.26 and 0.59 for Brazil. The weekly autocorrelation estimates are smaller than that from the original spread series, but remain large. The last column measures the explained variation, defined as one minus the ratio of the pricing error variance to the original data series variance. For Mexico, the model explains 55–59% of the variation. The model performs better on the CDS spreads on Brazil. The explained variations are 83–89% for the three series. Overall, the performance results suggest that there are additional systematic movements in the credit spreads that the estimated model fails to capture.

Table 3
Summary statistics of the pricing errors in CDS spreads and option implied volatilities from the CDFX model

Country		Mexico				Brazil			
Statistics		Mean	Std	Auto	VR	Mean	Std	Auto	VR
CDS	1y	−0.37	0.38	0.95	0.56	−0.30	0.59	0.93	0.83
CDS	3y	0.22	0.35	0.97	0.59	0.02	0.33	0.88	0.88
CDS	5y	0.32	0.31	0.98	0.55	0.11	0.26	0.84	0.89
IV(25c)	1m	0.39	0.16	0.38	0.99	−0.12	1.21	0.82	0.99
ATMV	1m	−0.32	0.18	0.55	0.99	−0.23	0.70	0.62	0.99
IV(25p)	1m	−0.16	0.20	0.57	0.99	−0.06	1.38	0.82	0.98
IV(25c)	2m	0.46	0.19	0.49	0.99	−0.49	1.93	0.90	0.97
ATMV	2m	−0.27	0.19	0.46	0.99	−0.11	0.48	0.41	1.00
IV(25p)	2m	−0.20	0.18	0.45	0.99	0.18	1.60	0.87	0.97
IV(25c)	3m	0.59	0.25	0.56	0.98	−0.81	2.38	0.92	0.94
ATMV	3m	−0.19	0.23	0.63	0.98	0.02	0.52	0.59	1.00
IV(25p)	3m	−0.10	0.23	0.63	0.97	0.58	1.85	0.90	0.95
IV(25c)	6m	0.43	0.30	0.69	0.97	−2.06	3.42	0.95	0.87
ATMV	6m	−0.08	0.25	0.63	0.96	0.02	0.47	0.70	1.00
IV(25p)	6m	−0.10	0.22	0.65	0.95	1.61	2.51	0.93	0.90
IV(25c)	1y	−0.06	0.27	0.63	0.97	−4.69	5.27	0.97	0.65
ATMV	1y	−0.20	0.23	0.59	0.96	−0.09	0.43	0.72	1.00
IV(25p)	1y	−0.06	0.40	0.89	0.75	3.45	3.34	0.97	0.79

Entries report the sample estimates of mean, standard deviation ('Std'), and weekly autocorrelation ('Auto') of the pricing errors from the CDFX model on the CDS spreads and options, defined as the difference between data and model-implied values. The pricing errors on credit swaps are based on spread quotes scaled by their sample mean for each series. The pricing errors on options are defined on implied volatilities in percentage points. The columns titled 'VR' report the explained variation, defined as one minus the variance of the pricing error over the variance of the data series. Model estimation is based on weekly sampled data (every Wednesday) from January 2, 2002 to March 2, 2005, 166 observations for each series.

The model captures the currency options behavior much better. For options on peso, the mean pricing errors on option implied volatilities do not show any obvious patterns, indicating that there is no systematic mispricing. Furthermore, the standard deviation of the pricing errors are all within half a percentage point. The autocorrelation estimates are also much lower than the original volatility series. The explained variation estimates are over 95% for all but one series.

For options on the Brazilian real, the pricing errors are small for all of the at-the-money options. The standard deviations for the pricing errors on at-the-money options are all less than one percentage point, and the explained variations are all 99% or higher. The pricing errors on out-of-the-money options become larger, especially at longer maturities.

5.2. The joint dynamics of diffusion variance and default arrival rates

Table 4 reports the maximum likelihood estimates and t-statistics of the structural parameters that control the joint dynamics of the diffusion variance rate and the default arrival rate. The joint dynamics from the two countries share several common features.

