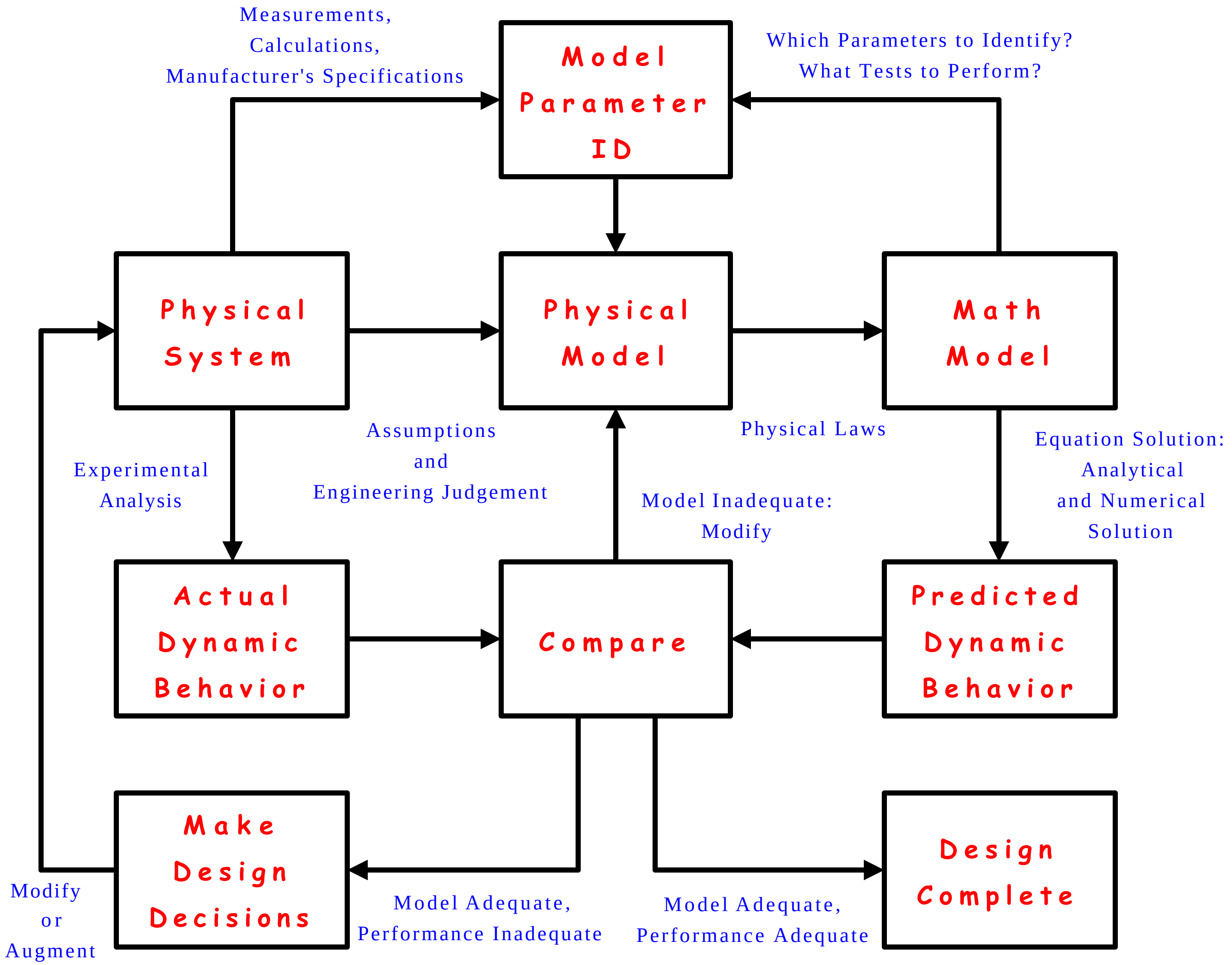
Modeling, Analysis, & Control of Dynamic Systems: Introduction

- Dynamic System Investigation
  - Stages of a Dynamic System Investigation
  - Balance: The Key to Success!
- Control Systems
  - Open-Loop and Closed-Loop
  - Analog and Digital



Measurements,
Calculations,
Manufacturer's Specifications

Model Parameter ID

Which Parameters to Identify?
What Tests to Perform?

