Electromechanical Motion Fundamentals

- Electric Machine – device that can convert either mechanical energy to electrical energy or electrical energy to mechanical energy
  - mechanical to electrical: generator
  - electrical to mechanical: motor
  - all practical motors and generators convert energy from one form to another through the action of a magnetic field
- Transformer – device that converts *ac* electric energy at one voltage level to *ac* electric energy at another voltage level

- It operates on the same principles as generators and motors, i.e., it depends on the action of a magnetic field to accomplish the change in voltage level

- Motors, Generators, and Transformers are ubiquitous in modern daily life. Why?
  - Electric power is:
    - Clean
    - Efficient
    - Easy to transmit over long distances
    - Easy to control
    - Environmental benefits

- Purpose of this Study
  - provide basic knowledge of electromechanical motion devices for mechatronic engineers
  - focus on electromechanical rotational devices commonly used in low-power mechatronic systems
    - permanent magnet dc motor
    - brushless dc motor
    - stepper motor
- Topics Covered:
  - Magnetic and Magnetically-Coupled Circuits
  - Principles of Electromechanical Energy Conversion

References

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Magnetic & Magnetically-Coupled Circuits

- Introduction
- Magnetic Field
- Magnetic Circuits
- Properties of Magnetic Materials
- Faraday's Law and Lenz's Law
- Production of an Induced Force on a Wire
- Induced Voltage on a Conductor Moving in a Magnetic Field

- Linear DC Machine – A Simple Example
- Stationary Magnetically-Coupled Circuits
- Magnetic Systems with Mechanical Motion
  - Elementary Electromagnet
  - Elementary Reluctance Machine
  - Windings in Relative Motion

Introduction

- Review concepts and terms for use in the study of electromechanical motion devices.
- In all electromechanical devices, mechanical motion must occur, either translational or rotational, and this motion is reflected into the electrical system either as a change of flux linkages (electromagnetic system) or as a change of charge (electrostatic system).
- Focus is primarily on electromagnetic systems.

- If the magnetic system is linear, then the change in flux linkages results owing to a change in the inductance, i.e., inductances of electric circuits associated with electromechanical motion devices are functions of the mechanical motion.
- Learn to express self- and mutual-inductances for simple translational and rotational electromechanical devices, and to handle these changing inductances in the voltage equations describing the electrical circuits associated with the electromechanical system.

Magnetic Field

- 10 facts about The Force
  - Known for Hundreds of Years
    - If free to rotate, permanent magnets point approximately north-south.
    - Like poles repel, unlike poles attract.
    - Permanent magnets attract some things (like iron and steel), but not others (like wood and glass). Magnetic forces attract only magnetic materials.
    - Magnetic forces act at a distance, and they can act through nonmagnetic barriers.
    - Things attracted to a permanent magnet become temporary magnets themselves.

– Known only since the 19th Century

- A coil of wire with an electric current running through it becomes a magnet.
- Putting iron inside a current-carrying coil greatly increases the strength of the electromagnet.
- A changing magnetic field induces an electric current in a conductor (like copper).
- A charged particle experiences no magnetic force when moving parallel to a magnetic field, but when it is moving perpendicular to the field it experiences a force perpendicular to both the field and the direction of motion.
- A current-carrying wire in a perpendicular magnetic field experiences a force perpendicular to both the wire and the field.

- Magnetic Fields are the fundamental mechanism by which energy is converted from one form to another in motors, generators, and transformers.
- Four Basic Principles describe how magnetic fields are used in these devices:
  - A current-carrying wire produces a magnetic field in the area around it.
  - A time-changing magnetic field induces a voltage in a coil of wire if it passes through that coil (basis of transformer action).
  - A current-carrying wire in the presence of a magnetic field has a force induced on it (basis of motor action).
  - A moving wire in the presence of a magnetic field has a voltage induced in it (basis of generator action).

- In the study of electricity, one learns that stationary charges produce an electric field.
- If the charges move with uniform velocity, a secondary effect takes place: magnetism
- If we accelerate charges, there is an additional effect; the accelerated charges now produce a radiating electromagnetic field, i.e., a field that can transport energy.
- Magnetism and electromagnetic fields are special cases of electricity!

- Since motion is relative, a given physical experiment which is purely electrostatic in one coordinate system can appear as electromagnetic in another coordinate system that is moving with respect to the first. Magnetic fields seem to appear and vanish merely by a change in the motion of the observer!
- A magnetic field is thus associated with moving charges. The sources of magnetic field are currents.

$$v_q = 0 \quad \Rightarrow \quad E \neq 0, \quad B = 0$$

$$v_q \neq 0 \quad \Rightarrow \quad E \neq 0, \quad B \neq 0$$

$$\frac{dv_q}{dt} \neq 0 \quad \Rightarrow \quad E \neq 0, \quad B \neq 0, \quad \text{Radiation Fields}$$

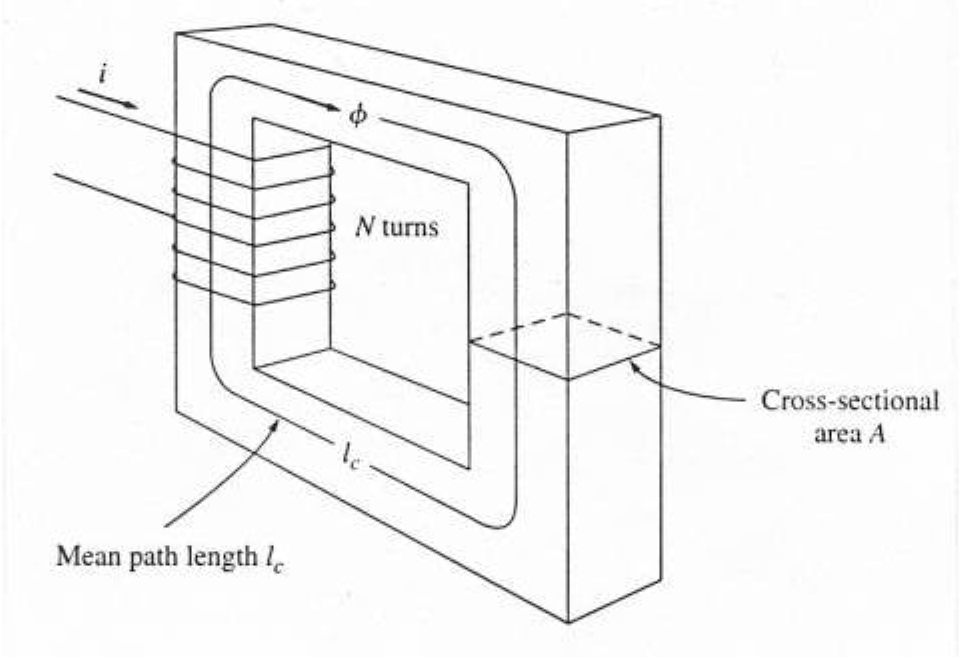
v_q = velocity of charge q

- Units of the Magnetic Field (SI and CGS)
  - Magnetic Flux Density B
    - Also called magnetic field and magnetic induction
    - 1 tesla (T) = 1 weber/meter^2 (1 Wb/m^2)
    - 1 T = 10^4 G (gauss)
    - Earth magnetic field is about 0.5 G
    - Small permanent magnet is about 100 G
    - Large electromagnet is about 20,000 G
  - Magnetic Field Intensity (or Strength) H
    - 1 ampere-turn/meter = $4\pi \times 10^{-3}$ oersted (Oe)
  - Magnetic Flux $\Phi = BA$
    - 1 weber (Wb) = 10^8 maxwell (Mx)

Magnetic Circuits

- Ampere's Law $\oint \vec{H} \bullet \vec{dL} = i_n$
  - The line integral of the magnetic field intensity (or magnetic field strength) about a closed path is equal to the net current enclosed within this closed path of integration.

Consider the elementary magnetic circuit shown.



- Rectangular ferromagnetic core with N turns of wire wrapped about one leg of the core.
- The net current passing within the path of integration is Ni.
- Assumptions
  - All the magnetic field produced by the current remains inside the core. Therefore the path of integration is the mean path length of the core.
  - The magnetic field intensity exists only in the direction of the given path of integration or, in other words, perpendicular to a cross section of the magnetic material (valid except in the vicinity of the corners).

- Carrying out the integration: $H\ell_c = Ni$
  - The right-hand side is referred to as ampere-turns (At) or magnetomotive force (mmf), analogous to electromotive force (emf) in electric circuits.
- The magnetic field intensity is a measure of the "effort" that a current is putting into the establishment of a magnetic field.
- The strength of the magnetic field flux produced in the core also depends on the material of the core. For linear, isotropic magnetic materials the magnetic flux density is related to the magnetic field intensity as:

$$\vec{B} = \mu\vec{H}$$

$\vec{H}$ = magnetic field intensity (At/m; 1 At/m = 0.0126 Oe)

μ = magnetic permeability of the material (Wb/A·m or H/m)

$\vec{B}$ = magnetic flux density (Wb/m^2 or T; 1 Wb/m^2 = 10^4 G)

- μ , the permeability of the medium, represents the relative ease of establishing a magnetic field in a given material.
- The permeability of any other material compared to the permeability of free space or air (μ_0) is called relative permeability.

$$\mu_r = \frac{\mu}{\mu_0} \quad \text{where } \mu_0 = 4\pi\times10^{-7} \ \text{H/m}$$

- Relative permeability is a convenient way to compare the magnetizability of materials.

- The surface integral of the flux density is equal to the total flux Φ (Wb) in a given area:

$$\Phi = \int_A \vec{B} \bullet \overrightarrow{dS}$$

- If the flux density vector is assumed perpendicular to a plane of area, and if the flux density is constant throughout the area, then: $\Phi = BA$
- In the elementary magnetic circuit: $\Phi = BA = \dfrac{\mu N i A}{\ell_c}$
- Electrical / Magnetic Circuit Analogy

$$V = iR$$

$$\mathfrak{I} = \Phi \mathfrak{R}$$

$$V \Leftrightarrow \mathfrak{I}$$

$$i \Leftrightarrow \Phi$$

$$R \Leftrightarrow \mathfrak{R}$$

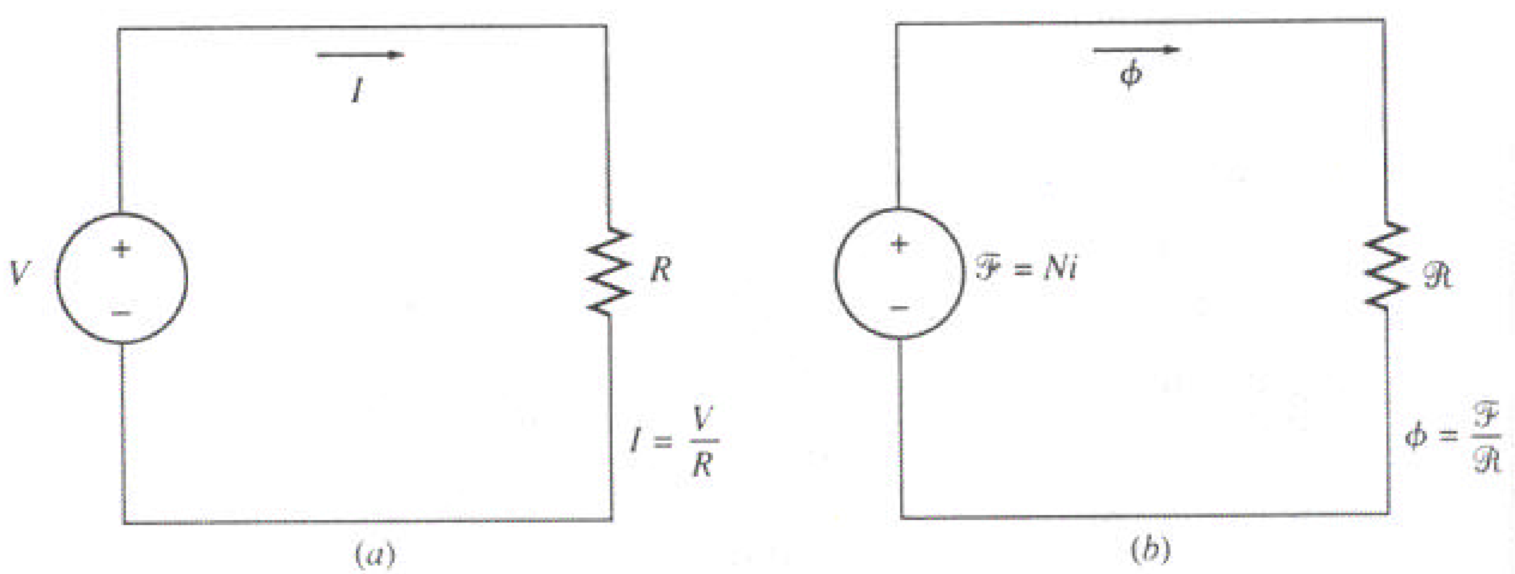
$\mathfrak{I} = Ni$ magnetomotive force (At)