First, the estimates for the risk-neutral mean-reverting coefficients (κ_v, κ_z) and their statistical counterparts $(\kappa_v^{\mathbb{P}}, \kappa_z^{\mathbb{P}})$ show that the default arrival rate is much more persistent than the diffusion variance rate under both the risk-neutral measure $\mathbb{Q}$ and the statistical measure $\mathbb{P}$. The persistence difference is larger under the risk-neutral measure than under the statistical measure.

The difference in statistical persistence suggests that the diffusion return variance rates are strongly mean-reverting and hence predictable, but it is difficult to predict the default

Table 4
Maximum likelihood estimates of the CDFX model parameters

	Country			
	Mexico		Brazil	
κ_v	4.2864	(23.58)	3.5594	(33.99)
κ_z	0.0030	(0.04)	0.0028	(0.08)
$\kappa_v^{\mathbb{P}}$	4.9818	(8.02)	1.0392	(1.89)
$\kappa_z^{\mathbb{P}}$	0.5613	(2.84)	0.4551	(0.79)
θ_v	0.0426	(25.06)	0.0972	(33.55)
θ_z	0.0045	(3.90)	0.0248	(4.18)
σ_v	0.1099	(17.29)	0.5892	(33.47)
σ_z	0.0804	(8.46)	0.4599	(26.62)
β	0.2986	(2.54)	0.8791	(5.75)
ρ^{sv}	−0.8998	(18.28)	−0.4332	(45.63)
μ_j	1.8465	(0.00)	0.1605	(37.12)
v_j	0.1268	(0.00)	0.1997	(78.82)
σ_s^2	0.2051	(11.25)	0.2082	(26.62)
σ_o^2	0.0012	(41.04)	0.0032	(66.01)
γ_v	−6.3253	(−1.13)	4.2775	(4.24)
γ_z	−6.9452	(−2.54)	−0.9834	(−0.79)
$\mathbb{E}^{\mathbb{P}}[\lambda]$	0.0106	(4.74)	0.1368	(4.03)
$\mathbb{E}^{\mathbb{Q}}[\lambda]$	1.5038	(0.68)	8.7358	(0.74)

Entries report the CDFX model parameter estimates and absolute values of the t-statistics (in parentheses). The estimation is based on weekly sampled data from January 2, 2002 to March 2, 2005.

arrival rate changes based on its past values. The difference in risk-neutral persistence dictates that the two factors have different impacts across the term structure of options and CDS spreads. Shocks on the diffusion variance rate affect the short-term options and CDS spreads, but dissipate quickly as the option and CDS maturity increases. Shocks on the more persistent default arrival rate last longer across the term structure of options and credit spreads.

For each risk factor, the difference in persistence under the two probability measures defines the market price of that factor's risk:

$$\gamma_v = (\kappa_v - \kappa_v^{\mathbb{P}})/\sigma_v, \quad \gamma_z = (\kappa_z - \kappa_z^{\mathbb{P}})/\sigma_z. \tag{25}$$

We compute the market prices (γ_v, γ_z) based on the parameter estimates and report them in the bottom panel of Table 4. For Mexico, both $(\kappa_z^{\mathbb{P}}, \kappa_v^{\mathbb{P}})$ are greater than their risk-neutral counterparts. Market prices for both sources of risks are negative. For Brazil, the market price is negative on the credit risk factor z, but positive on the diffusion risk v.

Under our specification, the market prices not only dictate the persistence difference of the risk factors under the two measures, but also create differences in the long-run means of the risk factors under the two measures. In particular, the statistical mean and the risk-neutral mean of the default arrival rate are given by

$$\mathbb{E}^{\mathbb{P}}[\lambda] = \mathbb{E}^{\mathbb{P}}[z + \beta v] = (\kappa_z^{\mathbb{P}})^{-1}\theta_z + \beta(\kappa_v^{\mathbb{P}})^{-1}\theta_v, \tag{26}$$

$$\mathbb{E}^{\mathbb{Q}}[\lambda] = \mathbb{E}^{\mathbb{Q}}[z + \beta v] = (\kappa_z)^{-1}\theta_z + \beta(\kappa_v)^{-1}\theta_v. \tag{27}$$