Physical System

Physical Model

Math Model

Experimental Analysis

Assumptions and Engineering Judgement

Physical Laws

Model Inadequate: Modify

Equation Solution: Analytical and Numerical Solution

Actual Dynamic Behavior

Compare

Predicted Dynamic Behavior

Modify or Augment

Make Design Decisions

Model Adequate, Performance Inadequate

Model Adequate, Performance Adequate

Design Complete

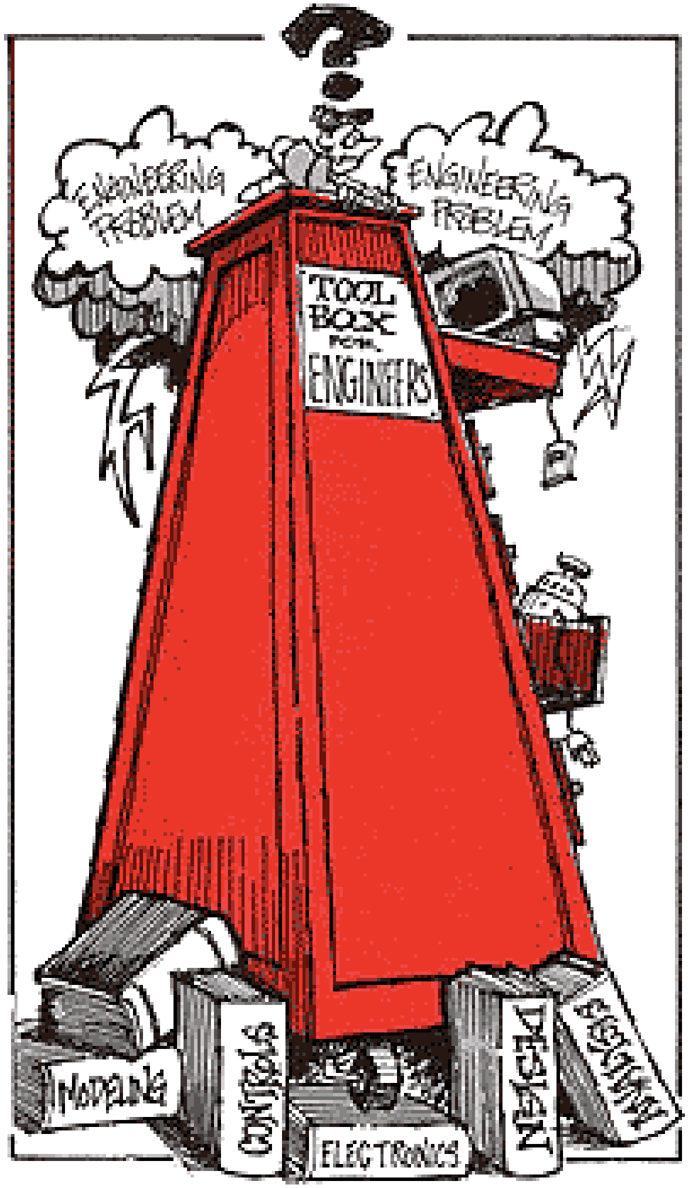
Dynamic System Investigation

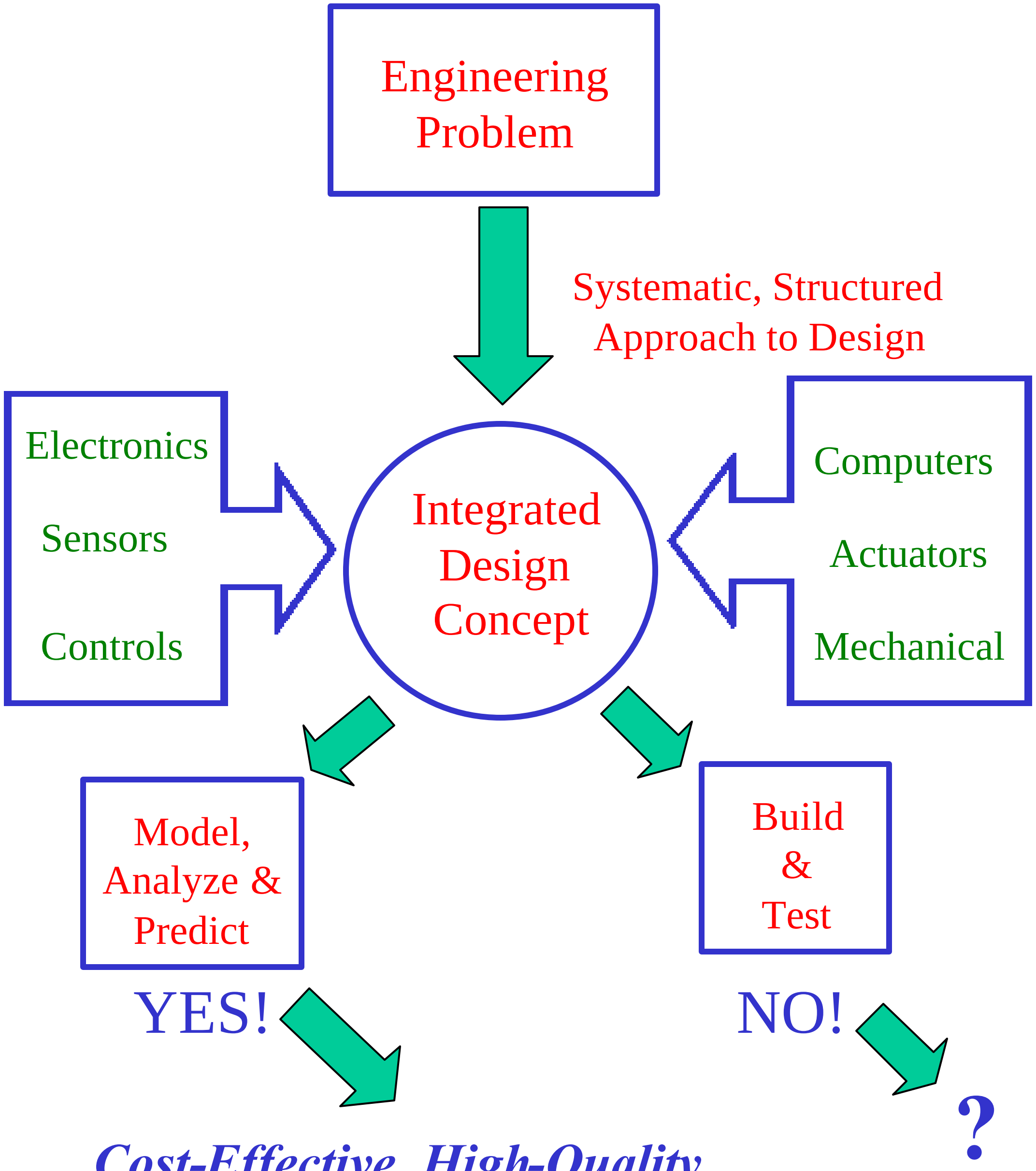
Dynamic System Investigation Overview

- The steps in this process should be applied not only when an actual physical system exists and one desires to understand and predict its behavior, but also when the physical system is a concept in the design process that needs to be analyzed and evaluated.
- After recognizing a need for a new product or service, one uses past experience (personal and vicarious), awareness of existing hardware, understanding of physical laws and creativity to generate design concepts.

- The importance of modeling and analysis in the design process has never been more important.
- These design concepts can no longer be evaluated by the build-and-test approach because it is too costly and time consuming.
- Validating the predicted dynamic behavior in this case, when no actual physical system exists, then becomes even more dependent on one's past hardware and experimental experience.

Do you ever feel like this?





Engineering Problem

Systematic, Structured Approach to Design

Electronics

Sensors

Controls

Integrated Design Concept

Computers

Actuators

Mechanical

Model, Analyze & Predict

Build & Test

YES!

NO!

?

Cost-Effective, High-Quality, Timely, Robust Design

- A dynamic physical system is any collection of interacting elements for which there are cause-and-effect relationships among the time-dependent variables. The present output of the system depends on past inputs.
- The analysis of the dynamic behavior of physical systems has become the cornerstone of modern technology and, more than any other field, links the engineering disciplines.
- The purpose of a dynamic system investigation is to understand and predict the dynamic behavior of a given system and to modify and/or control the system, if necessary.

- Essential Features of the Study of Dynamic Systems
  - Deals with entire operating machines and processes rather than just isolated components.
  - Treats dynamic behavior of mechanical, electrical, fluid, thermal, and mixed systems.
  - Emphasizes the behavioral similarity between systems that differ physically and develops general analysis and design tools useful for all kinds of physical systems.
  - Sacrifices detail in component descriptions so as to enable understanding of the behavior of complex systems made from many components.

- Uses methods which accommodate component descriptions in terms of experimental measurements, when accurate theory is lacking or is not cost-effective, and develops universal lab test methods for characterizing component behavior.
- Serves as a unifying foundation for many practical application areas, e.g., vibrations, measurement systems, control systems, acoustics, vehicle dynamics, etc.
- Offers a wide variety of computer software to implement its methods of analysis and design.

- Stages of a Dynamic System Investigation
  - Physical System
    - Define the physical system to be studied, along with the system boundaries, input variables, and output variables.
  - Physical System to Physical Model
    - A physical model is an imaginary physical system which resembles an actual system in its salient features, but which is simpler, more ideal, and is thereby more amenable to analytical studies.
    - Not oversimplified, not overly complicated - *a slice of reality.*

- The astuteness with which approximations are made at the onset of an investigation is the very crux of engineering analysis.
- Develop a set of performance specifications for the model based on the specific purpose of the model. What features must be included? How accurately do they need to be represented?
- Truth Model vs. Design Model
- Engineering Judgment is the Key!

- Physical Model to Mathematical Model
  - We derive a mathematical model to represent the physical model, i.e., write down the differential equations of motion of the physical model.
  - The goal is a generalized treatment of dynamic systems, including mechanical, electrical, electromechanical, fluid, and thermal systems.
    - Define System: Boundary, Inputs, Outputs
    - Define Variables: Through and Across Variables
    - Write System Relations: Dynamic Equilibrium Relations and Compatibility Relations
    - Write Physical Relations for Each Element
    - Combine: Generate State Equations

- Study Dynamic Behavior and Compare to Measured Dynamics
  - Study the dynamic behavior of the mathematical model by solving the differential equations of motion either through mathematical analysis or computer simulation.
  - Dynamic behavior is a consequence of the system structure - don’t blame the input!
  - Seek a relationship between physical model structure and behavior.
  - Develop insight into system behavior.

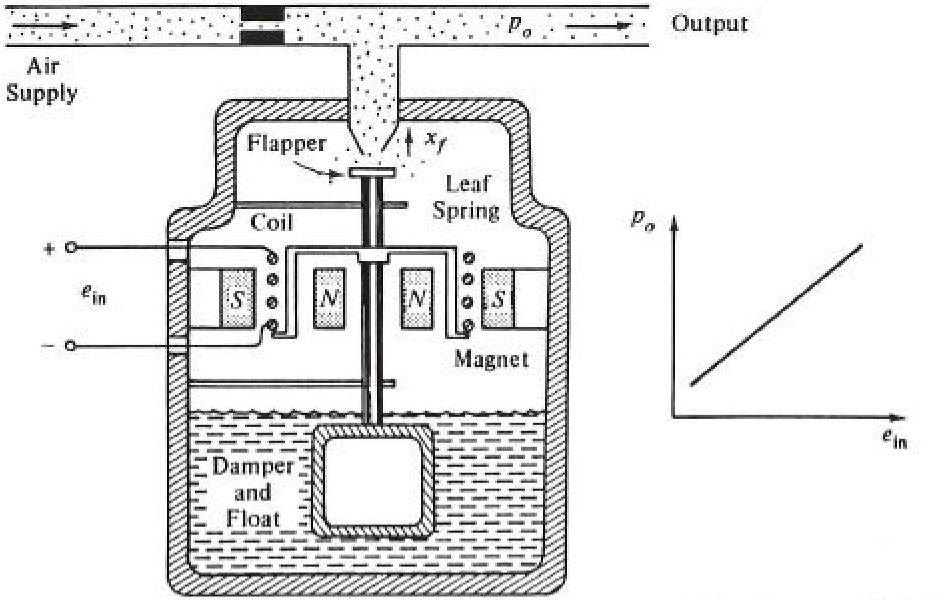
- Compare the predicted dynamic behavior to the measured dynamic behavior from tests on the actual physical system; make physical model corrections, if necessary.