$\mathfrak{R} = \dfrac{\ell_c}{\mu A}$ reluctance (At/Wb)

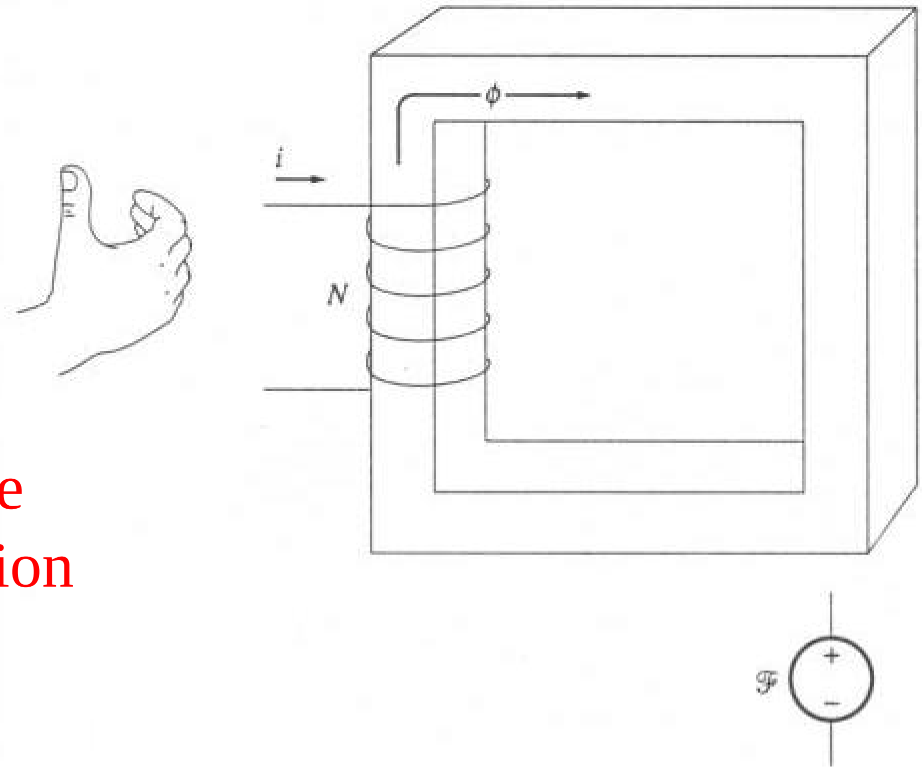
$\dfrac{1}{\mathfrak{R}} = \text{permeance}$

$$R = \frac{\ell}{\sigma A}$$

- The magnetic circuit model of magnetic behavior is often used in the design of electric machines and transformers to simplify the complex design process.
- (a) Electric Circuit and (b) Magnetic Circuit



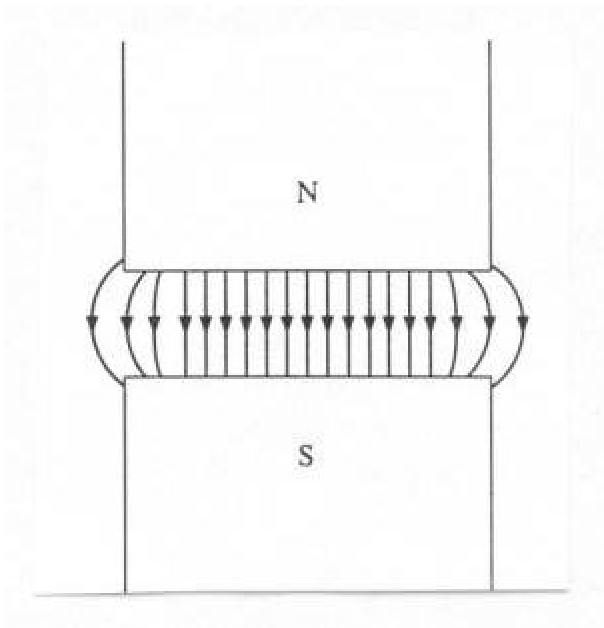
- The magnetomotive force, like voltage, has a polarity associated with it.



Modified right-hand rule for determining the direction of the positive mmf.

- Reluctances in a magnetic circuit obey the same rules for parallel and series combinations as resistances in an electric circuit.

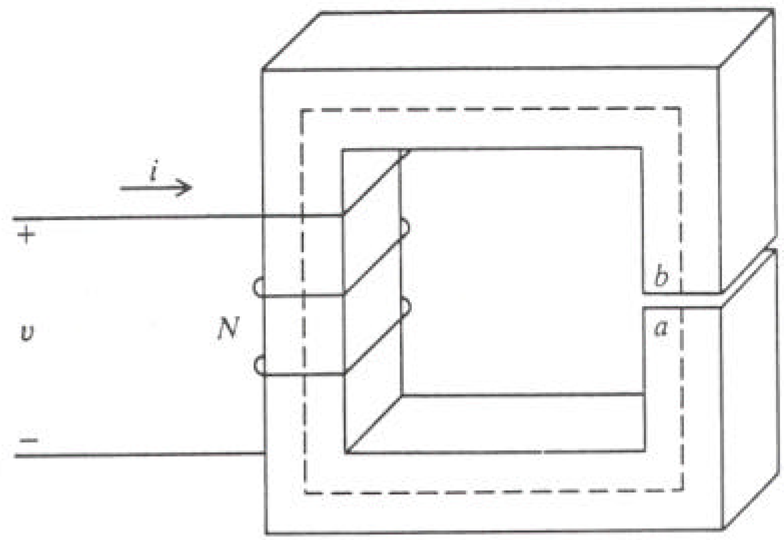
- Calculations of the flux in a core using magnetic circuit concepts are always approximations – accurate to within 5% at best! Why?
  - It is not true that all the flux is confined within the magnetic core. Flux outside the core is called *leakage flux*.
  - Calculation of reluctance assumes a certain mean path length and cross-sectional area for the core. These assumptions are not very good, especially at corners.
  - In ferromagnetic materials, the permeability varies with the amount of flux already in the material. This is a nonlinear effect. Reluctances depend on the permeability of the material.
  - "Fringing Effect" of the magnetic field at an air gap causes an increased effective cross-sectional area of the air gap.



Fringing Effect
of the magnetic field
at an air gap

- “Corrected” mean path lengths and cross-sectional areas can be used to offset these inherent sources of error.
- Magnetic circuit concept is still the easiest design tool available for calculation of fluxes.

- Consider the magnetic system shown.



- Assume that the magnetic system (circuit) consists only of the magnetic member and the air gap.
- Apply Ampere's Law to the elementary magnetic system:

$$\int_a^b H_i dL + \int_b^a H_g dL = Ni$$

- Assume that the field intensity exists only in the direction of the given path of integration or, in other words, perpendicular to a cross section of the magnetic material taken in the same sense as the air gap is cut through the material (valid except in the vicinity of the corners where the field intensity changes gradually rather than abruptly)
- Path of integration is taken as the mean length about the magnetic member ("mean length approximation")

- Carrying out the integration: $H_i \ell_i + H_g \ell_g = Ni$
- Assume that the flux density is uniformly distributed over the cross-sectional area and also perpendicular to the cross-sectional area:

$$\Phi_i = B_i A_i$$

$$\Phi_g = B_g A_g$$

- Streamlines of flux density are closed, hence the flux in the air gap is equal to the flux in the core:

$$\Delta \bullet \vec{B} = 0 \quad \text{Maxwell's } 4^{th} \text{ Equation} \qquad \Phi_i = \Phi_g \equiv \Phi$$

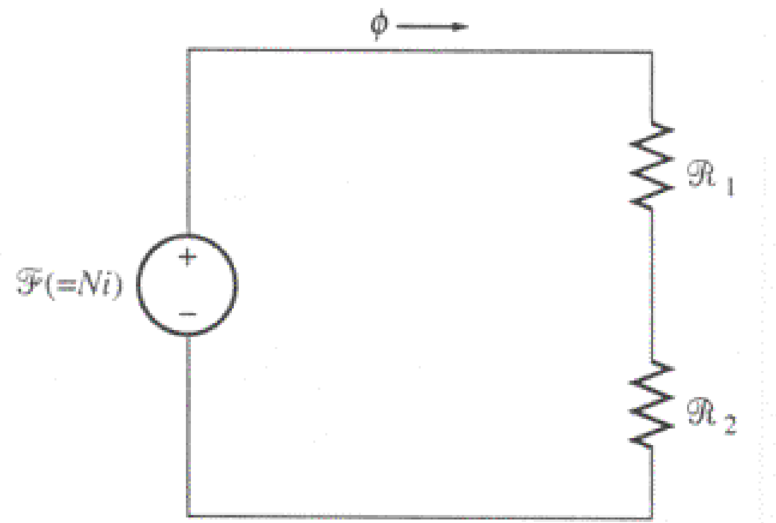
- Assume $A_g = A_i$, even though we know $A_g = kA_i$ where k > 1 due to the fringing effect.
- Results:

$$\frac{\ell_i}{\mu_i A_i}\Phi + \frac{\ell_g}{\mu_g A_g}\Phi = Ni$$

$$\mu_i = \mu_{ri}\mu_0 = (500 \rightarrow 4000)\mu_0$$

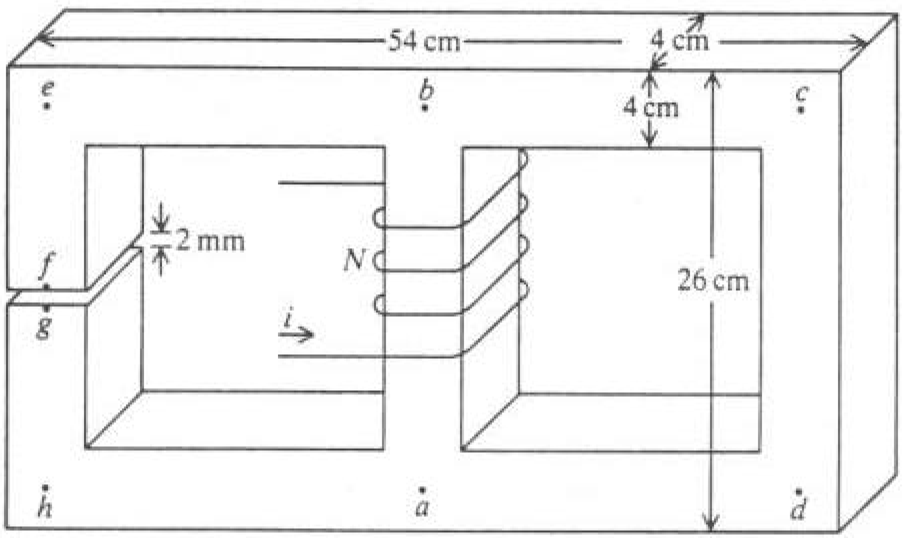
$$\mu_g = \mu_{rg}\mu_0 = (1)\mu_o$$

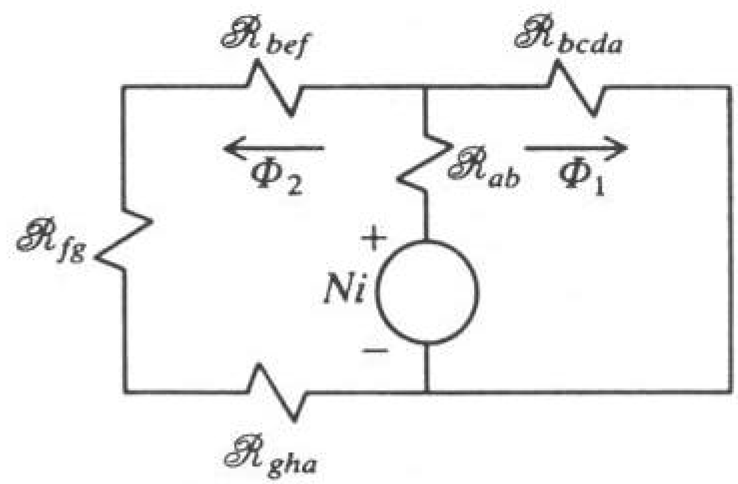
$$(\Re_i + \Re_g)\Phi = Ni$$



Example Problem

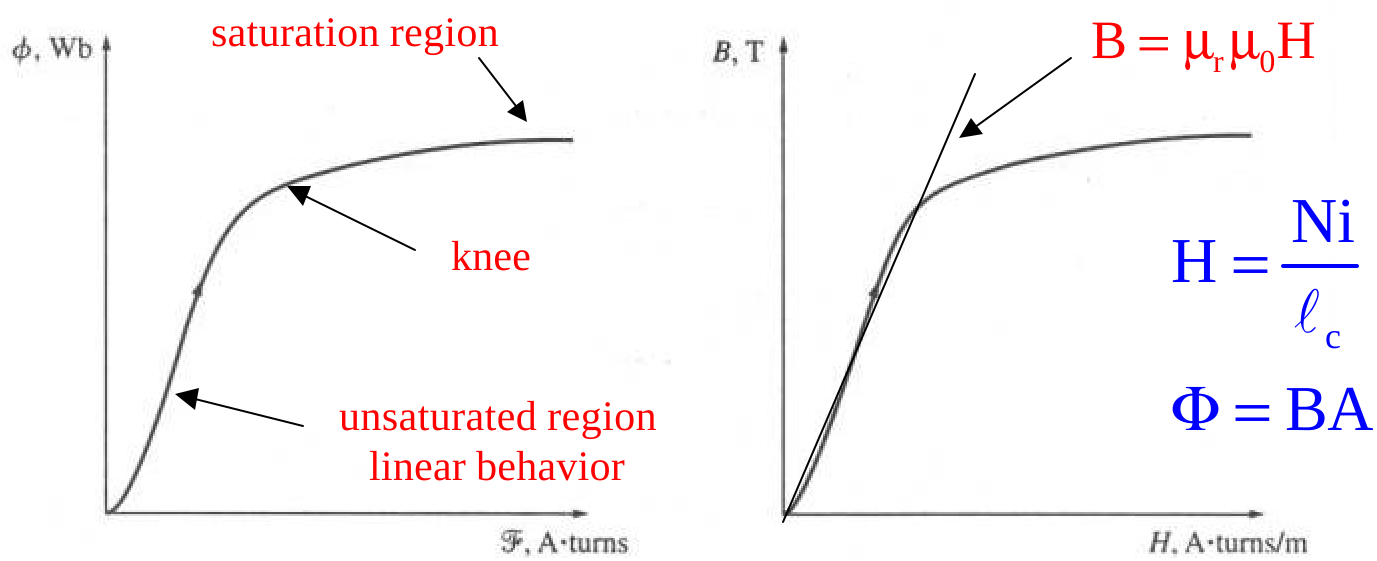
- In the magnetic system shown, the total number of turns is 100, the relative permeability of the iron is 1000, and the current is 10A. Calculate the total flux in the center leg.





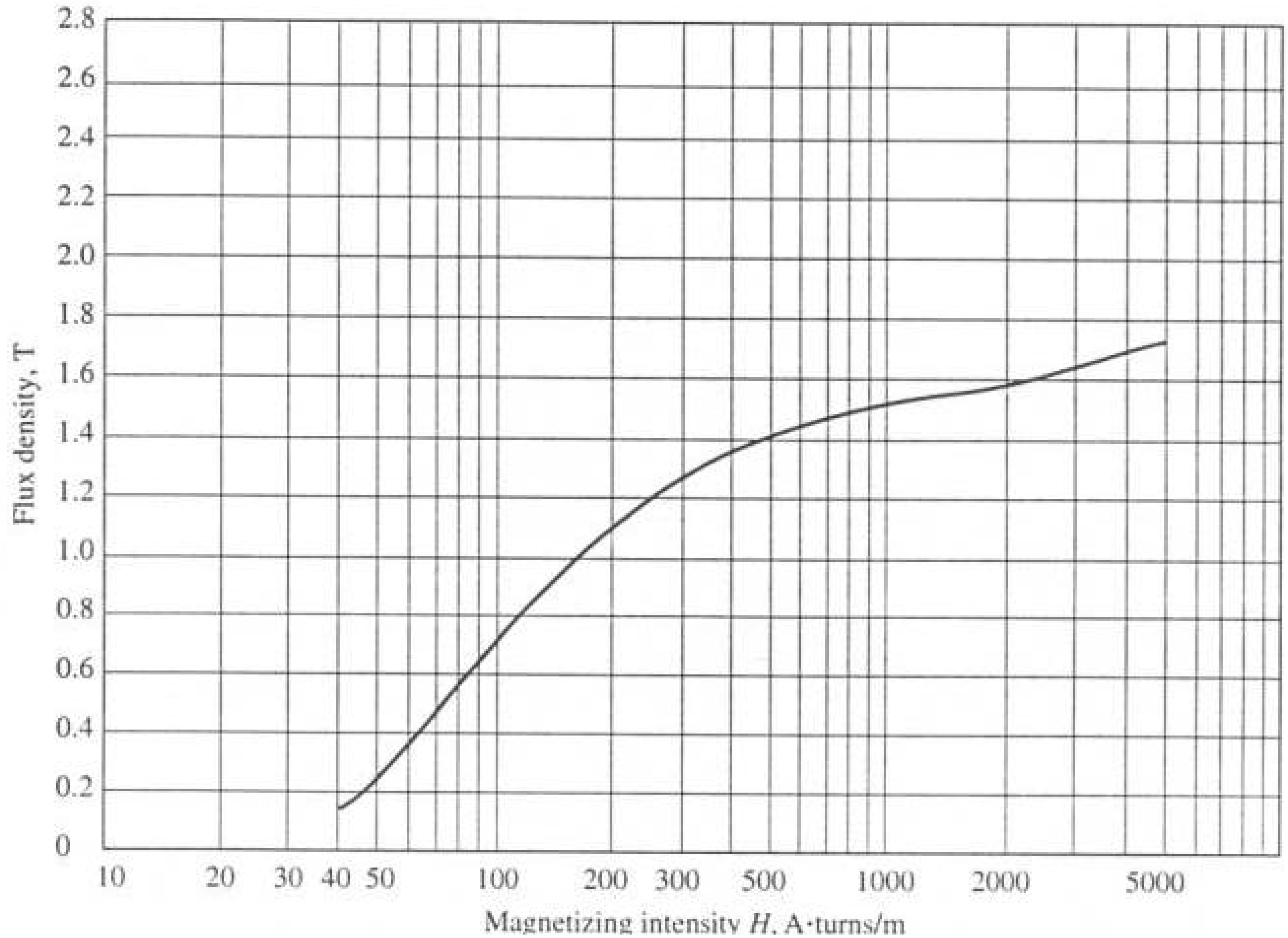
Properties of Magnetic Materials

- The permeability of free space μ_0 is constant.
- The permeability of ferromagnetic materials (e.g., iron, nickel, cobalt) is very high (500 to 4000 times that of free space) but it is not constant. It depends on the mmf applied to the material.
- <u>Experiment</u>:
  - Apply a direct current to the elementary magnetic circuit previously discussed, starting with 0 A and slowly working up to the maximum permissible current. Assume that B and H are initially zero.
  - Plot flux produced in the core vs. mmf

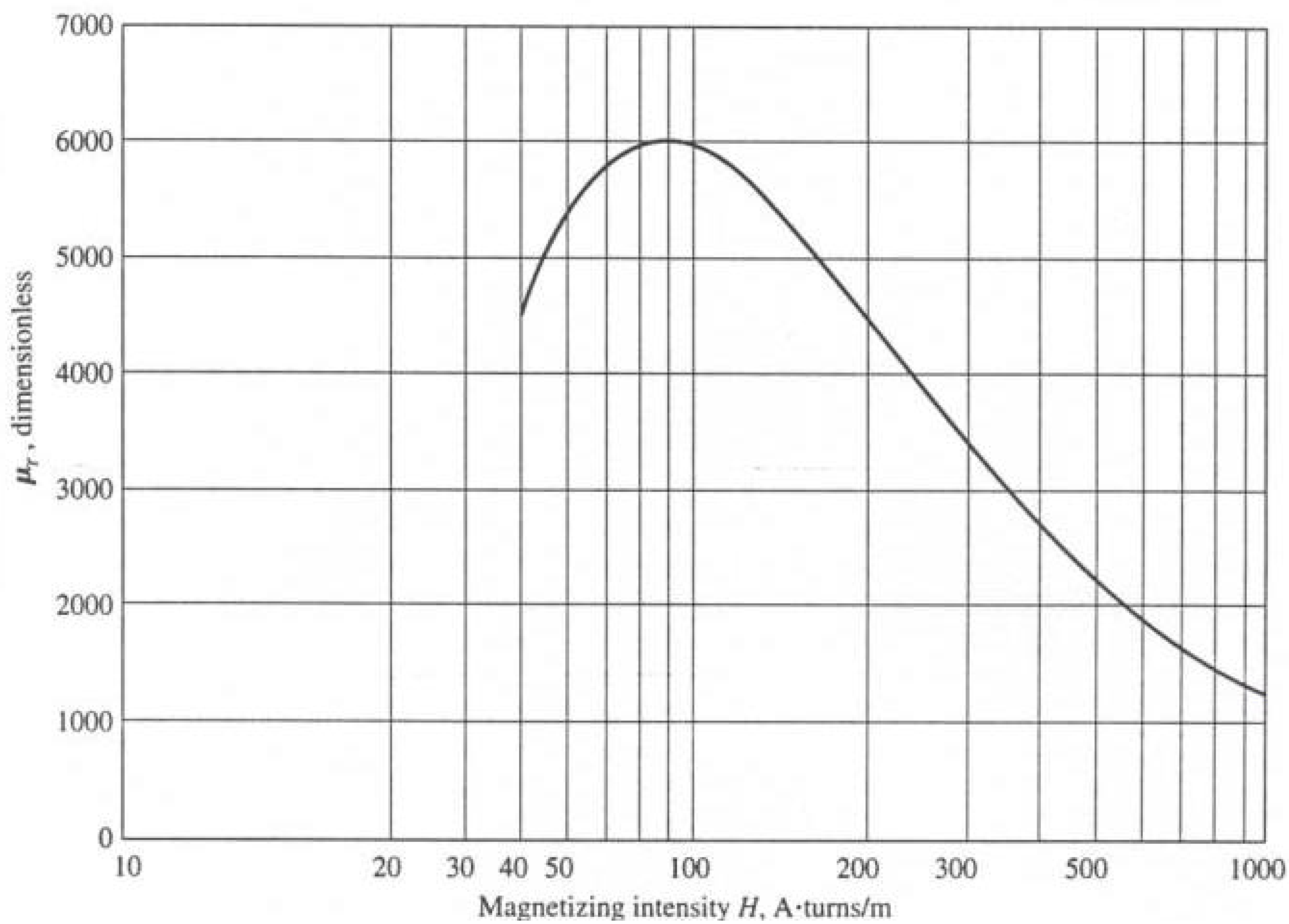


Saturation Curve or Magnetization Curve

- The relationship between B and H has the same shape as the relationship between flux and mmf.
- The slope of the B vs. H curve at any value of H is, by definition, the permeability of the core at that H.



Magnetization Curve for a Typical Piece of Steel



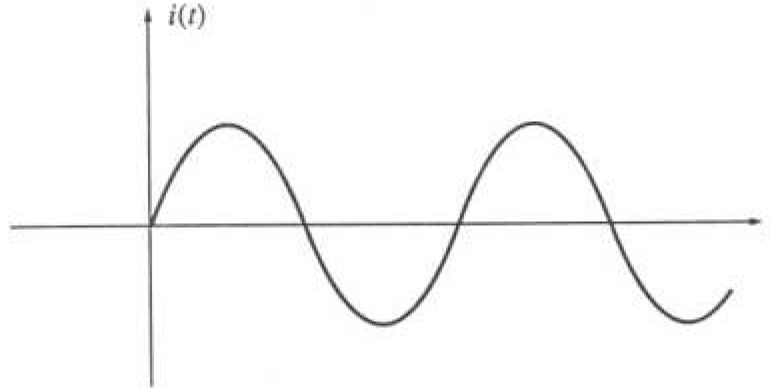
Relative Permeability vs. H Curve for a Typical Piece of Steel

- Most real machines operate near the knee of the magnetization curve; the flux in their cores is not linearly related to the mmf producing it.
- Why does the magnetization curve have this shape?
  - Microscopically, ferromagnetic materials have been found to be divided into magnetic domains wherein all magnetic moments (dipoles) are aligned. Each domain acts as a small permanent magnet. The direction of this alignment will differ from one domain to another; domains are oriented randomly within the material.
  - When a ferromagnetic material is subjected to an external field, it causes domains that happen to point in the direction of the field to grow at the expense of domains pointed in other directions. It is a positive feedback effect!