The bottom panel of Table 4 also reports the two mean estimates based on the parameter estimates. The mean default arrival rate is much lower under the statistical measure $\mathbb{P}$ than under the risk-neutral $\mathbb{Q}$. For Mexico, the mean default arrival rate is 0.0106 under $\mathbb{P}$, much smaller than its risk-neutral counterpart at 1.5038. The same ranking holds for Brazil, with the statistical mean at 0.1368 being much smaller than its risk-neutral measure counterpart at 8.7358. These estimates are consistent with the empirical findings in the corporate bond literature that the historical average default probabilities are much lower than those implied from the corporate bond credit spreads.[5]

If we define the credit spread at a maturity τ as the difference between the continuously compounded spot rate on the sovereign debt with zero recovery upon default and the corresponding spot rate in the benchmark Eurodollar market, then under our dynamic model specification, this spread is affine in the state vector,

$$\mathsf{Spread}(t,\tau) = \left[\frac{a_\lambda(\tau)}{\tau}\right] + \left[\frac{b_\lambda(\tau)}{\tau}\right]^{\top} x_t, \tag{28}$$

with the coefficients $[a_\lambda(\tau), b_\lambda(\tau)]$ given by the ordinary differential equations in (17). The vector $b_\lambda(\tau)/\tau$ measures the contemporaneous response of the credit spread term structure to unit shocks on the two risk factors. Fig. 1 plots the contemporaneous response $b_\lambda(\tau)/\tau$ across different maturities. The solid lines denote the response to the diffusion risk factor v and the dashed lines to the default risk factor z. Our model normalizes the response of the instantaneous spread to the default risk factor z to one, but lets the response to the diffusion risk factor be a free parameter β. For Mexico, the estimate for β is relatively small at

[5] See, for example, Huang and Huang (2003), Eom et al. (2004), Elton et al. (2001) and Collin-Dufresne et al. (2003).

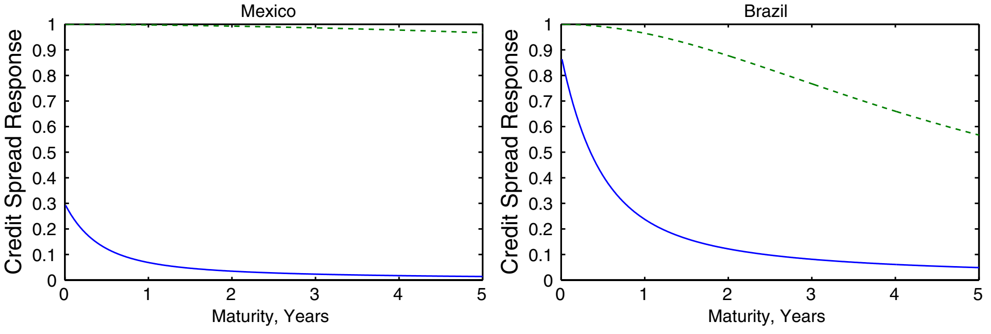

Fig. 1. Contemporaneous response of credit spread term structures to unit shocks on risk factors. Lines denote the contemporaneous response of the credit spread, defined as the difference between continuously compounded spot rate of a reference sovereign country and the corresponding spot rate for the US dollar, to unit shocks on the two sources of risks: diffusion variance rate v (solid lines) and default risk factor z (dashed lines).

0.2986. For Brazil, the estimate for β is larger at 0.8791. These numbers determine the response of the credit spreads at very short maturities. The risk-neutral persistence of the two factors $[\kappa_v, \kappa_z]$ determines how fast the responses decay with increasing maturities. Given the larger estimates for κ_v than for κ_z for both countries, the solid lines decline much faster with increasing maturities than the dashed lines. Thus, at long maturities, the movements of the credit spreads are mostly driven by the independent default risk factor z.

Conditional on default occurring, the mean impact on currency return is captured by μ_j, with variance being v_j. The estimates on Brazil are 16.05% for the mean and 19.97% for the variance. The estimates on Mexico, however, are estimated with large errors. For both countries, the estimates for the instantaneous correlation between currency diffusion return and return variance ρ^{sv} are strongly negative, consistent with Black's (1976) classic argument of leverage effect in stocks and the analogy between equities and fiat money for an economy.