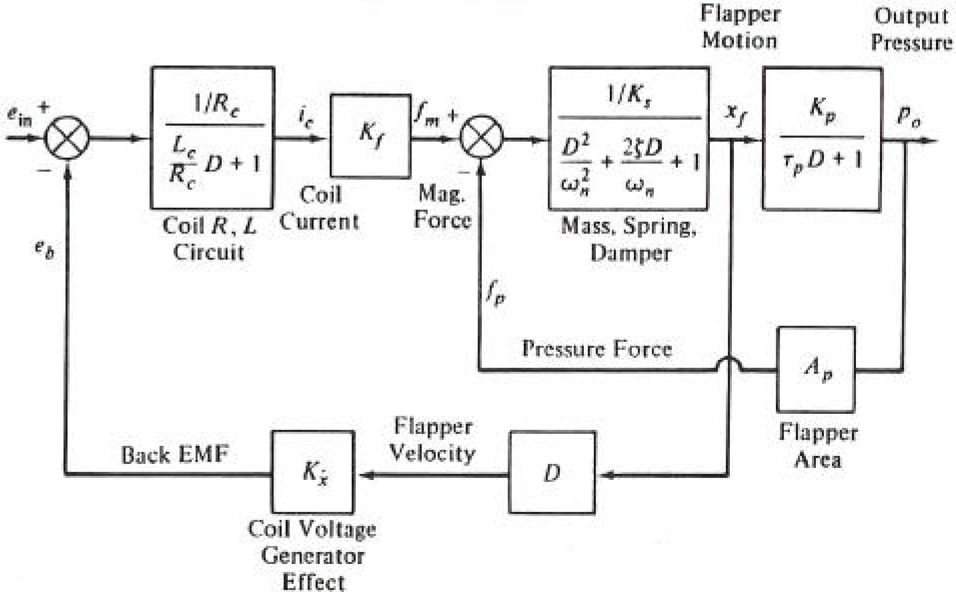
- Make Design Decisions
  - Make design decisions so that the system will behave as desired:
    - modify the system (e.g., change the physical parameters of the system)
    - control the system (e.g., augment the system, typically by adding a dynamic system called a compensator or controller)

- Comments on Truth Model vs. Design Model
  - In modeling dynamic systems, we use engineering judgment and simplifying assumptions to develop a physical model. The complexity of the physical model depends on the particular need, and the intelligent use of simple physical models requires that we have some understanding of what we are missing when we choose the simpler model over the more complex model.
  - The truth model is the model that includes all the known relevant characteristics of the real system. This model is often too complicated for use in engineering design, but is most useful in verifying design changes or control designs prior to hardware implementation.

- The design model captures the important features of the process for which a control system is to be designed or design iterations are to be performed, but omits the details which you believe are not significant.
- In practice, you may need a hierarchy of models of varying complexity: a very detailed truth model for final performance evaluation before hardware implementation, several less complex truth models for use in evaluating particular effects, and one or more design models.

Example of a Dynamic System: Electro-Pneumatic Transducer





This system can be collapsed into a simplified approximate overall model when numerical values are properly chosen:

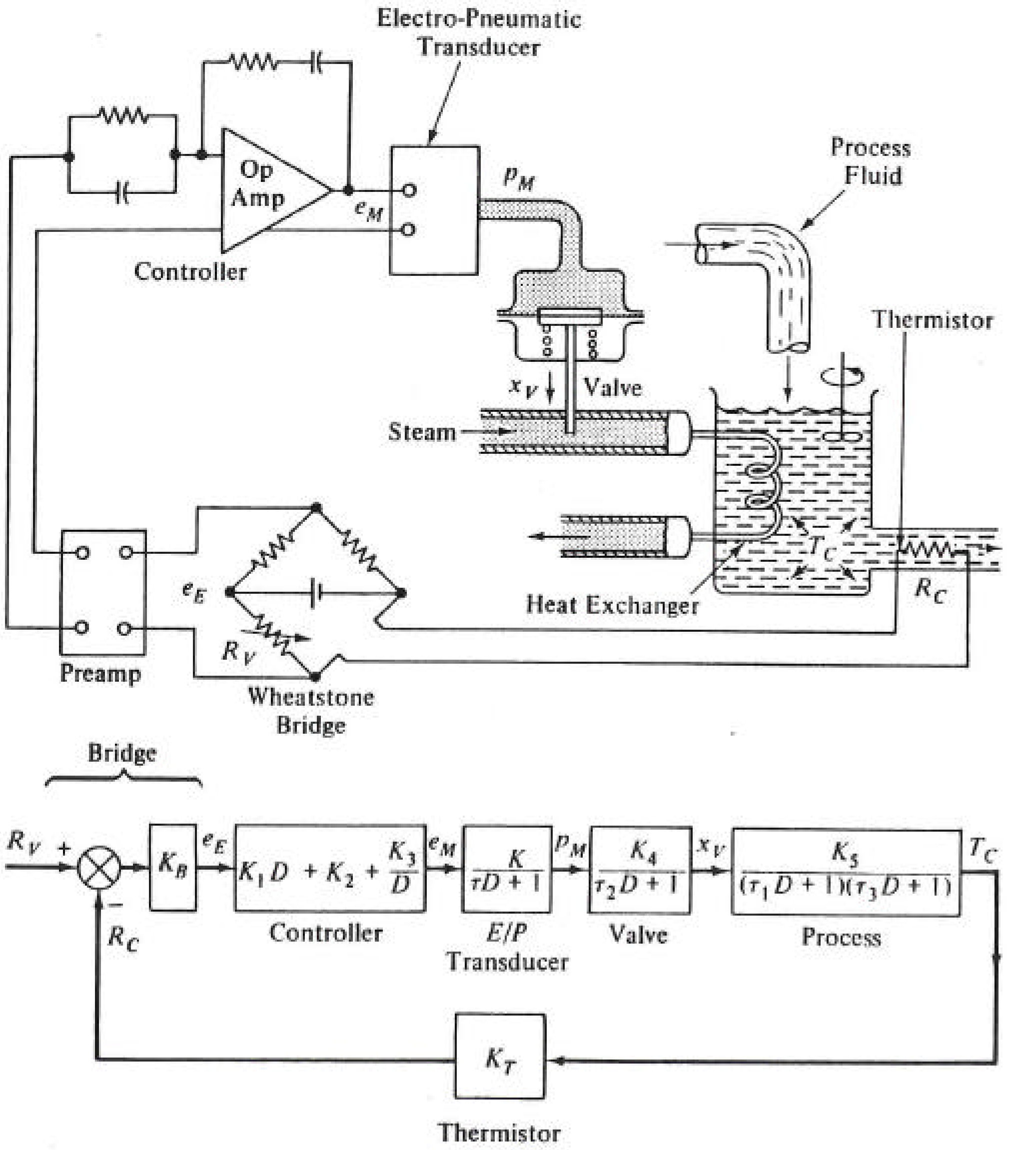
$$\frac{p_o}{e_{in}} = \frac{K}{\tau D + 1}$$

$$\text{Differential Operator } D \equiv \frac{d}{dt}$$

- It is interesting to note here that while the block diagram shows one input for the system, i.e., command voltage e_{in}, there are possible undesired inputs that must also be considered.
- For example, the ambient temperature will affect the electric coil resistance, the permanent magnet strength, the leaf-spring stiffness, the damper-oil viscosity, the air density, and the dimensions of the mechanical parts. All these changes will affect the system output pressure p_o in some way, and the cumulative effects may not be negligible.

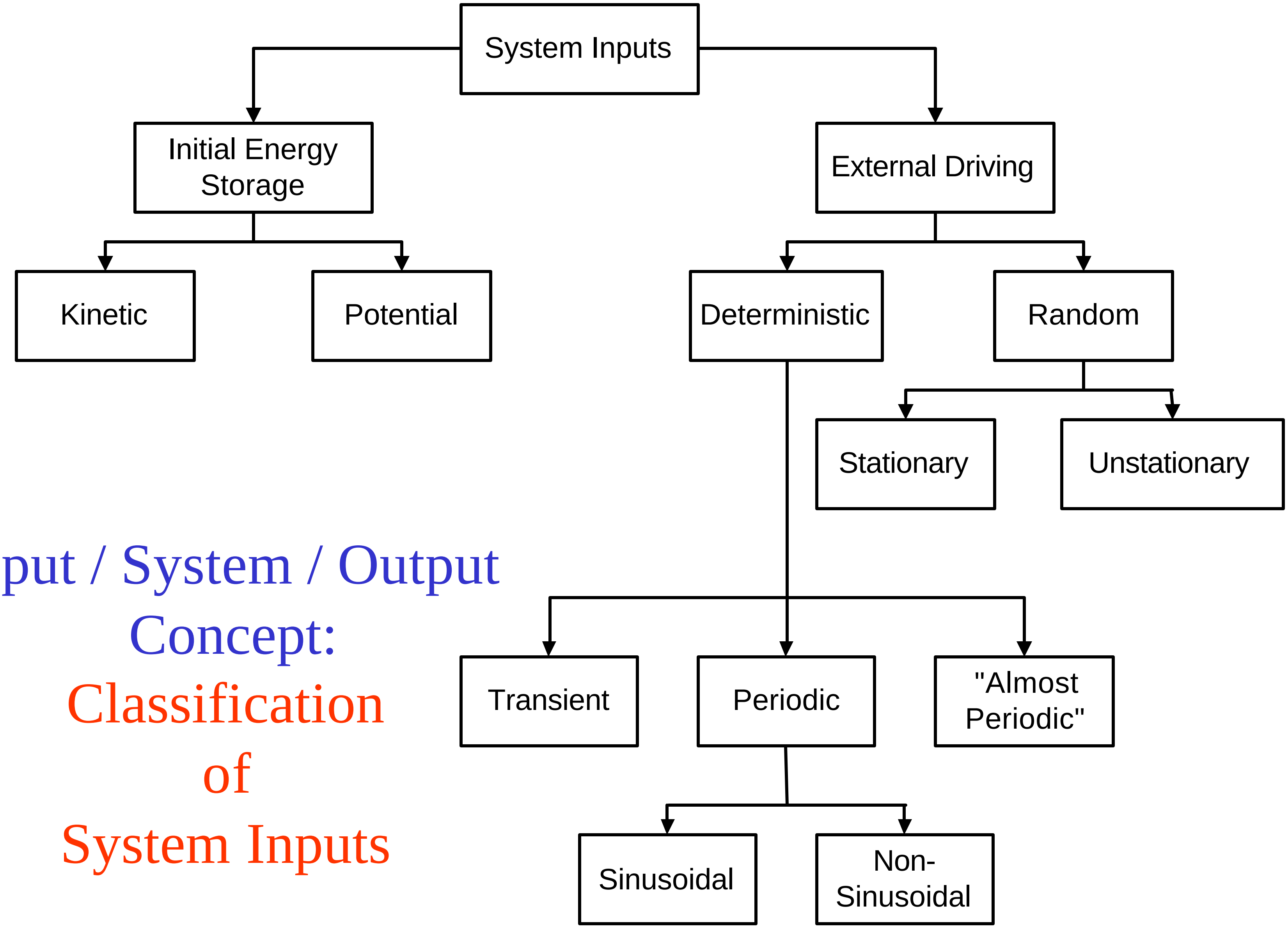
Example of a Dynamic System: Temperature Control System