- This is known as domain wall motion. As the strength of the magnetic field increases, the aligned domains continue to grow in a nearly linear fashion. Whole domains that are aligned in the wrong direction eventually reorient themselves as a unit to line up with the field. A nearly linear B-H curve results.
- Soon the ability of the aligned domains to take from the unaligned domains starts to slow. This gives rise to the knee of the B-H curve and saturation is beginning.
- Finally, when nearly all the atoms and domains in the iron are lined up with the external field, any further increase in the mmf can cause only the same flux increase that it would in free space. Once everything is aligned, there can be no more feedback effect to strengthen the field. The material is saturated with flux. Slope of B-H curve is μ_0 .

New Experiment

Instead of applying a direct current to the windings on the core, apply an alternating current and observe what happens. Assume that B and H are initially both zero.



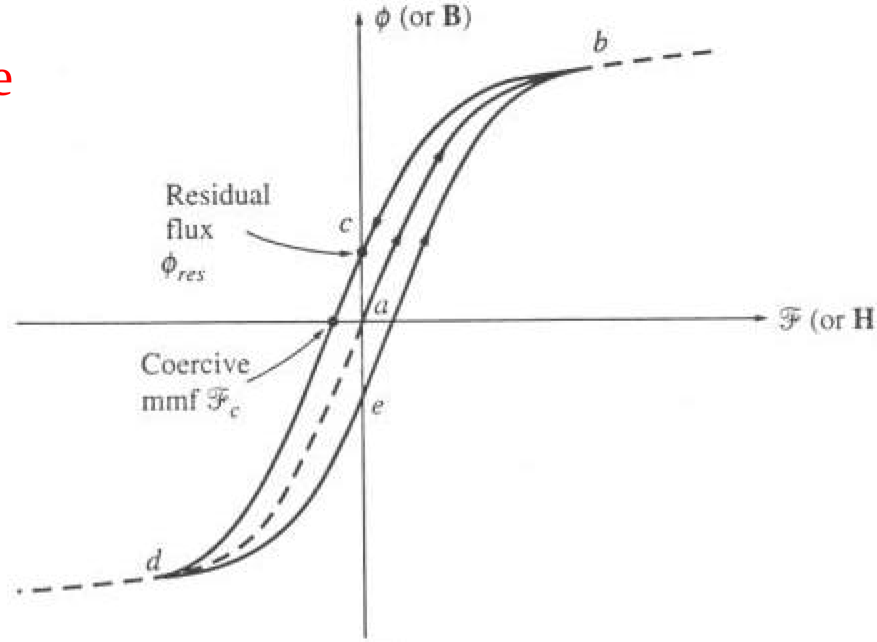
(a)

After several cycles, a steady-state condition is reached.

Hysteresis Loop

Path b-c-d-e-b

(double-valued function)



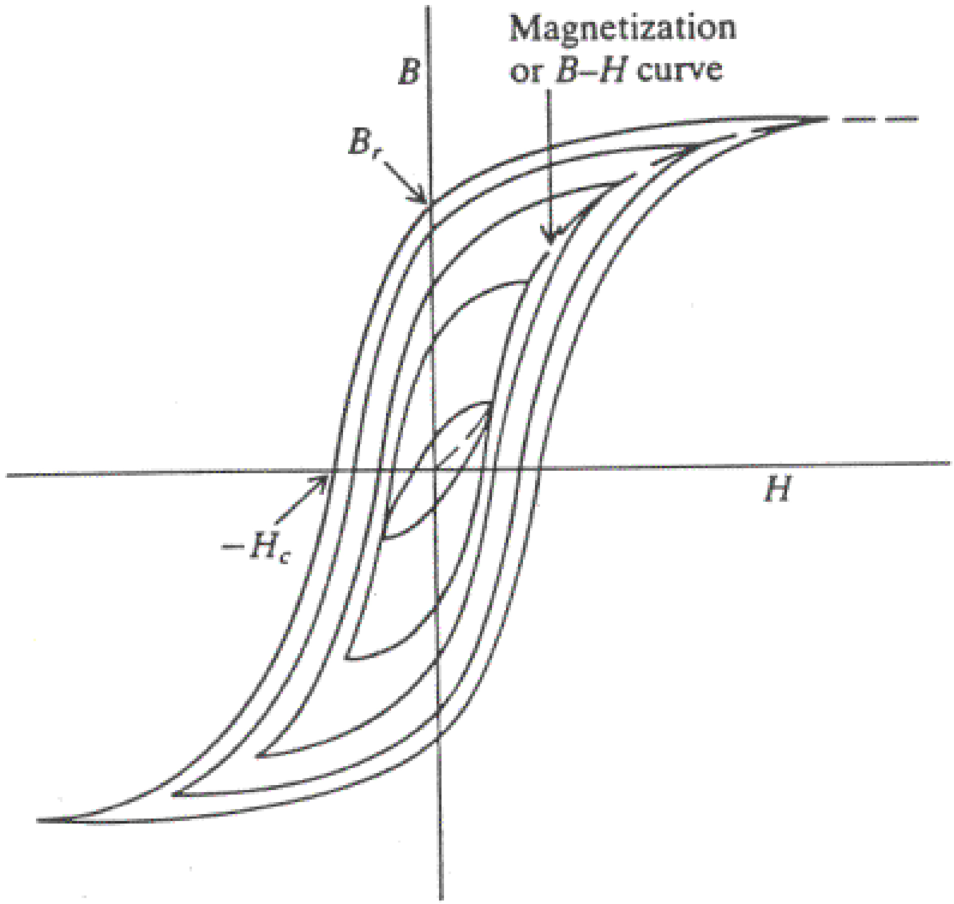
(b)

- Observations
  - The amount of flux present in the core depends not only on the amount of current applied to the windings of the core, but also on the previous history of the flux in the core.
  - The dependence on the preceding flux history and the resulting failure to retrace flux paths is called *hysteresis*.
  - Path *bcdeb* traced out on the magnetization curve as the applied current changes is called a *hysteresis loop*.
  - If a large mmf is first applied to the core and then removed, the flux path in the core will be *abc*. When the mmf is removed, the flux in the core does not go to zero. Instead, a magnetic field is left in the core. This magnetic field is called the *residual flux*. It is in precisely this manner that permanent magnets are produced.

- To force the flux to zero, an amount of mmf known as the *coercive magnetomotive force* must be applied to the core in the opposite direction.

- Why does hysteresis occur?
  - In simple terms, the growth of aligned domains for an incremental change in H in one direction is not equal to the growth of oppositely aligned domains if this change in H were suddenly reversed.

The diagram shows the effect of the size of mmf excursions on the magnitude of the hysteresis loop.



Family of Steady-State Hysteresis Loops

- Hysteresis Loss
  - The fact that turning domains in a ferromagnetic material requires energy leads to a common type of energy loss in all machines and transformers.
  - The hysteresis loss in an iron core is the energy required to accomplish the reorientation of domains during each cycle of the alternating current applied to the core. The area enclosed in the hysteresis loop formed by applying an alternating current to the core is directly proportional to the energy lost in a given *ac* cycle.
  - The smaller the applied mmf excursions on the core, the smaller the area of the resulting hysteresis loop and so the smaller the resulting losses.

- This energy causes a rise in the temperature of the magnetic material and the power associated with this energy loss is called *hysteresis loss*.

- Eddy Currents
  - The mechanism of eddy current losses is explained by Faraday's Law. A time-changing flux induces voltage within a ferromagnetic core.
  - When a solid block of magnetic material is subjected to an alternating field intensity, the resulting alternating flux induces current in the solid magnetic material which will circulate in a loop perpendicular to the flux density inducing it. These are called *eddy currents*.
  - There are two undesirable side effects from eddy currents:

- First, the mmf established by these circulating currents opposes the mmf produced by the winding, and this opposition is greatest at the center of the material because that tends to be also the center of the current loops. Thus, the flux would tend not to flow through the center of the solid magnetic member, thereby not utilizing the full benefits of the ferromagnetic material.
- Second, there is a I^2R loss associated with these eddy currents flowing in a resistive material, called eddy current loss, which is dissipated as heat.

– These two adverse effects can be minimized in several ways; the most common way is to build the ferromagnetic core of laminations insulated from each other and oriented in the direction of the magnetic field. These thin strips offer a much smaller area in which the eddy currents can flow, hence smaller currents and smaller losses result.

- Core Losses
  - The core losses associated with ferromagnetic materials are the combination of the hysteresis and eddy current losses.
  - Electromagnetic devises are designed to minimize these losses; however, they are always present.
  - We can often take them into account in a linear system analysis by representing their effects on the system by a resistance.

Faraday's Law and Lenz's Law

- Now focus on the various ways in which an existing magnetic field can effect its surroundings.
- Faraday's Law

  Maxwell's 1st Equation $\Delta \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

  – If a flux passes through a turn of a coil of wire, a voltage will be induced in the turn of wire that is directly proportional to the rate of change in the flux with respect to time.

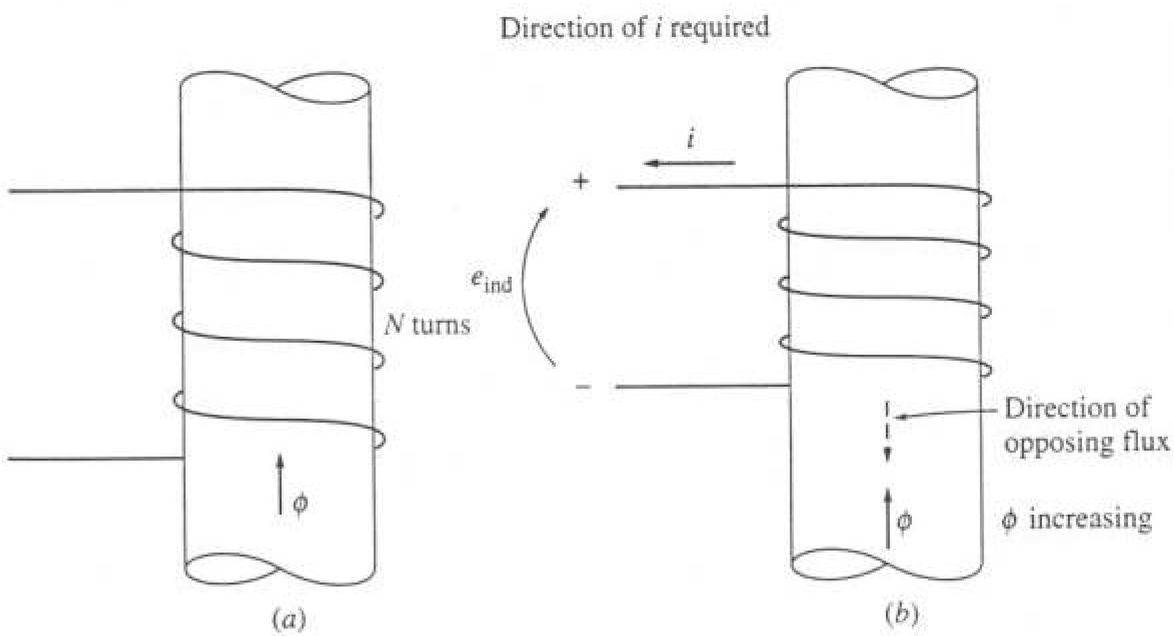
  $$e_{ind} = -\frac{d\phi}{dt}$$

  – If the coil has N turns and the same flux passes through all of them, then

  $$e_{ind} = -N\frac{d\phi}{dt}$$

- It is the fundamental law of transformer operation.
- The minus sign is an expression of Lenz's Law.
  - The direction of the voltage buildup in the coil is such that if the coil ends were short-circuited, it would produce current that would cause a flux opposing the original flux change. Since the induced voltage opposes the change that causes it, a minus sign is included.
  - The minus sign is often left out, as the polarity of the resulting voltage can be determined from physical considerations.
- Practical Problem
  - Equation assumes that exactly the same flux is present in each turn of the coil. What about leakage flux?

Meaning of Lenz's Law



(a) A coil enclosing an increasing magnetic flux
(b) Determining the resulting voltage polarity

– If the windings are tightly coupled, so that the vast majority of the flux passing through one turn of the coil does indeed pass through all of them, then the equation will give valid answers.