5.3. Robustness analysis

For Mexico in particular, our model generates better performance on currency options than on CDS spreads. This performance difference can potentially come from the fact that we use 15 currency option series, but only three CDS spread series for estimating the model. As a robustness check, we repeat the estimation exercise three additional times on each economy, each time using a different subset of the data: (I) the three currency options at one-month maturity and the three CDS series, (II) the 15 currency option series only, and (III) the three CDS spread series only. Table 5 reports the parameter estimates and the average explained variation on CDS and currency options for each exercise.

In the first case, we use the same number of observations on options and on CDS spreads. The idea is to use the variance rate v_t to capture the short-term options and use the default risk factor z_t to capture the term structure of CDS spreads. Under this exercise, the model performs well on Brazil for both options and CDS spreads, but for Mexico, there remains some tension in fitting the term structure of CDS spreads, with the average explained variation being 77.5%. This tension suggests that we need richer default arrival

Table 5
Robustness analysis of parameter estimates

Country	Mexico						Brazil					
Data	I		II		III		I		II		III	
κ_v	2.2965	(0.57)	4.3264	(19.12)	1.3318	(4.25)	2.9696	(1.19)	4.0107	(26.53)	1.7763	(7.67)
κ_z	0.0009	(0.03)	0.0030	(0.02)	0.0000	(0.00)	0.0027	(0.07)	0.0028	(0.08)	0.1286	(10.49)
$\kappa_v^{\mathbb{P}}$	29.7744	(5.29)	4.9814	(8.21)	0.8027	(1.26)	1.8313	(2.45)	0.9706	(1.98)	0.3443	(0.45)
$\kappa_z^{\mathbb{P}}$	1.0083	(2.37)	0.5612	(2.57)	0.2791	(1.78)	1.0601	(1.32)	0.4240	(1.02)	0.2822	(0.93)
θ_v	0.1065	(6.35)	0.0423	(23.56)	0.0085	(0.00)	0.1930	(10.60)	0.0798	(28.13)	0.0449	(0.00)
θ_z	0.0131	(2.94)	0.0045	(1.68)	0.0090	(6.77)	0.0825	(4.21)	0.0446	(5.14)	0.0783	(11.52)
σ_v	0.5843	(6.12)	0.1095	(18.20)	0.0864	(0.00)	0.7953	(11.85)	0.4350	(26.45)	0.3905	(0.00)
σ_z	0.0739	(5.06)	0.0804	(4.34)	0.0781	(8.16)	0.4403	(13.66)	0.6298	(20.30)	0.3477	(25.43)
μ_j	19.6553	(0.00)	1.8411	(0.00)	1.8465	(0.00)	0.2163	(8.82)	0.1466	(37.43)	0.1605	(0.00)
v_j	0.1215	(3.05)	0.1268	(0.00)	0.1268	(0.00)	0.1022	(2.55)	0.1770	(40.35)	0.1997	(0.00)
β	−0.2217	(1.74)	0.2984	(1.88)	−2.6416	(0.00)	−0.8777	(2.65)	2.1826	(10.64)	−3.5515	(0.00)
ρ^{sv}	−0.2588	(5.93)	−0.8997	(18.65)	−0.8998	(0.00)	−0.3558	(13.29)	−0.4833	(32.02)	−0.4332	(0.00)
σ_s^2	0.0652	(10.87)	0.2051	(0.00)	0.0021	(15.31)	0.0132	(20.79)	0.2082	(0.00)	0.0009	(12.65)
σ_o^2	0.0007	(9.17)	0.0012	(44.61)	0.0012	(0.00)	0.0045	(6.55)	0.0032	(74.77)	0.0032	(0.00)
Performance	CDS	Currency	CDS	Currency	CDS	Currency	CDS	Currency	CDS	Currency	CDS	Currency
VR	0.775	0.991	–	0.963	0.995	–	0.988	0.990	–	0.932	0.999	–

Entries report the CDFX model parameter estimates, the absolute values of the *t*-statistics (in parentheses), and the aggregate explained variation for CDS and currency options when the model is estimated using three different subsets of the data: (I) the three one-month options and the three CDS series, (II) the 15 currency options series only, and (III) the three CDS series only. The estimation is based on weekly sampled data from January 2, 2002 to March 2, 2005. The last panel reports the average explained variation (VR) on the CDS series and the option implied volatilities that are used in the model estimation.

dynamics to capture the term structure of CDS spreads on Mexico, which we will do in the next section.