Here, the electro-pneumatic transducer is a component in the overall dynamic system.



- Note that in the block diagram of this system, the detailed operation of the electropneumatic transducer is not made apparent; only its overall input/output relation is included.
- The designer of the temperature feedback-control dynamic system would consider the electropneumatic transducer an off-the-shelf component with certain desirable operating characteristics.
- The methods of system dynamics are used by both the electropneumatic transducer designer and the designer of the larger temperature feedback-control system.

Input / System / Output Concept: Classification of System Inputs



- System Inputs
  - Initial Energy Storage
    - Kinetic
    - Potential
  - External Driving
    - Deterministic
      - Transient
      - Periodic
        - Sinusoidal
        - Non-Sinusoidal
      - "Almost Periodic"
    - Random
      - Stationary
      - Unstationary

Classification of System Inputs

- Input – some agency which can cause a system to respond.
- Initial energy storage refers to a situation in which a system, at time = 0, is put into a state different from some reference equilibrium state and then released, free of external driving agencies, to respond in its characteristic way. Initial energy storage can take the form of either kinetic energy or potential energy.
- External driving agencies are physical quantities which vary with time and pass from the external environment, through the system interface or boundary, into the system, and cause it to respond.

- We often choose to study the system response to an assumed ideal source, which is unaffected by the system to which it is coupled, with the view that practical situations will closely correspond to this idealized model.
- External inputs can be broadly classified as deterministic or random, recognizing that there is always some element of randomness and unpredictability in all real-world inputs.
- Deterministic input models are those whose complete time history is explicitly given, as by mathematical formula or a table of numerical values. This can be further divided into:
  - transient input model: one having any desired shape, but existing only for a certain time interval, being constant before the beginning of the interval and after its end.

  - periodic input model: one that repeats a certain wave form over and over, ideally forever, and is further classified as either sinusoidal or non-sinusoidal.
  - almost periodic input model: continuing functions which are completely predictable but do not exhibit a strict periodicity, e.g., amplitude-modulated input.
- Random input models are the most realistic input models and have time histories which cannot be predicted before the input actually occurs, although statistical properties of the input can be specified.
- When working with random inputs, there is never any hope of predicting a specific time history before it occurs, but statistical predictions can be made that have practical usefulness.

- If the statistical properties are time-invariant, then the input is called a stationary random input. Unstationary random inputs have time-varying statistical properties. These are often modeled as stationary over restricted periods of time.

Dynamic System Investigation Example: Spring-Mass System

Mechanical oscillations are very important physical phenomena in mathematics, engineering, and the physical sciences.

A spring-mass system is used in the modeling and analysis of many engineering systems. It is often embedded in real systems, or a real system may sometimes be modeled as a spring-mass system due to its similar dynamic behavior.

- Physical System
  - A mass hanging at the tip of a tension spring that is attached to a stationary rigid support constitutes the dynamic system for investigation.
  - The motion of the spring-mass system is constrained by a linear bearing on the side of the support so that the mass oscillates only in one direction, the vertical direction.
  - A non-contact optical (infrared) sensor is used to measure the position of the mass.
  - The free oscillation of the spring-mass system is considered here; there is no externally-applied driving force acting on the system.

- Physical-Model Simplifying Assumptions
  - The support to which the spring is attached is rigid. This assumption in effect says that the environment is independent of system motions.
  - The spring is pure, i.e., it only has the characteristic (elasticity) for which it is named. A pure spring has negligible mass and damping. This, of course, is an idealization as all springs have mass and dissipate energy upon cycling. If the spring mass is less than 10% of the mass attached to it, this is a reasonable assumption (except in high-speed applications). The energy dissipation in the spring is very small compared to other dissipation mechanisms in the system, so neglecting it is also reasonable.

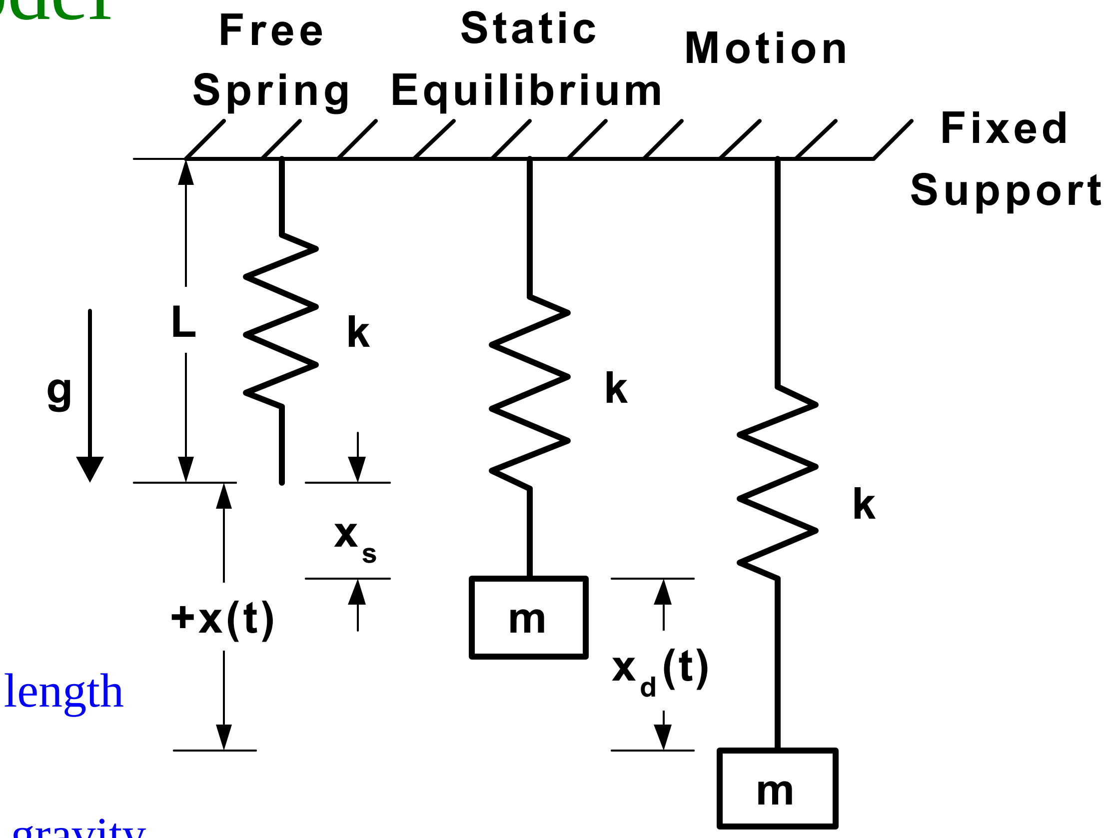
- The spring is ideal, i.e., there is a linear relationship between spring force and spring displacement in the range of mass motion considered. This can be experimentally verified. The actual spring is a tension spring with some pretension (a force that pulls the coils of the spring together) and the motion of the mass must be restricted to the range during which the spring is in tension, i.e., large amplitudes of motion of the mass are excluded from consideration.
- The attached mass can be treated as a rigid body.
- The mass moves with one degree of freedom in pure translation in the vertical plane. There is no out-of-plane motion and there is no rotational motion of the mass. The mass then can be treated as a point mass.