– If the leakage is quite high or if extreme accuracy is required, a different expression is needed.

$$e_{ind} = \frac{d(\phi_i)}{dt} \quad i^{th} \text{ turn of the coil}$$

$$e_{ind} = \sum_{i=1}^{N} e_i = \sum_{i=1}^{N} \frac{d(\phi_i)}{dt} = \frac{d}{dt}\left(\sum_{i=1}^{N} \phi_i\right)$$

$$e_{ind} = \frac{d\lambda}{dt} \quad \text{where } \lambda = \sum_{i=1}^{N} \phi_i = \text{ flux linkage of coil (Wb-turns)}$$

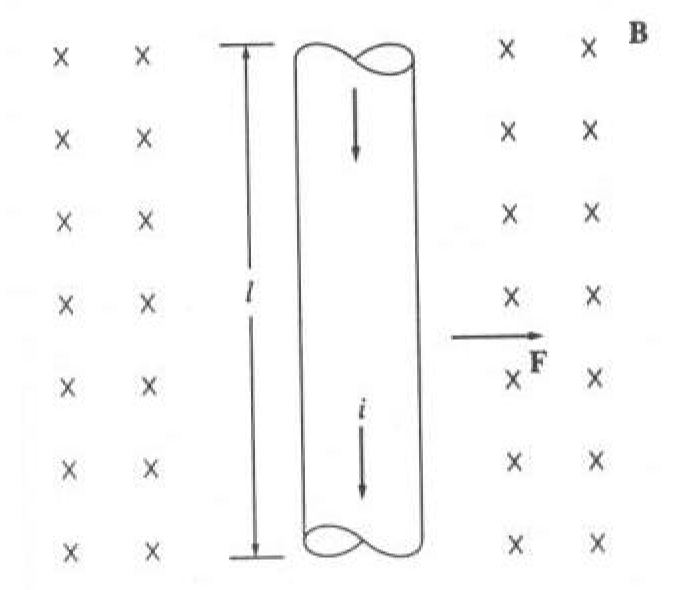
Production of an Induced Force on a Wire

- A magnetic field induces a force on a current-carrying wire within the field. $\vec{F} = i\left(\vec{\ell} \times \vec{B}\right)$
- The direction of the force is given by the right-hand rule.
- The magnitude of the force is given by $F = i\ell B \sin\theta$ where θ is the angle between the wire and the flux density vector.
- The induction of a force in a wire by a current in the presence of a magnetic field is the basis of motor action.

$\vec{B}$

points into the page

A current-carrying wire in the presence of a magnetic field



$$\vec{F} = i\left(\vec{\ell} \times \vec{B}\right)$$

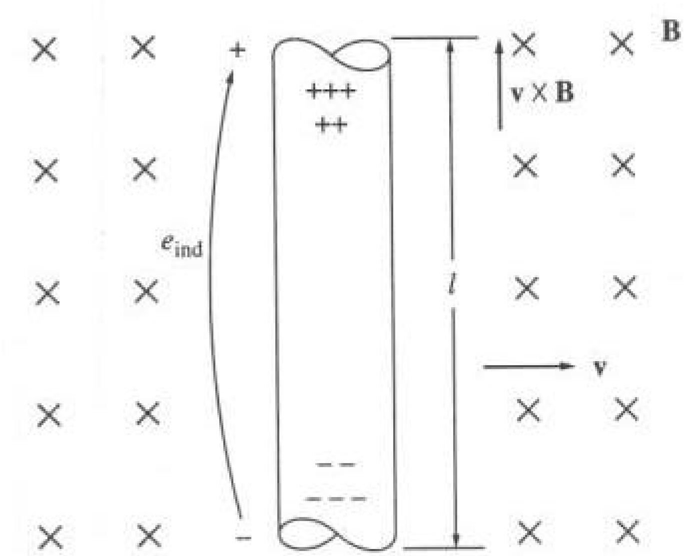
Induced Voltage on a Conductor Moving in a Magnetic Field

- If a wire with the proper orientation moves through a magnetic field, a voltage is induced in it. The voltage induced in the wire is given by $e_{ind} = \left(\vec{v} \times \vec{B}\right) \bullet \vec{\ell}$
- Vector $\vec{\ell}$ points along the direction of the wire toward the end making the smallest angle with respect to the vector $\vec{v} \times \vec{B}$.
- The voltage in the wire will be built up so that the positive end is in the direction of the vector $\vec{v} \times \vec{B}$.
- The induction of voltages in a wire moving in a magnetic field is the basis of generator action.

$\vec{B}$

points into the page

A conductor moving in the presence of a magnetic field



$$e_{ind} = (\vec{v} \times B) \bullet \vec{\ell}$$

Linear DC Machine – A Simple Example

- A linear dc machine is about the simplest and easiest-to-understand version of a dc machine, yet it operates according to the same principles and exhibits the same behavior as real generators and motors.

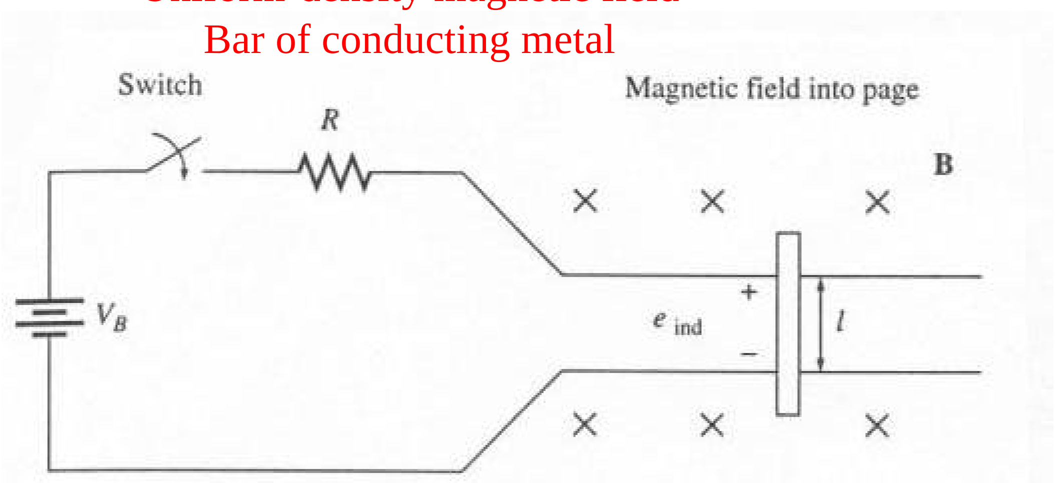
$$\vec{F} = i\left(\vec{\ell} \times \vec{B}\right)$$

$$e_{ind} = \left(\vec{v} \times \vec{B}\right) \bullet \vec{\ell}$$

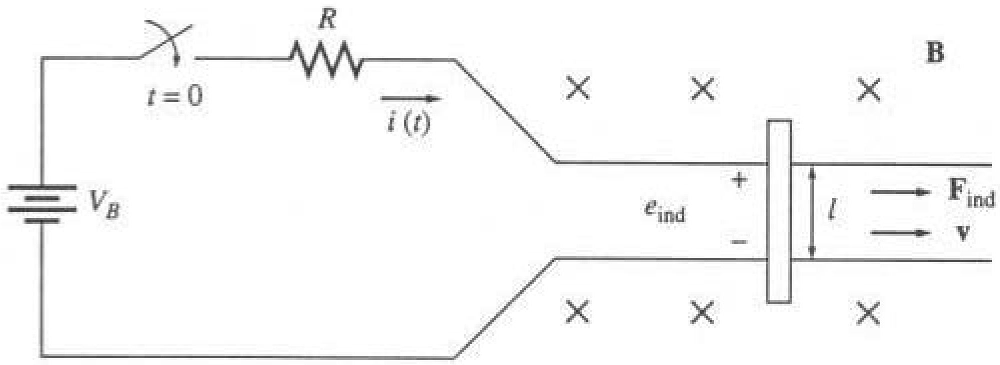
$$V_B - iR - e_{ind} = 0$$

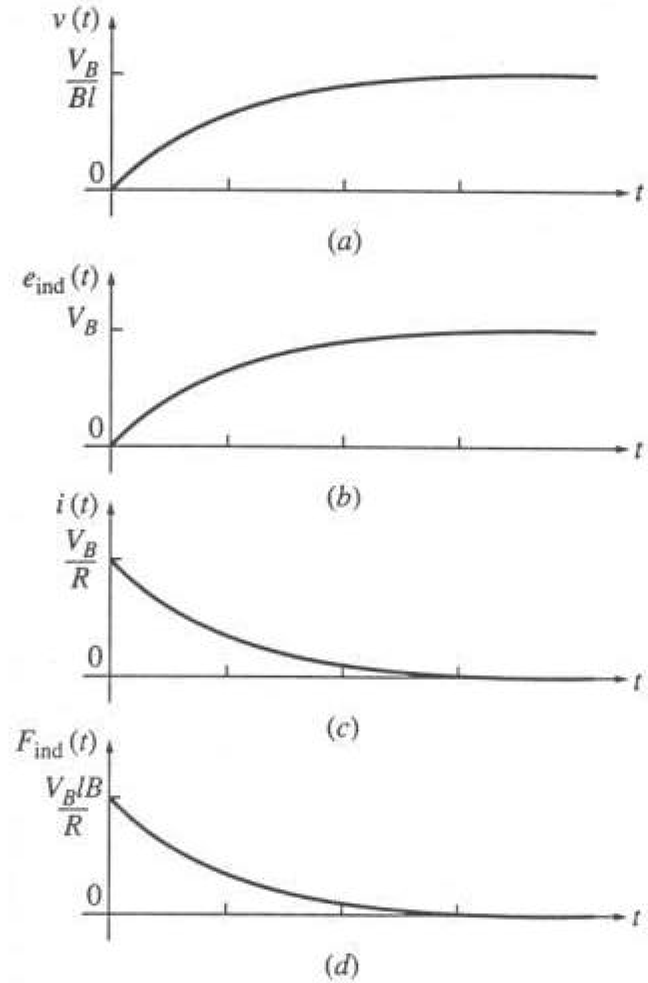
$$F_{net} = ma$$

Smooth frictionless rails
Uniform-density magnetic field
Bar of conducting metal



Starting the Linear DC Machine





Closing the switch produces a current flow $i = \frac{V_B}{R}$

The current flow produces a force on the bar given by $F = i\ell B$

The bar accelerates to the right, producing an induced voltage e_{ind} as it speeds up.

This induced voltage reduces the current flow $i = \frac{(V_B - e_{ind} \uparrow)}{R}$

The induced force is thus decreased until eventually F = 0. At that point, $e_{ind} = V_B$ and i = 0, and the bar moves at a constant no-load speed.

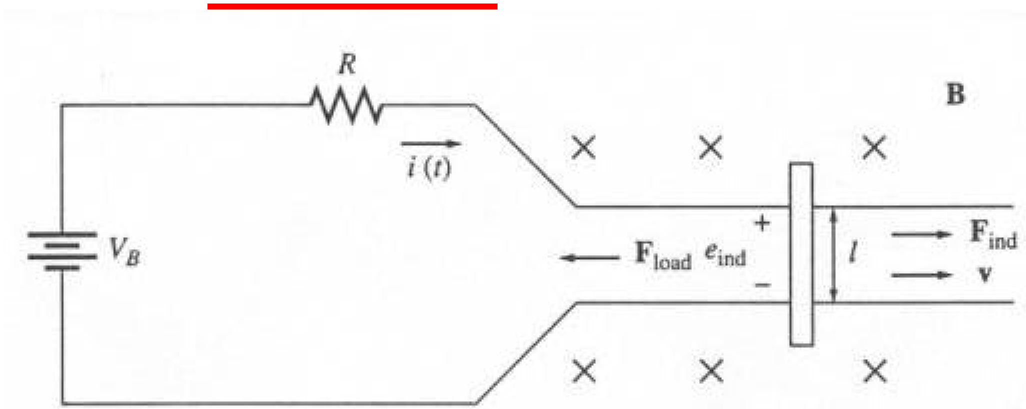
$F = i \downarrow \ell B$

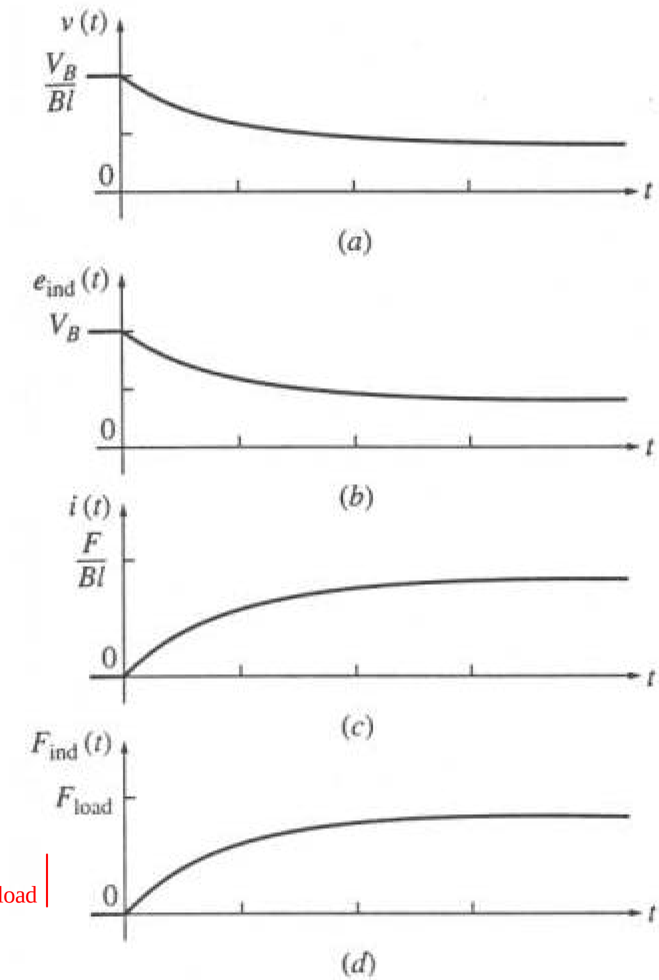
$v_{ss} = \frac{V_B}{B\ell}$

The Linear DC Machine as a Motor

Apply an external load

Assume machine is initially running at no-load SS conditions





A force F_{load} is applied opposite to the direction of motion, which causes a net force F_{net} opposite to the direction of motion.