When we use currency options only (II) or CDS spreads only (III) to estimate the model, we are essentially using the two risk factors to capture the movements in one market. Not surprisingly, the model performs well in the market that it is calibrated to. Nevertheless, we experience some identification issues when we estimate the model using the three CDS series only. For example, the conditional jump size distribution in currency returns upon default does not affect the CDS pricing and hence cannot be identified using CDS data alone.

As expected, using different subsets of data also generates variations on the parameter estimates. Nevertheless, our main conclusions on the risk dynamics and market pricing remain the same. In all cases, the default risk factor z_t is significantly more persistent than the variance rate v_t under both the statistical measure and the risk-neutral measure. Furthermore, while the evidence on the market price of variance rate risk is mixed, the market price estimates on the default risk factor are significantly negative in all cases.

6. Stochastic central tendency in default arrivals

Estimation shows that the joint dynamics specification in Eqs. (5)–(8) can capture the behavior of currency options well, but have some difficulties in capturing the term

Table 6
Summary statistics of the pricing errors under the CDFX stochastic central tendency model

Country		Mexico				Brazil			
Statistics		Mean	Std	Auto	VR	Mean	Std	Auto	VR
CDS	1y	−0.06	0.31	0.94	0.71	−0.28	0.58	0.94	0.83
CDS	3y	0.12	0.19	0.91	0.88	−0.01	0.25	0.84	0.93
CDS	5y	−0.06	0.17	0.93	0.87	0.05	0.19	0.78	0.94
IV(25c)	1m	0.15	0.22	0.58	0.99	−0.18	1.25	0.84	0.99
ATMV	1m	−0.50	0.25	0.64	0.98	−0.30	0.69	0.57	0.99
IV(25p)	1m	−0.49	0.21	0.57	0.98	−0.04	1.38	0.82	0.98
IV(25c)	2m	0.31	0.22	0.55	0.99	−0.51	1.92	0.90	0.97
ATMV	2m	−0.27	0.23	0.51	0.98	−0.17	0.47	0.41	1.00
IV(25p)	2m	−0.44	0.16	0.36	0.99	0.22	1.62	0.88	0.97
IV(25c)	3m	0.48	0.23	0.65	0.99	−0.80	2.36	0.91	0.95
ATMV	3m	−0.11	0.23	0.67	0.98	−0.02	0.52	0.62	1.00
IV(25p)	3m	−0.28	0.19	0.57	0.98	0.62	1.89	0.92	0.95
IV(25c)	6m	0.33	0.28	0.72	0.98	−2.00	3.35	0.95	0.88
ATMV	6m	0.04	0.24	0.62	0.97	0.00	0.51	0.73	1.00
IV(25p)	6m	−0.18	0.20	0.58	0.96	1.63	2.55	0.93	0.90
IV(25c)	1y	−0.22	0.25	0.67	0.97	−4.67	5.29	0.97	0.65
ATMV	1y	−0.15	0.23	0.63	0.96	−0.16	0.41	0.69	1.00
IV(25p)	1y	−0.03	0.38	0.85	0.79	3.38	3.32	0.96	0.79

Entries report the sample estimates of mean, standard deviation (‘Std’), and weekly autocorrelation (‘Auto’) of the pricing errors from the CDFX stochastic central tendency model on the CDS spreads and options, defined as the difference between data and model-implied values. The pricing errors on credit swaps are based on spread quotes scaled by their sample mean for each series. The pricing errors on options are defined on implied volatilities in percentage points. The columns titled ‘VR’ report the explained variation, defined as one minus the variance of the pricing error over the variance of the data series. Model estimation is based on weekly sampled data (every Wednesday) from January 2, 2002 to March 2, 2005, 166 observations for each series.