- The friction in the system is parasitic, i.e., no energy dissipation mechanism has been designed into the system. Any air damping due to the motion of the mass in the air is negligible. The friction in the linear bearing is the main source of energy dissipation and, based on engineering experience, is a combination of viscous damping (proportional to the velocity of the mass and directed opposite to the mass motion) and dry-friction, or Coulomb damping, as it is called, (essentially constant in magnitude, independent of mass velocity, and directed opposite to the mass motion). Coulomb friction leads to a nonlinear mathematical model, while viscous friction leads to a linear mathematical model.

- The desire to have a linear mathematical model does not justify the assumption of viscous damping and the omission of Coulomb damping. If this assumption is not based on sound engineering judgment, then the resulting mathematical model will not predict the actual behavior of the dynamic system. However, initially this parasitic damping will be neglected so as to keep the analysis simpler. If it is determined that this assumption is invalid, then a viscous damping model will be assumed with an experimentally-determined viscous damping coefficient.

- The system is vertical with the acceleration due to gravity pointing downward and constant in value.
- All parameters (mass m, spring constant k, viscous damping coefficient B) are constant, i.e., do not change with time or temperature, for example.

- # Physical Model



$m =$ mass

$m_s =$ spring mass

$L =$ unstretched spring length

$k =$ spring constant

$g =$ acceleration due to gravity

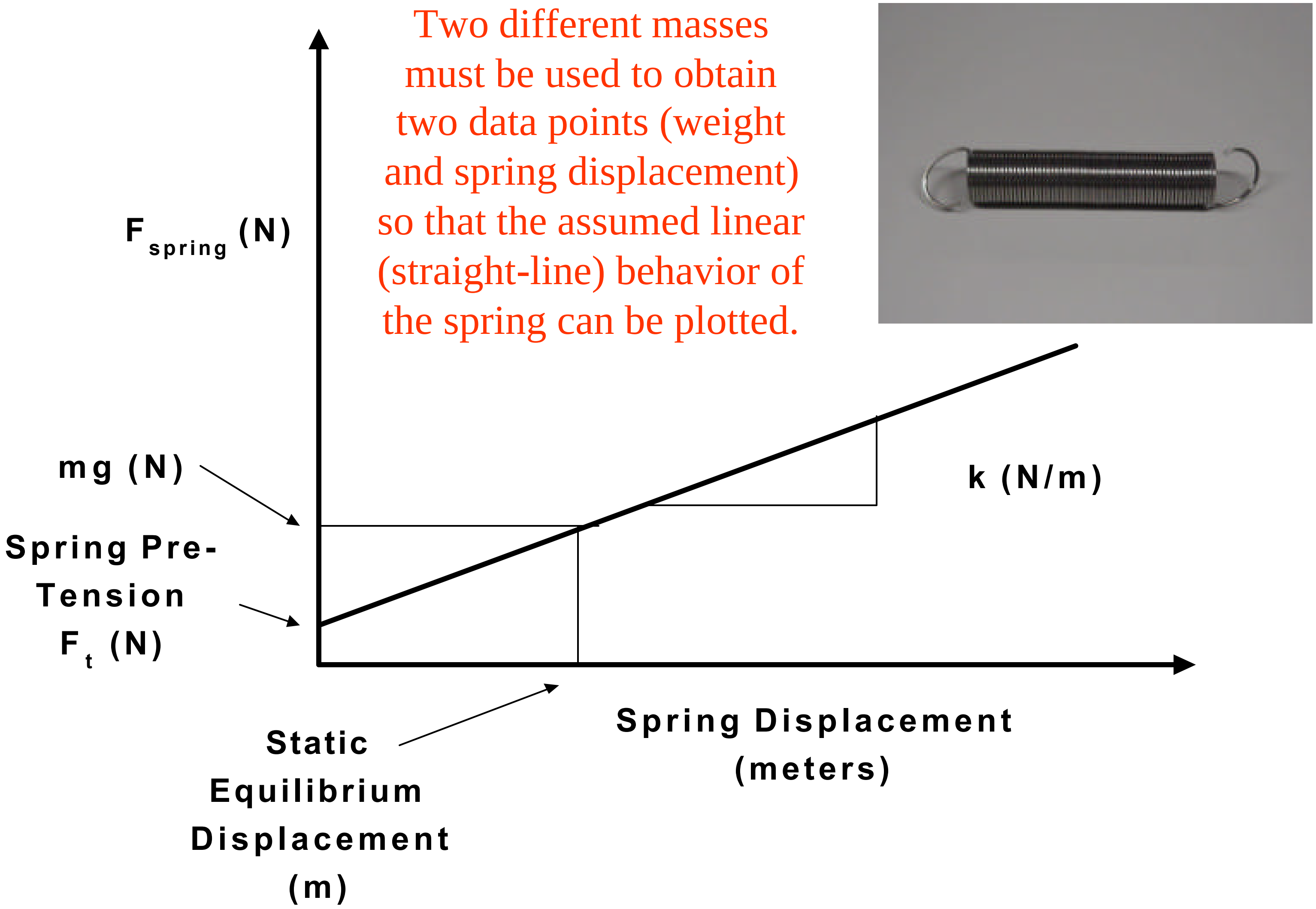
$x_s =$ static spring stretch

$x_d =$ dynamic spring stretch

$x =$ total spring stretch $= x_s + x_d$

- Parameter Identification
  - Parameters in the physical model that need to be identified are:
    - Mass m (kg) of the attached block - obtained by weighing the block
    - Mass m_s (kg) of the spring - obtained by weighing the spring
    - Unstretched length of the spring L (m) - obtained by direct measurement
    - Spring constant k (N/m) of the spring
    - Pretension force F_t (N) of the spring

Tension Spring

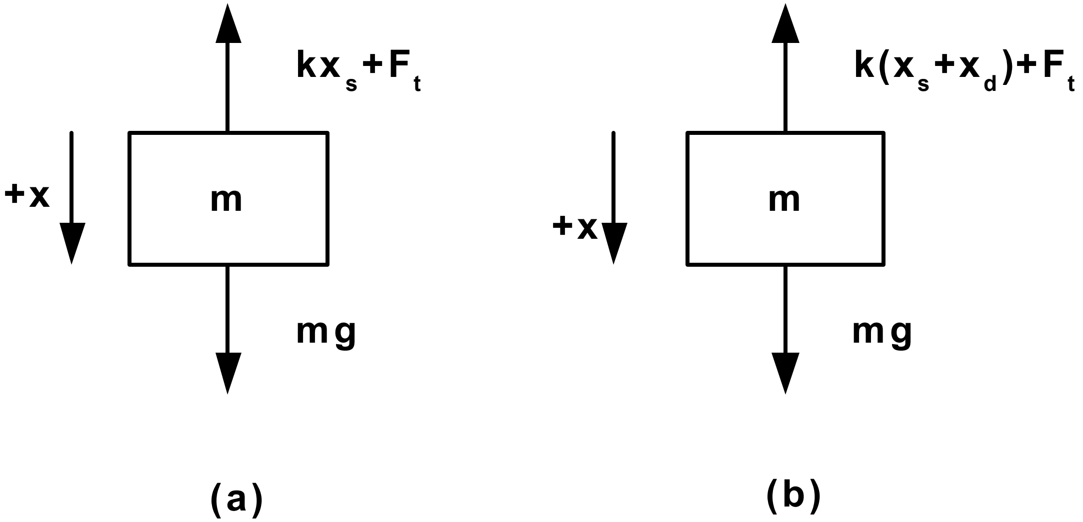


Two different masses must be used to obtain two data points (weight and spring displacement) so that the assumed linear (straight-line) behavior of the spring can be plotted.

Experimental Determination of Spring Constant k and Pretension Force F_t

- Parameter Identification Test Results
  - Unstretched spring length L = 0.127 m
  - Spring constant k = 491.1 N/m
  - Spring pretension force F_t = 9.521 N
  - Mass m = 5.231 kg
  - Spring mass m_s = 0.08 kg
  - Static equilibrium displacement = 0.0851 m
  - Equation of force F_{spring} (N) vs. displacement x (m) curve is: $F_{spring} = 491.1x + 9.5206$ and this linear curve is valid in the range 0 to 0.120 meters spring displacement.