The resulting acceleration is negative, so the bar slows down. $a = \dfrac{F_{net}}{m}$

$$e_{ind} = v \downarrow \ell B$$

The voltage e_{ind} falls, and so i increases.

$$i = \frac{\left(V_B - e_{ind} \downarrow\right)}{R}$$

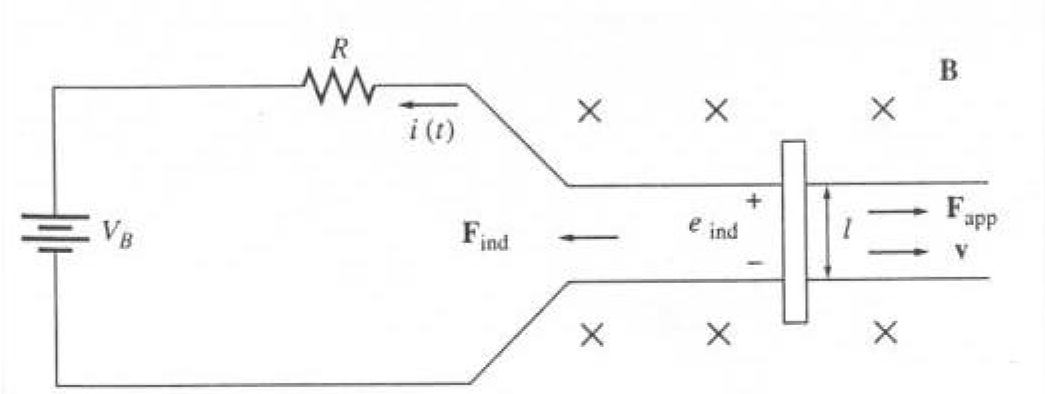
The induced force F_{ind} increases until, at a lower speed, $\left|F_{ind}\right| = \left|F_{load}\right|$

$$F_{ind} = i \uparrow \ell B$$

An amount of electric power equal to $e_{ind}i$ is now being converted to mechanical power equal to $F_{ind}v$.

The Linear DC Machine as a Generator

Apply a force in the direction of motion
Assume machine is initially running at no-load SS conditions



A force F_{app} is applied in the direction of motion; F_{net} is in the direction of motion.
Acceleration is positive, so the bar speeds up. $a = \frac{F_{net}}{m}$

$$e_{ind} = v \uparrow B\ell$$

The voltage e_{ind} increases, and so i increases.

$$i = \frac{\left(e_{ind} \uparrow - V_B\right)}{R}$$

The induced force F_{ind} increases until, at a higher speed, $\left|F_{ind}\right| = \left|F_{app}\right|$

$$F_{ind} = i \uparrow \ell B$$

An amount of mechanical power equal to $F_{ind}v$ is now being converted to electric power $e_{ind}i$, and the machine is acting a a generator.

- Observations
  - The same machine acts as both motor and generator.
    - Generator: externally applied forces are in the direction of motion
    - Motor: externally applied forces are opposite to the direction of motion
  - Electrically
    - $e_{ind} > V_B$, machine acts as a generator
    - $e_{ind} < V_B$, machine acts as a motor
  - Whether the machine is a motor or a generator, both induced force (motor action) and induced voltage (generator action) are present at all times.
  - This machine was a generator when it moved rapidly and a motor when it moved more slowly, but whether it was a motor or a generator, it always moved in the same direction.

Example Problem

- A linear dc machine has a battery voltage of 120 V, an internal resistance of 0.3 Ω, and a magnetic flux density of 0.1 T.
  - What is the machine's maximum starting current? What is its steady-state velocity at no load?
  - Suppose that a 30-N force pointing to the right were applied to the bar. What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the battery be producing or consuming? Explain the difference between these two figures. Is this machine acting as a motor or as a generator?

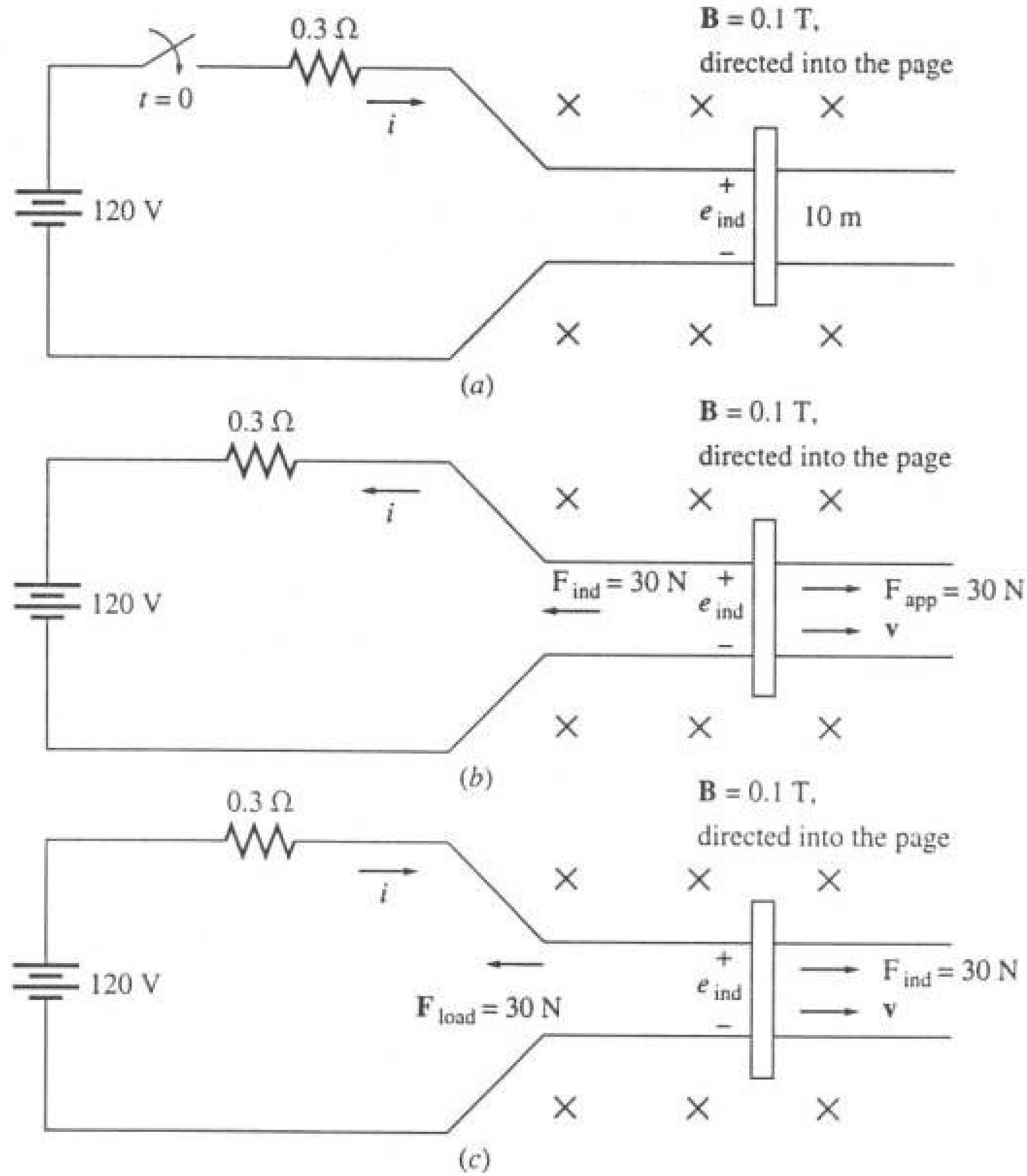
- Now suppose a 30-N force pointing to the left were applied to the bar. What would the new steady-state speed be? Is this machine a motor or a generator now?
- Assume that a force pointing to the left is applied to the bar. Calculate the speed of the bar as a function of the force for values from 0 N to 50 N in 10-N steps. Plot the velocity of the bar versus the applied force.
- Assume that the bar is unloaded and that it suddenly runs into a region where the magnetic field is weakened to 0.08 T. How fast will the bar go now?

Example Problem

Starting Conditions

Operating as a Generator

Operating as a Motor

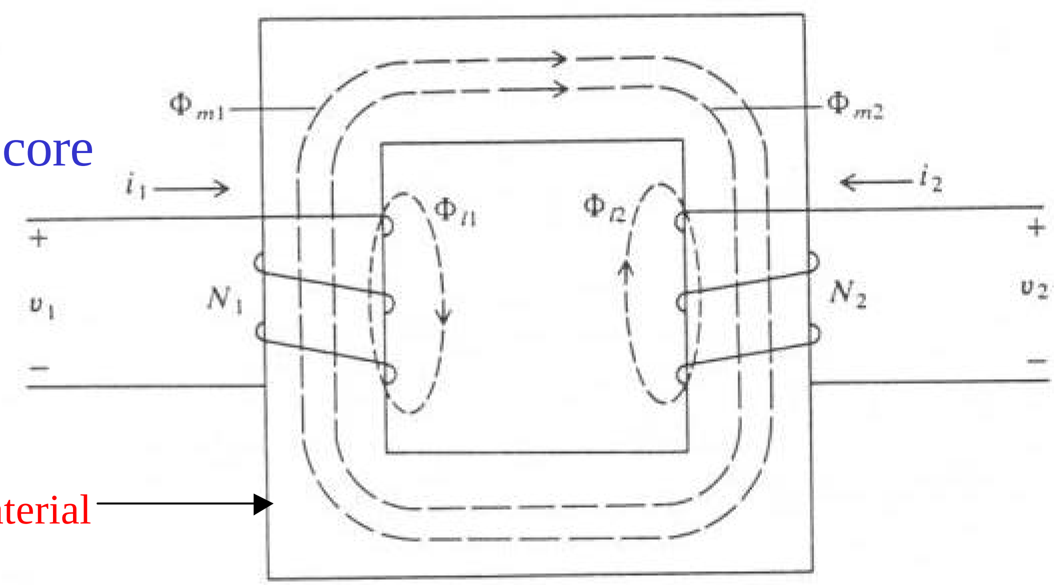


Stationary Magnetically Coupled Circuits

- Magnetically-coupled electric circuits are central to the operation of transformers and electromechanical motion devices.
- In transformers, stationary circuits are magnetically coupled for the purpose of changing the voltage and current levels.
- In electromechanical motion devices, circuits in relative motion are magnetically coupled for the purpose of transferring energy between the mechanical and electrical systems.

- Goal:
  - Establish the equations that describe the behavior of magnetically coupled circuits
  - Express the equations in a form convenient for analysis
- Consider first two stationary electric circuits which are magnetically coupled.