structure of the CDS spreads, especially for Mexico. To generate richer behaviors on the CDS term structure, we extend the original specification and allow the mean of the default arrival rate factor to be also stochastic:

$$\mathrm{d}z(t) = (m(t) - \kappa_z z(t))\,\mathrm{d}t + \sigma_z\sqrt{z(t)}\mathrm{d}W_t^z, \tag{29}$$

$$\mathrm{d}m(t) = (\theta_m(t) - \kappa_m m(t))\,\mathrm{d}t + \sigma_m\sqrt{m(t)}\mathrm{d}W_t^m. \tag{30}$$

In Eq. (29), we replace the constant term (θ_z) in the drift for the default arrival factor z in Eq. (7) by a stochastic quantity $m(t)$, the dynamics of which are given in Eq. (30), with W^m denoting yet another standard Brownian motion that is independent of all other Brownian motions. The extension allows the default arrival factor z to revert to a central level that is stochastic by itself. Following Balduzzi et al. (1998), we label this extended model as the CDFX stochastic central tendency model. Under this model, the default arrival rate is controlled by a three-factor dynamic structure.

Under this specification, the generalized Fourier transform on the log currency return remains exponential affine in the expanded state vector, $x_t \equiv [v_t, z_t, m_t]$. The solutions to the credit default swap spreads also take analogous forms. Hence, the analytical tractability remains. We estimate this model on the same data set. Table 6 reports the summary statistics of the pricing errors. By allowing the default arrival rate to revert to a stochastic mean level, the model performs better than in the original specification. The improvement is most obvious on CDS spreads for Mexico. The explained variation for the three CDS spread series increase from 55–59% to 71–88%. Furthermore, the systematic pricing bias observed

Table 7
Maximum likelihood estimates of parameters under the CDFX stochastic central tendency model

	Country			
	Mexico		Brazil	
κ_v	3.0654	(29.21)	3.2199	(37.44)
κ_z	0.0028	(0.02)	0.0029	(0.06)
κ_m	0.0023	(0.02)	0.0025	(0.01)
$\kappa_v^{\mathbb{P}}$	3.2143	(0.96)	1.1235	(1.54)
$\kappa_z^{\mathbb{P}}$	1.1211	(1.66)	0.6389	(0.91)
$\kappa_m^{\mathbb{P}}$	0.4977	(0.90)	0.5966	(1.19)
θ_v	0.0391	(45.85)	0.0917	(34.35)
θ_m	0.0049	(3.69)	0.0128	(1.78)
σ_v	0.3660	(31.17)	0.5640	(33.04)
σ_z	0.0726	(6.65)	0.5715	(18.13)
σ_m	0.1229	(2.73)	0.4012	(6.68)
β	0.0286	(0.29)	1.3418	(7.79)
ρ^{sv}	−0.3635	(44.43)	−0.4308	(49.92)
μ_j	1.2740	(0.00)	0.1670	(38.78)
v_j	0.0439	(0.00)	0.1960	(67.97)
σ_s^2	0.0643	(10.92)	0.1639	(19.03)
σ_o^2	0.0009	(57.26)	0.0031	(65.30)
γ_v	−0.4067	(−0.04)	3.7172	(2.70)
γ_z	−15.4122	(−1.98)	−1.1128	(−0.91)
γ_m	−4.0301	(−0.95)	−1.4807	(−1.12)

Entries report the CDFX stochastic central tendency model parameter estimates and absolute values of the t-statistics (in parentheses). The estimation is based on weekly sampled data from January 2, 2002 to March 2, 2005.