- Mathematical Model
  - Newton's 2nd Law of Motion $$\sum F_x = m\frac{d^2x}{dt^2} = m\ddot{x}$$
  - Free-Body Diagrams



Static Equilibrium Motion

- Equations of Motion

$$mg - \left(kx_s + F_t\right) = 0$$

$$x_s = \frac{mg - F_t}{k}$$ Static Equilibrium Position

$$mg - \left[k\left(x_s + x_d\right) + F_t\right] = m\ddot{x}$$

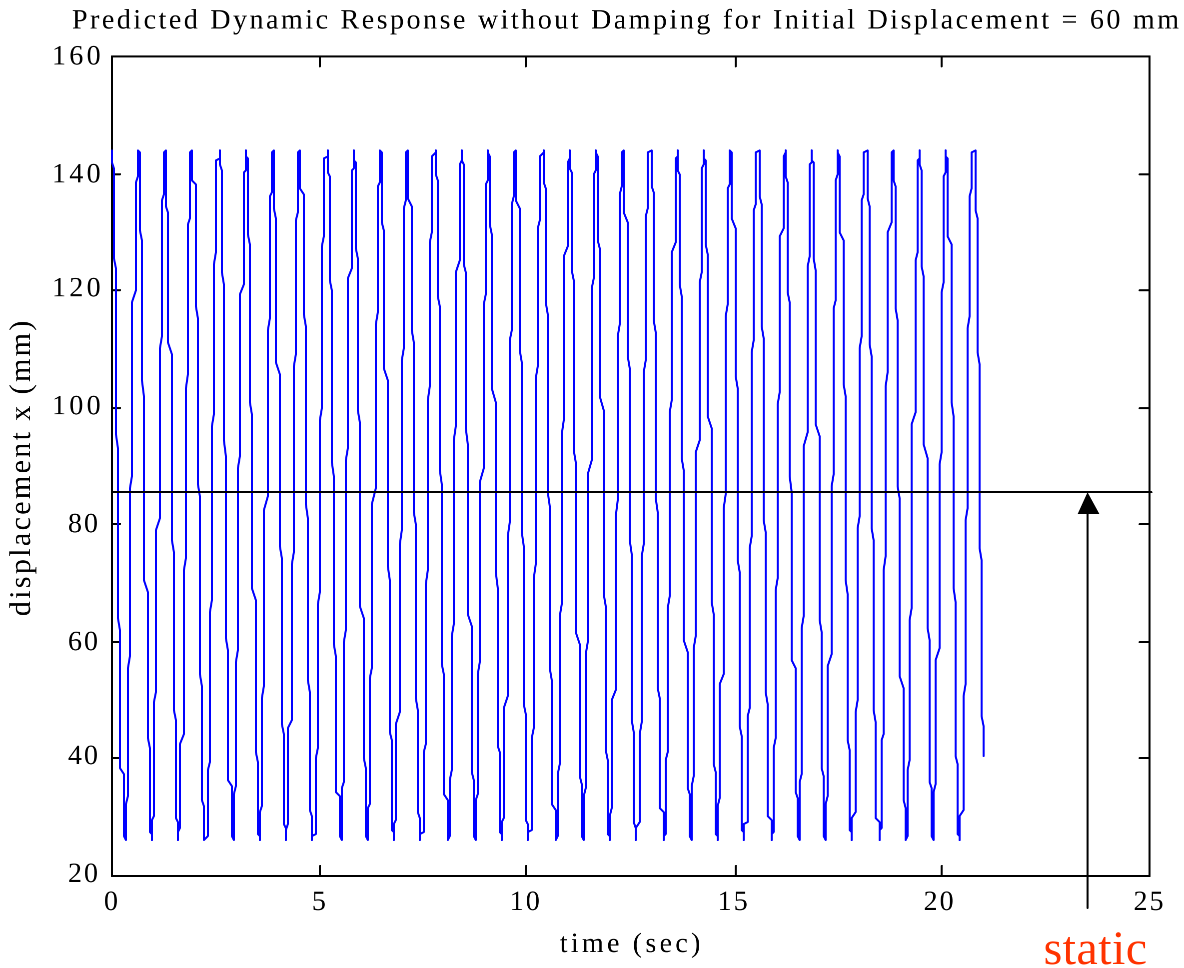
$$m\ddot{x} + k\left(x_s + x_d\right) = mg - F_t$$