Magnetically-Coupled Circuits:
two windings on a common core



- Some comments:
  - Ideal transformer
    - only the turns ratio is considered

$$v_2 = \frac{N_2}{N_1} v_1$$

$$i_2 = -\frac{N_1}{N_2} i_1$$

  - This treatment is often not sufficient for a detailed analysis of transformers, and it is seldom appropriate in the analysis of electromechanical motion devices, since an air gap is necessary for motion to occur; hence, windings are not as tightly coupled and leakage flux must be taken into account.
- Flux produced by each winding can be separated into two components:
  - leakage component and magnetizing component

- Leakage Flux
  - The leakage flux associated with a given winding links only that winding
- Magnetizing Flux
  - The magnetizing flux, whether it is due to current in winding 1 or winding 2, links both windings
- The flux linking each winding is expressed as:

$$\phi_1 = \phi_{\ell 1} + \phi_{m1} + \phi_{m2}$$

$$\phi_2 = \phi_{\ell 2} + \phi_{m2} + \phi_{m1}$$

  - Leakage flux is produced by current flowing in a winding and it links only the turns of that winding
  - Magnetizing flux is produced by current flowing in a winding and it links all the turns of both windings

- This is an idealization of the actual magnetic system
  - All of the leakage flux will not link all the turns of the winding producing it; so the leakage fluxes are really “equivalent” leakage fluxes
  - All the magnetizing flux of one winding will not link all of the turns of the other winding; N_1 and N_2 are often considered to be “equivalent” number of turns rather than the actual number.
- The voltage equations may be expressed as:

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

$$\lambda_1 = N_1 \phi_1$$

$$\lambda_2 = N_2 \phi_2$$

- Assume that the magnetic system is linear; apply Ohm's Law for magnetic circuits to express the fluxes.

Typically $\Re_{\ell 1} \gg \Re_m$, $\Re_{\ell 2} \gg \Re_m$

$$\phi_{\ell 1} = \frac{N_1 i_1}{\Re_{\ell 1}}$$

$$\phi_{m1} = \frac{N_1 i_1}{\Re_m}$$

$$\phi_{\ell 2} = \frac{N_2 i_2}{\Re_{\ell 2}}$$

$$\phi_{m2} = \frac{N_2 i_2}{\Re_m}$$

$$\phi_1 = \phi_{\ell 1} + \phi_{m1} + \phi_{m2} = \frac{N_1 i_1}{\Re_{\ell 1}} + \frac{N_1 i_1}{\Re_m} + \frac{N_2 i_2}{\Re_m}$$

$$\phi_2 = \phi_{\ell 2} + \phi_{m2} + \phi_{m1} = \frac{N_2 i_2}{\Re_{\ell 2}} + \frac{N_2 i_2}{\Re_m} + \frac{N_1 i_1}{\Re_m}$$

$\Re_m$ can be computed with sufficient accuracy

$\Re_\ell$ is usually approximated from test data

$$\lambda_1 = N_1\phi_1 = \frac{N_1^2}{\Re_{\ell 1}} i_1 + \frac{N_1^2}{\Re_m} i_1 + \frac{N_1 N_2}{\Re_m} i_2$$

$$\lambda_2 = N_2\phi_2 = \frac{N_2^2}{\Re_{\ell 2}} i_2 + \frac{N_2^2}{\Re_m} i_2 + \frac{N_2 N_1}{\Re_m} i_1$$

- When the magnetic system is linear, the flux linkages are generally expressed in terms of inductances and the currents.
  - The self inductances are:

$$L_{11} = \frac{N_1^2}{\Re_{\ell 1}} + \frac{N_1^2}{\Re_m} = L_{\ell 1} + L_{m1}$$

$$L_{22} = \frac{N_2^2}{\Re_{\ell 2}} + \frac{N_2^2}{\Re_m} = L_{\ell 2} + L_{m2}$$

– We see that: $$\frac{L_{m2}}{N_2^2} = \frac{L_{m1}}{N_1^2}$$

– The mutual inductances are:

$$\left.\begin{aligned} L_{12} &= \frac{N_1 N_2}{\Re_m} \\ L_{21} &= \frac{N_2 N_1}{\Re_m} \end{aligned}\right\} \quad L_{12} = L_{21}$$

– In this situation, with the assumed positive direction for current flow and the manner in which the windings are wound, the mutual inductances are positive. If, however, the assumed positive directions of current were such that φ_{m1} opposed φ_{m2} , then the mutual inductances would be negative.

– We see that: $$L_{12} = \frac{N_2}{N_1} L_{m1} = \frac{N_1}{N_2} L_{m2}$$

- Therefore

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$
$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$

  - L_{11} and L_{22} are always positive
  - $L_{12} = L_{21}$ may be positive or negative
- The voltage equations

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt}$$
$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt}$$

  (already derived) may be used for purposes of analysis. However, it is customary to perform a change of variables.

$$\lambda_1 = L_{11}i_1 + L_{12}i_2$$
$$= \left(L_{\ell 1} + L_{m1}\right)i_1 + L_{12}i_2$$
$$= L_{\ell 1}i_1 + L_{m1}\left(i_1 + \frac{N_2}{N_1}i_2\right)$$

$$\lambda_2 = L_{21}i_1 + L_{22}i_2$$
$$= \left(L_{\ell 2} + L_{m2}\right)i_2 + L_{21}i_1$$
$$= L_{\ell 2}i_2 + L_{m2}\left(\frac{N_1}{N_2}i_1 + i_2\right)$$

Two possibilities for substitute variables: $\frac{N_2}{N_1}i_2$ or $\frac{N_1}{N_2}i_1$

Let $i_2' = \frac{N_2}{N_1}i_2$ and then $N_1 i_2' = N_2 i_2$

This current i_2', when flowing through winding 1, produces the same mmf as the actual i_2 flowing through winding 2. This is said to be referring the current in winding 2 to winding 1 or to a winding with N_1 turns, whereupon winding 1 becomes the reference winding.

- We want the instantaneous power to be unchanged by this substitution of variables. Therefore

$$v_2' i_2' = v_2 i_2$$

$$v_2' = v_2 \frac{i_2}{i_2'}$$

$$v_2' = \frac{N_1}{N_2} v_2$$

- Flux linkages, which have units of volts-second, are related to the substitute flux linkages in the same way as voltages.

$$\lambda_2' = \frac{N_1}{N_2} \lambda_2$$

$$\lambda_1 = L_{\ell 1} i_1 + L_{m1}\left(i_1 + \frac{N_2}{N_1} i_2\right)$$

$$i_2' = \frac{N_2}{N_1} i_2$$

$$\boxed{\lambda_1 = L_{\ell 1} i_1 + L_{m1}\left(i_1 + i_2'\right)}$$

$$L_{11} = L_{\ell 1} + L_{m1}$$

$$\boxed{\lambda_1 = L_{11} i_1 + L_{m1} i_2'}$$

$$\lambda_2 = L_{\ell 2} i_2 + L_{m2}\left(\frac{N_1}{N_2} i_1 + i_2\right)$$

$$i_2' = \frac{N_2}{N_1} i_2 \qquad \frac{L_{m2}}{N_2^2} = \frac{L_{m1}}{N_1^2}$$

$$\lambda_2' = \frac{N_1}{N_2}\lambda_2 \qquad L_{\ell 2}' \equiv \left(\frac{N_1}{N_2}\right)^2 L_{\ell 2}$$

$$\boxed{\lambda_2' = L_{\ell 2}' i_2' + L_{m1}\left(i_1 + i_2'\right)}$$

$$L_{22} = L_{\ell 2} + L_{m2}$$

$$\frac{L_{m2}}{N_2^2} = \frac{L_{m1}}{N_1^2}$$

$$L_{22}' \equiv \left(\frac{N_1}{N_2}\right)^2 L_{22} = L_{\ell 2}' + L_{m1}$$

$$\boxed{\lambda_2' = L_{m1} i_1 + L_{22}' i_2'}$$

$$\lambda_1 = L_{11} i_1 + L_{m1} i_2'$$

$$\lambda_2' = L_{m1} i_1 + L_{22}' i_2'$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

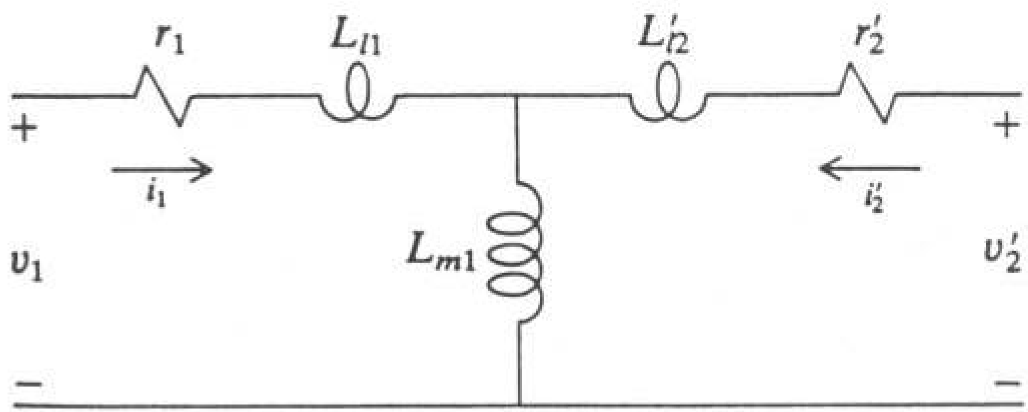
$$i_2' = \frac{N_2}{N_1} i_2$$

$$v_2' = \frac{N_1}{N_2} v_2$$

$$\begin{bmatrix} v_1 \\ v_2' \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2' \end{bmatrix} \begin{bmatrix} i_1 \\ i_2' \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_1 \\ \lambda_2' \end{bmatrix}$$

$$r_2' \equiv \left(\frac{N_1}{N_2} \right)^2 r_2$$

Equivalent T circuit with winding 1 selected as the reference winding



This method may be extended to include any number of windings wound on the same core.

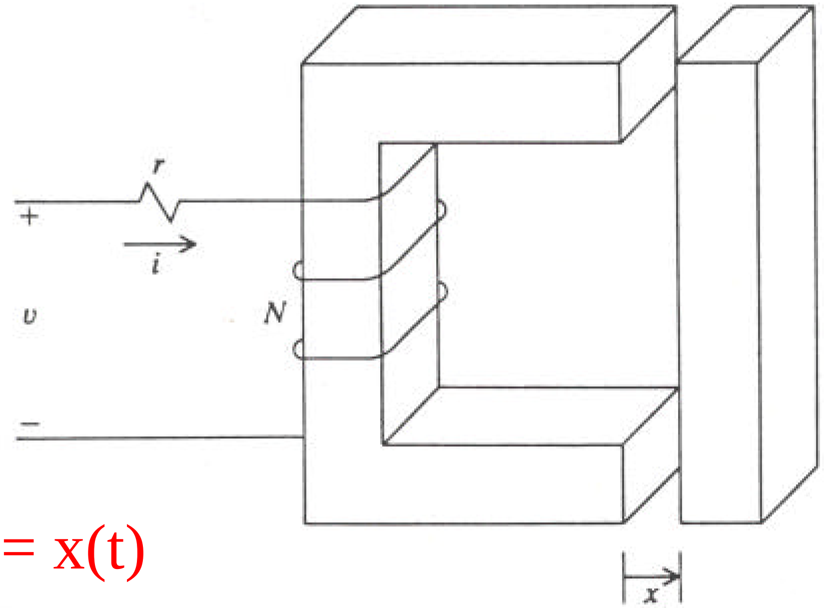
- We can now more fully appreciate the assumption that for an ideal transformer only the turns-ratio is considered.
  - the resistances and the leakage inductances are neglected
  - It is assumed that the magnetizing inductance is large so that the magnetizing current $i_1 + i_2'$ is negligibly small
- This section forms the basis of the equivalent circuits for many types of electric machines. Using a turns-ratio to refer the voltages and currents of rotor circuits of electric machines to a winding with the same number of turns as the stator windings is common practice.

Magnetic Systems with Mechanical Motion

- We introduce three elementary electromechanical systems for the purpose of establishing the voltage equations and expressions for the self- and mutual-inductances, thereby setting the stage for deriving the relationships for determining the electromagnetic force or torque established in electromechanical systems.
- The three systems are:
  - elementary version of an electromagnet
  - rotational device referred to as a reluctance machine
  - rotational device which has two windings

Elementary Electromagnet

- The system consists of:
  - stationary core with a winding of N turns
  - block of magnetic material is free to slide relative to the stationary member



x = x(t)

$$v = ri + \frac{d\lambda}{dt}$$

voltage equation that describes the electric system

$$\lambda = N\phi$$

$$\phi = \phi_\ell + \phi_m$$

ϕ_ℓ = leakage flux

ϕ_m = magnetizing flux

flux linkages

(the magnetizing flux is common to both stationary and rotating members)

$$\phi_\ell = \frac{Ni}{\Re_\ell}$$

$$\phi_m = \frac{Ni}{\Re_m}$$

If the magnetic system is considered to be linear (saturation neglected), then, as in the case of stationary coupled circuits, we can express the fluxes in terms of reluctances.

$$\lambda = \left(\frac{N^2}{\Re_\ell} + \frac{N^2}{\Re_m} \right) i = \left(L_\ell + L_m \right) i$$

flux linkages

$L_\ell =$ leakage inductance

$L_m =$ magnetizing inductance

$$\Re_m = \Re_i + 2\Re_g$$

reluctance of the magnetizing path

$\Re_i$ — total reluctance of the magnetic material of the stationary and movable members

$\Re_g$ — reluctance of one of the air gaps

$$\Re_i = \frac{\ell_i}{\mu_{ri}\mu_0 A_i}$$

$$\Re_g = \frac{x}{\mu_0 A_g}$$

Assume that the cross-sectional areas of the stationary and movable members are equal and of the same material

$$A_g = A_i$$

This may be somewhat of an oversimplification, but it is sufficient for our purposes.

$$\Re_m = \Re_i + 2\Re_g$$

$$= \frac{1}{\mu_0 A_i}\left(\frac{\ell_i}{\mu_{ri}} + 2x\right)$$

$$L_m = \frac{N^2}{\frac{1}{\mu_0 A_i}\left(\frac{\ell_i}{\mu_{ri}} + 2x\right)}$$

Assume that the leakage inductance is constant.