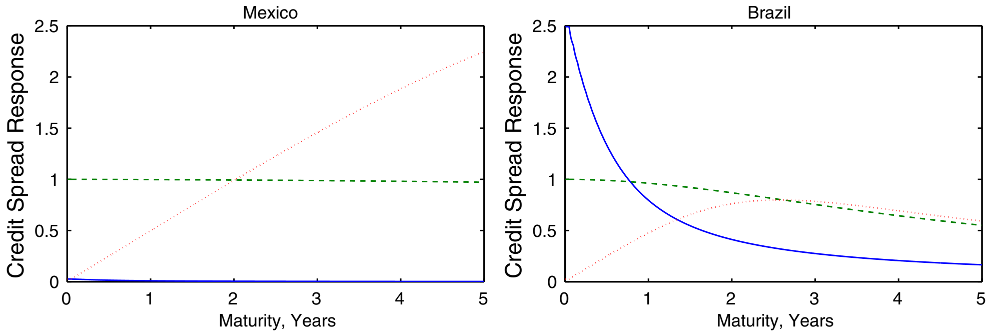

Fig. 2. The mean credit spread term structure and factor loadings implied from the central tendency model. Lines denote the contemporaneous response of the credit spread, defined as the difference between continuously compounded spot rate of a reference sovereign country and the corresponding spot rate for the US dollar, to unit shocks on the three sources of risks: currency diffusion variance v (solid lines), default risk factor z (dashed lines), and the central tendency factor m (dotted lines).

from the original model no longer exists in this stochastic central tendency model. The performance improvement on currency options and on the series for Brazil are less significant given the much better performance on these series from the original model.

Table 7 reports the maximum likelihood estimates and t-statistics of the structural parameters. The estimates for the parameters that control the dynamics of v and z are similar to those obtained in Table 4 for the original model. Hence, the original discussion on the joint dynamics remains valid. The new model incorporates a stochastic central tendency factor. The parameter estimates on the dynamics of this factor suggest that for both countries, the central tendency factor, m, is more persistent than both the diffusion variance v and the default risk factor z under both the risk-neutral measure and the statistical measure. The time-series persistence indicates that the central tendency factor behaves close to a random walk. Its movements are the hardest to predict among the three factors.

Based on the parameter estimates in Table 7, we compute the contemporaneous response of the credit spread term structure to unit shocks on the three dynamic factors, which are plotted in Fig. 2. The solid lines denote the response to the diffusion risk factor v, the dashed lines to the default risk factor z, and the dotted lines to the central tendency factor m. The credit spread responses to v and z under the new model are similar to those under the original model. The response to the stochastic central tendency factor m starts at zero at zero maturity, but becomes positive as maturity increases. When the maturity is over three years, the impact of the central tendency factor becomes the largest of the three factors.

7. Concluding remarks

As noted before, economic or political instability often leads to both worsened sovereign credit quality and aggravated currency return volatility in a sovereign country, thus generating positive co-movements between sovereign credit spreads and currency return implied volatilities. This co-movement also has its support from an analogy relating fiat money of a country to the shares of a corporation. In this paper, we examine the relation between sovereign CDS spreads and currency implied volatilities using data from two emerging markets: Mexico and Brazil. The analysis confirms our conjecture: Credit spreads and currency option implied volatilities show strongly positive contemporaneous correlations.

Guided by the evidence, we propose an integrated valuation framework that allows us to jointly model and estimate the dynamics of currency return variance risk and sovereign credit risk. Under this framework, sovereign default arrives via a compound Poisson process with stochastic arrival rate. Upon default, the currency price, quoted in dollars per unit of the local currency, drops by a sizable random amount. In the absence of a default event, the currency price moves continuously with a stochastic instantaneous variance rate. We model the joint dynamics of the diffusion variance rate and the default arrival rate via a two-dimensional vector process that captures the contemporaneous co-movements between the currency options market and the sovereign CDS market.

Estimation using over three years of data shows that the default arrival rate is much more persistent than the diffusion variance rate of currency returns under both the statistical measure and the risk-neutral measure. The persistence difference under the statistical measure suggests that the default arrival rate is more difficult to predict than the diffusion return variance rate. The difference under the risk-neutral measure implies that shocks in the instantaneous default arrival rate have a larger impact on long-term contracts than shocks of the same size in the instantaneous diffusion variance. The market price estimates on the two sources of risk also explain the well-documented evidence that historical average default probabilities are lower than those implied from credit spreads.

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