$$m\ddot{x} + kx = mg - F_t$$ Equation of Motion in terms of $x = x_s + x_d$

$$m\ddot{x}_d + kx_d = 0$$ Equation of Motion in terms of x_d

Predicted Dynamic Response

Initial Mass Displacement is 60 mm from the Static Equilibrium Position



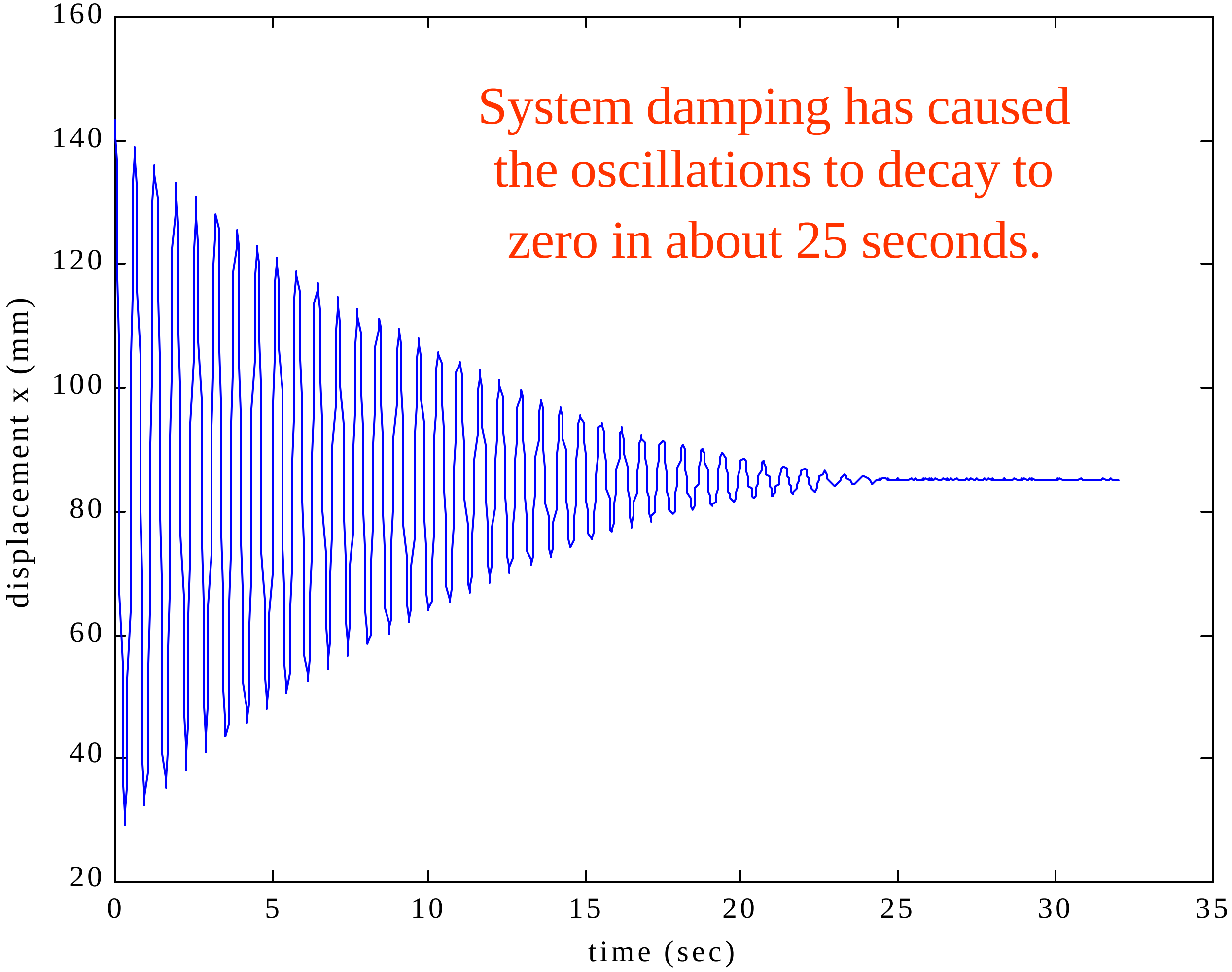
static equilibrium position

$$\omega_n = \sqrt{\frac{k}{m}} = 9.69 \text{ rad/sec}$$

$$f_n = \frac{\omega_n}{2\pi} = 1.54 \text{ cycles/sec or Hz}$$

Actual Dynamic Response

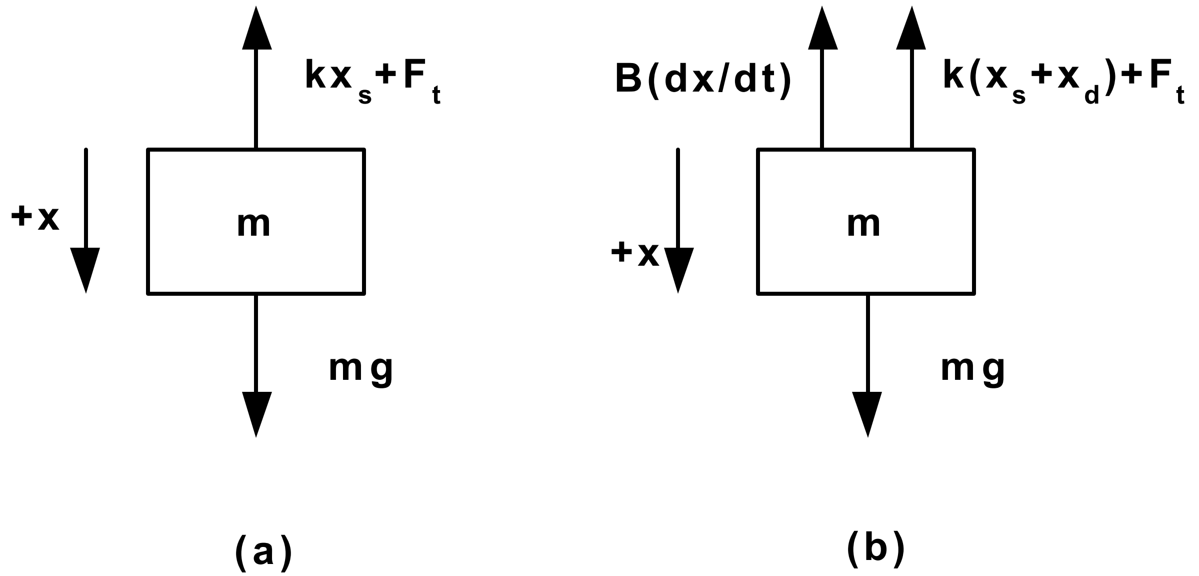
Initial Mass Displacement is 60 mm from the Static Equilibrium Position



System damping has caused the oscillations to decay to zero in about 25 seconds.

- Comparison: Actual vs. Predicted
  - The frequency of the oscillations for both responses is approximately the same, i.e., 1.5 Hz. This is as expected from engineering experience since parasitic damping has little effect on the natural frequency of oscillations of the system.
  - The oscillations of the mass in both the predicted response and the actual response is about the static equilibrium position, i.e., $x_s = 85.1$ mm.
  - The amplitude of the oscillations for both responses does not agree well at all. This also is as expected from engineering experience because even a small amount of damping will reduce the amplitude of oscillations significantly over time. The greater the damping, the faster the oscillations decay to zero.

- Physical Model Modification
  - Viscous damping is now included in the physical model.
  - Revised Free-Body Diagrams



Static Equilibrium

Motion

– Revised Equations of Motion

$$mg - B\dot{x} - \left[k\left(x_s + x_d\right) + F_t\right] = m\ddot{x}$$

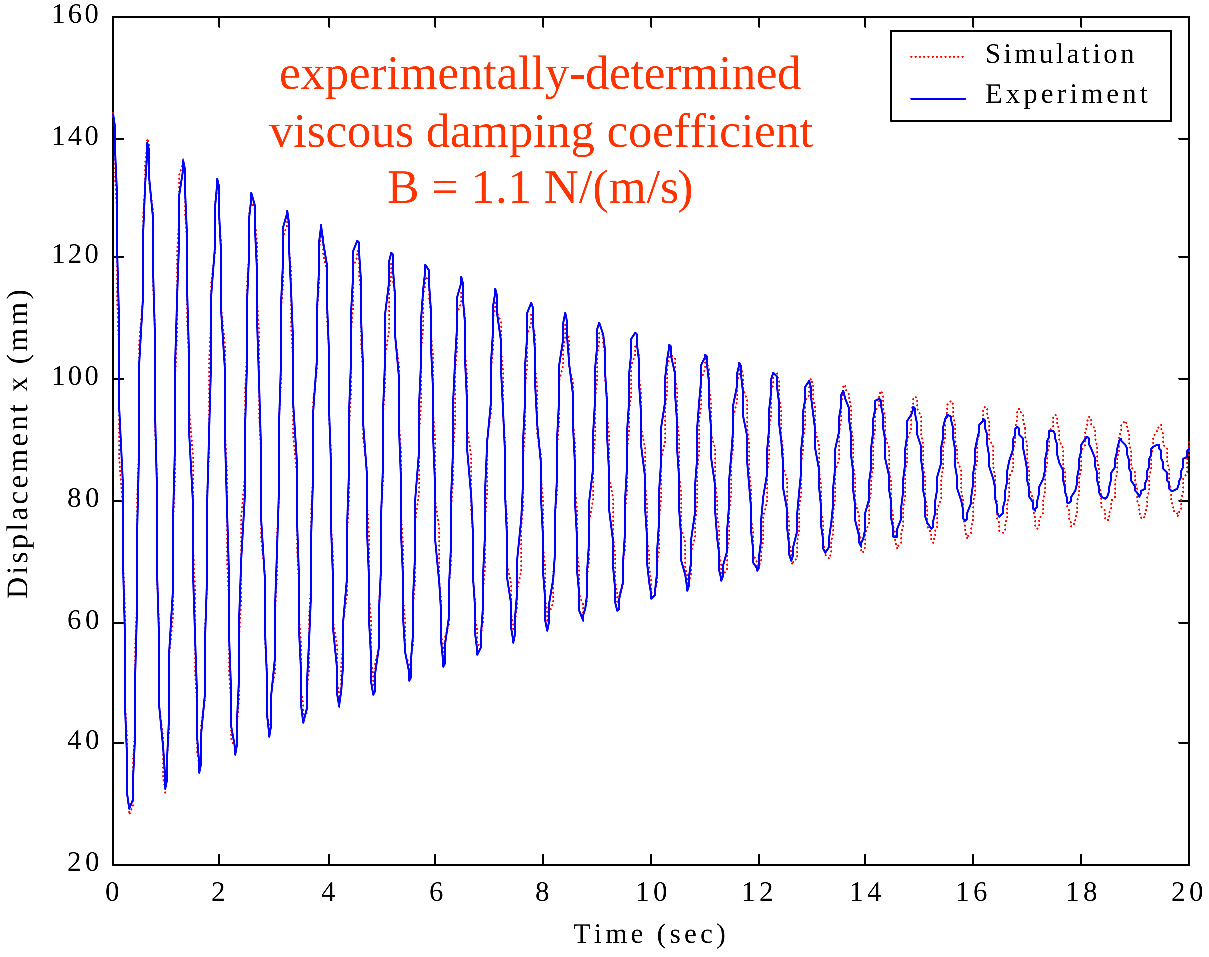
$$m\ddot{x} + B\dot{x} + k\left(x_s + x_d\right) = mg - F_t$$

$$m\ddot{x} + B\dot{x} + kx = mg - F_t$$ Equation of Motion in terms of $x = x_s + x_d$

$$m\ddot{x}_d + B\dot{x}_d + kx_d = 0$$ Equation of Motion in terms of x_d

Revised Comparison: Actual vs. Predicted

Initial Mass Displacement is 60 mm from the Static Equilibrium Position



- The agreement between the predicted response and the actual response is now quite good.
- The difference between the two responses becomes more noticeable as the oscillations diminish.
- This is expected because, again from engineering experience, Coulomb friction will begin to dominate the response over viscous friction when the system begins to slow down, and the physical model does not contain a Coulomb-friction term.

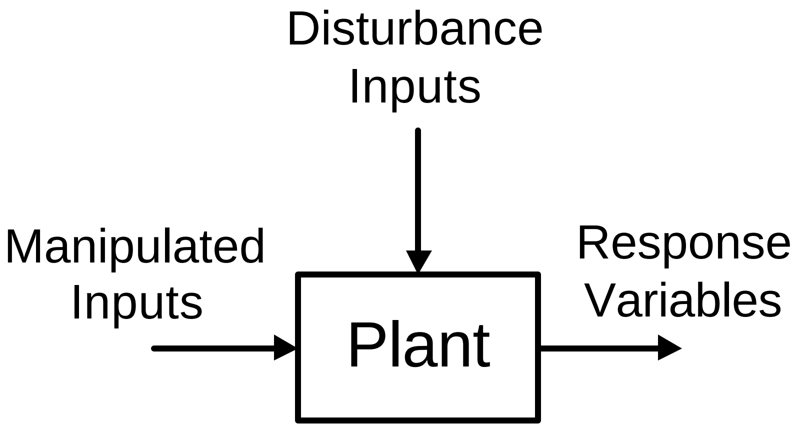
- Conclusion
  - The dynamic system investigation process has been demonstrated for the simple spring-mass system. Even for such a simple mechanical system, the process is still quite involved.
  - Many simplifying assumptions must be made and sound engineering judgment used. The result of the investigation is a physical and mathematical model that together predict quite well the behavior of the actual physical system.