The magnetizing inductance is clearly a function of displacement.

$x = x(t)$ and $L_m = L_m(x)$

When dealing with linear magnetic circuits wherein mechanical motion is not present, as in the case of a transformer, the change of flux linkages with respect to time was simply L(di/dt). This is not the case here.

$$\lambda(i,x) = L(x)i = \left[L_\ell + L_m(x)\right]i$$

$$\frac{d\lambda(i,x)}{dt} = \frac{\partial\lambda}{\partial i}\frac{di}{dt} + \frac{\partial\lambda}{\partial x}\frac{dx}{dt}$$

The inductance is a function of x(t).

$$v = ri + \left[L_\ell + L_m(x)\right]\frac{di}{dt} + i\frac{dL_m(x)}{dx}\frac{dx}{dt}$$

The voltage equation is a nonlinear differential equation.

$$L_m(x) = \frac{N^2}{\frac{1}{\mu_0 A_i}\left(\frac{\ell_i}{\mu_{ri}} + 2x\right)}$$

Let's look at the magnetizing inductance again.

$$L_m(x) = \frac{k}{k_0 + x} \qquad \begin{cases} k = \dfrac{N^2\mu_0 A_i}{2} \\ k_0 = \dfrac{\ell_i}{2\mu_{ri}} \end{cases}$$

$$L_m(0) = \frac{k}{k_0} = \frac{N^2\mu_0\mu_{ri}A_i}{\ell_i}$$

$$L_m(x) \cong \frac{k}{x} \quad \text{for } x > 0$$

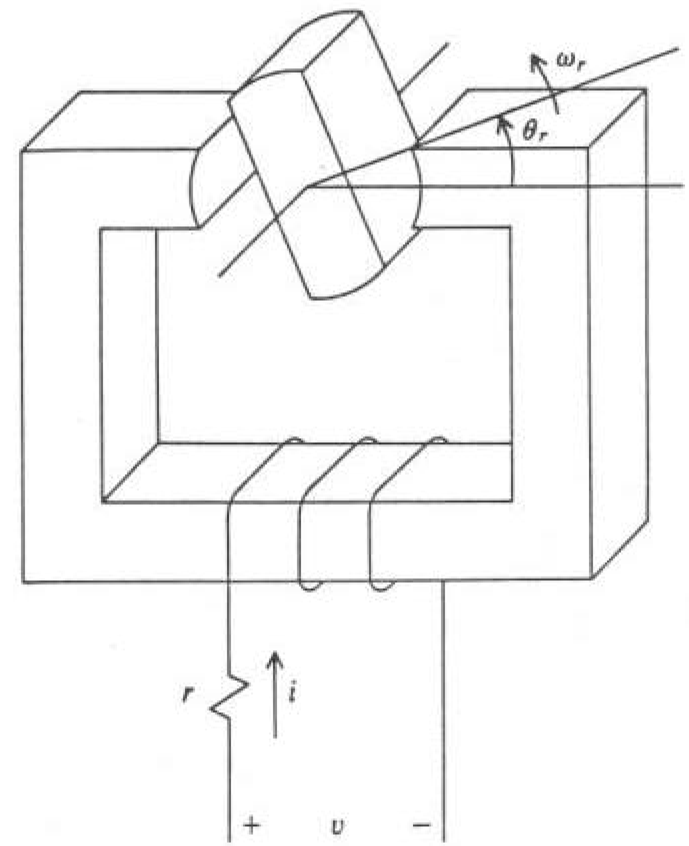
Elementary Reluctance Machine

- The machine consists of:
  - stationary core with a winding of N turns
  - moveable member which rotates

$\theta_r =$ angular displacement

$\omega_r =$ angular velocity

$$\theta_r = \int_0^t \omega_r(\xi)\,d\xi + \theta_r(0)$$



$$v = ri + \frac{d\lambda}{dt}$$

voltage equation

$$\phi = \phi_\ell + \phi_m$$

$\phi_\ell =$ leakage flux

$\phi_m =$ magnetizing flux

$$\lambda = (L_\ell + L_m)i$$

It is convenient to express the flux linkages as the product of the sum of the leakage inductance and the magnetizing inductance and the current in the winding.

$L_\ell =$ constant (independent of θ_r)

$L_m =$ periodic function of θ_r

$$L_m = L_m(\theta_r)$$

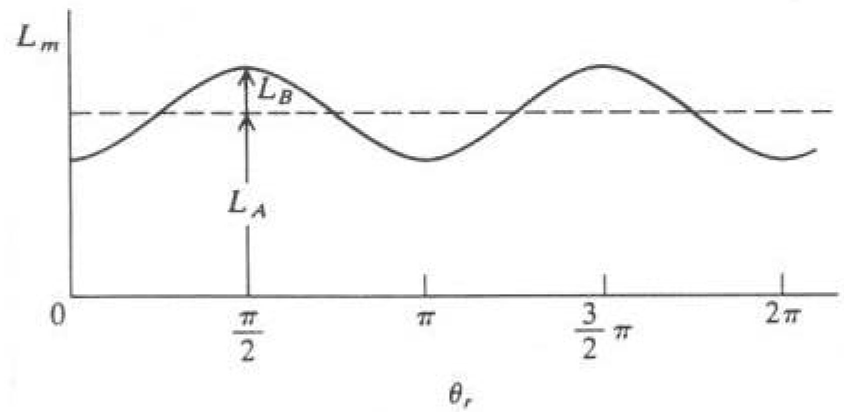
$$L_m(0) = \frac{N^2}{\Re_m(0)} \quad \Rightarrow \quad \begin{cases} \Re_m \text{ is maximum} \\ L_m \text{ is minimum} \end{cases}$$

$$L_m\left(\frac{\pi}{2}\right) = \frac{N^2}{\Re_m\left(\frac{\pi}{2}\right)} \quad \Rightarrow \quad \begin{cases} \Re_m \text{ is minimum} \\ L_m \text{ is maximum} \end{cases}$$

The magnetizing inductance varies between maximum and minimum positive values twice per revolution of the rotating member.

Assume that this variation may be adequately approximated by a sinusoidal function.

$$L_m(\theta_r) = L_A - L_B\cos(2\theta_r)$$



$$L_m(0) = L_A - L_B$$

$$L_m\left(\frac{\pi}{2}\right) = L_A + L_B$$

$$L_A > L_B$$

$$L_A = \text{ average value}$$

$$L(\theta_r) = L_\ell + L_m(\theta_r)$$
$$= L_\ell + L_A - L_B\cos(2\theta_r)$$

$$v = ri + \left[L_\ell + L_m(\theta_r)\right]\frac{di}{dt} + i\frac{dL_m(\theta_r)}{d\theta_r}\frac{d\theta_r}{dt}$$

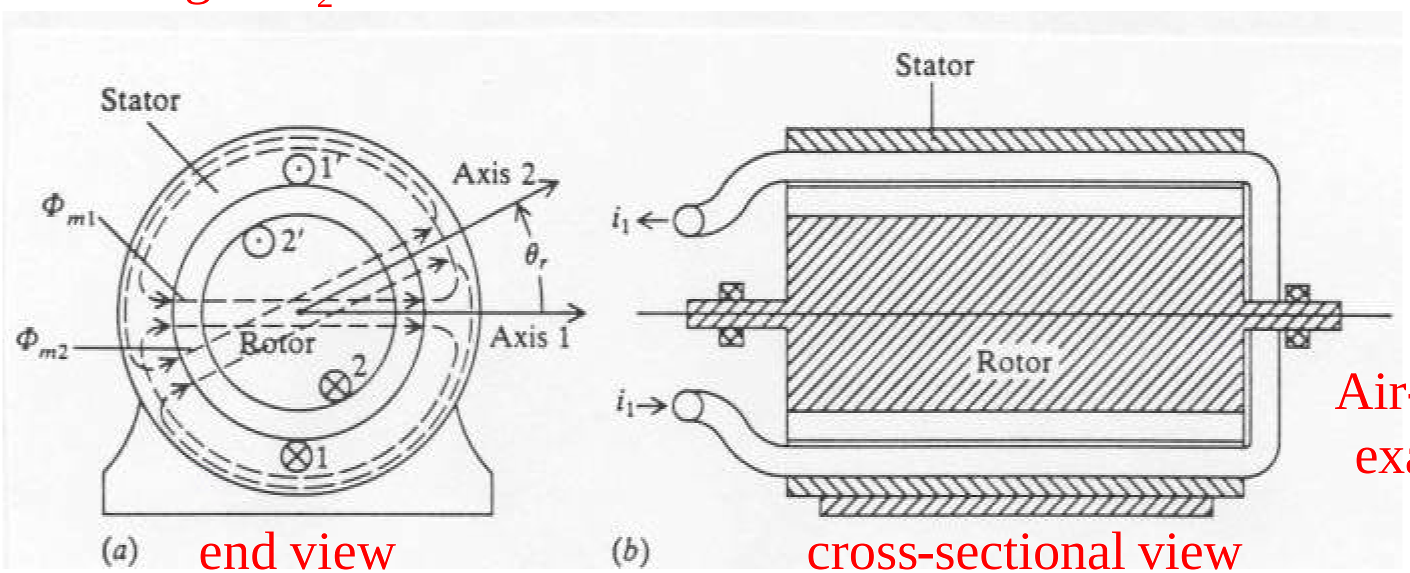
voltage equation

Windings in Relative Motion

- The rotational device shown will be used to illustrate windings in relative motion.

Winding 1: N_1 turns on stator
Winding 2: N_2 turns on rotor

Assume that the turns are concentrated in one position.



end view

cross-sectional view

Air-gap size is exaggerated.

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt}$$

voltage equations

$$\lambda_1 = L_{11} i_i + L_{12} i_2$$

$$\lambda_2 = L_{21} i_i + L_{22} i_2$$

The magnetic system is assumed linear.

$$L_{11} = L_{\ell 1} + L_{m1} = \frac{N_1^2}{\Re_{\ell 1}} + \frac{N_1^2}{\Re_m}$$

$$L_{22} = L_{\ell 2} + L_{m2} = \frac{N_2^2}{\Re_{\ell 2}} + \frac{N_2^2}{\Re_m}$$

The self-inductances L_{11} and L_{22} are constants and may be expressed in terms of leakage and magnetizing inductances.

$\Re_m$ is the reluctance of the complete magnetic path of φ_{m1} and φ_{m2}, which is through the rotor and stator iron and twice across the air gap.

Let's now consider L_{12}.

$$\theta_r = \text{ angular displacement}$$

$$\omega_r = \text{ angular velocity}$$

$$\theta_r = \int_0^t \omega_r(\xi)\,d\xi + \theta_r(0)$$

When θ_r is zero, then the coupling between windings 1 and 2 is maximum. The magnetic system of winding 1 aids that of winding 2 with positive currents assumed. Hence the mutual inductance is positive.

$$L_{12}(0) = \frac{N_1 N_2}{\Re_m}$$

When θ_r is $\pi/2$, the windings are orthogonal. The mutual coupling is zero.

$$L_{12}\left(\frac{\pi}{2}\right) = 0$$

Assume that the mutual inductance may be adequately predicted by:

$$L_{12}\left(\theta_r\right) = L_{sr}\cos\left(\theta_r\right)$$

$$L_{sr} = \frac{N_1 N_2}{\Re_m}$$

L_{sr} is the amplitude of the sinusoidal mutual inductance between the stator and rotor windings.

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt}$$

In writing the voltage equations, the total derivative of the flux linkages is required.