- If a more detailed model is needed to more accurately predict the behavior of the actual system, the simplifying assumptions need to be reexamined and modified, if necessary, e.g., include Coulomb friction from the linear bearing, account for the mass of the spring.

Introduction to Control Systems

Everything Needs Controls
for Optimum Functioning!

- Process or Plant
- Process Inputs
  - Manipulated Inputs
  - Disturbance Inputs
- Response Variables



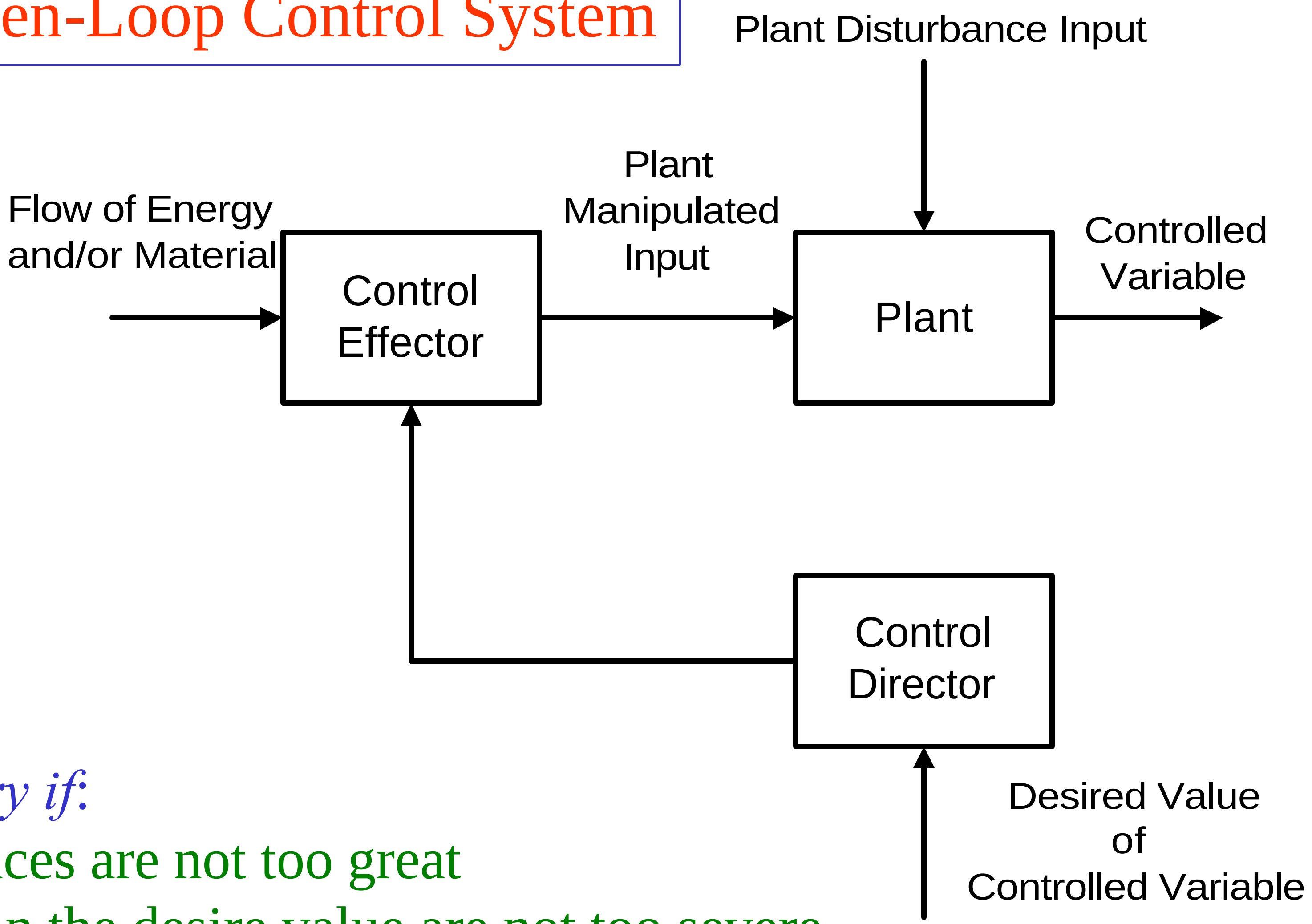
Control systems are an integral part of the overall system and ***not*** *after-thought add-ons!*

Why Controls?

- Command Following
- Disturbance Rejection

- Classification of Control System Types
  - Open-Loop
    - Basic
    - Input-Compensated Feedforward
      - Disturbance-Compensated
      - Command-Compensated
  - Closed-Loop (Feedback)
    - Classical
      - Root-Locus
      - Frequency Response
    - Modern (State-Space)
    - Advanced
      - e.g., Adaptive, Nonlinear, Fuzzy Logic

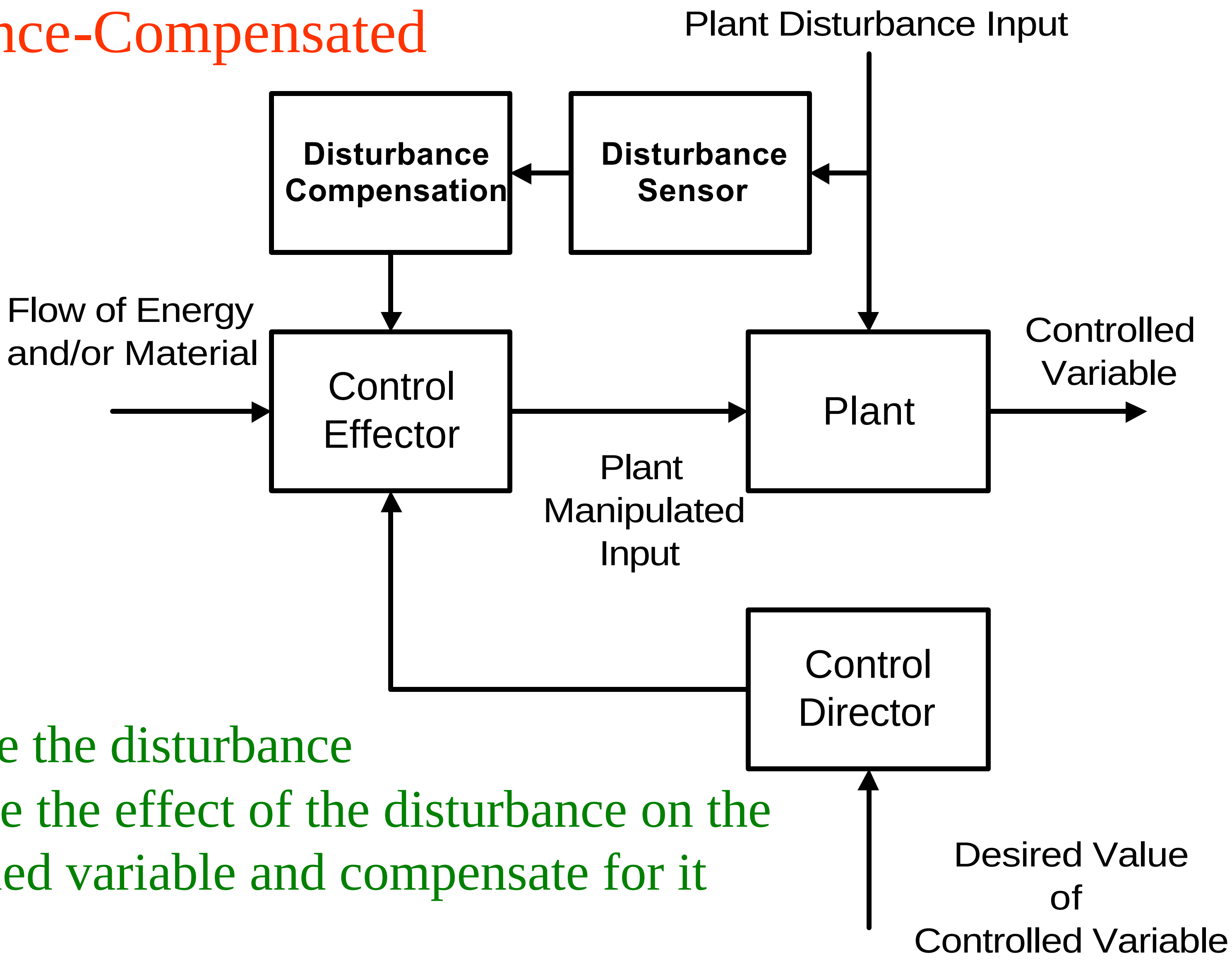
Basic Open-Loop Control System



Satisfactory if:

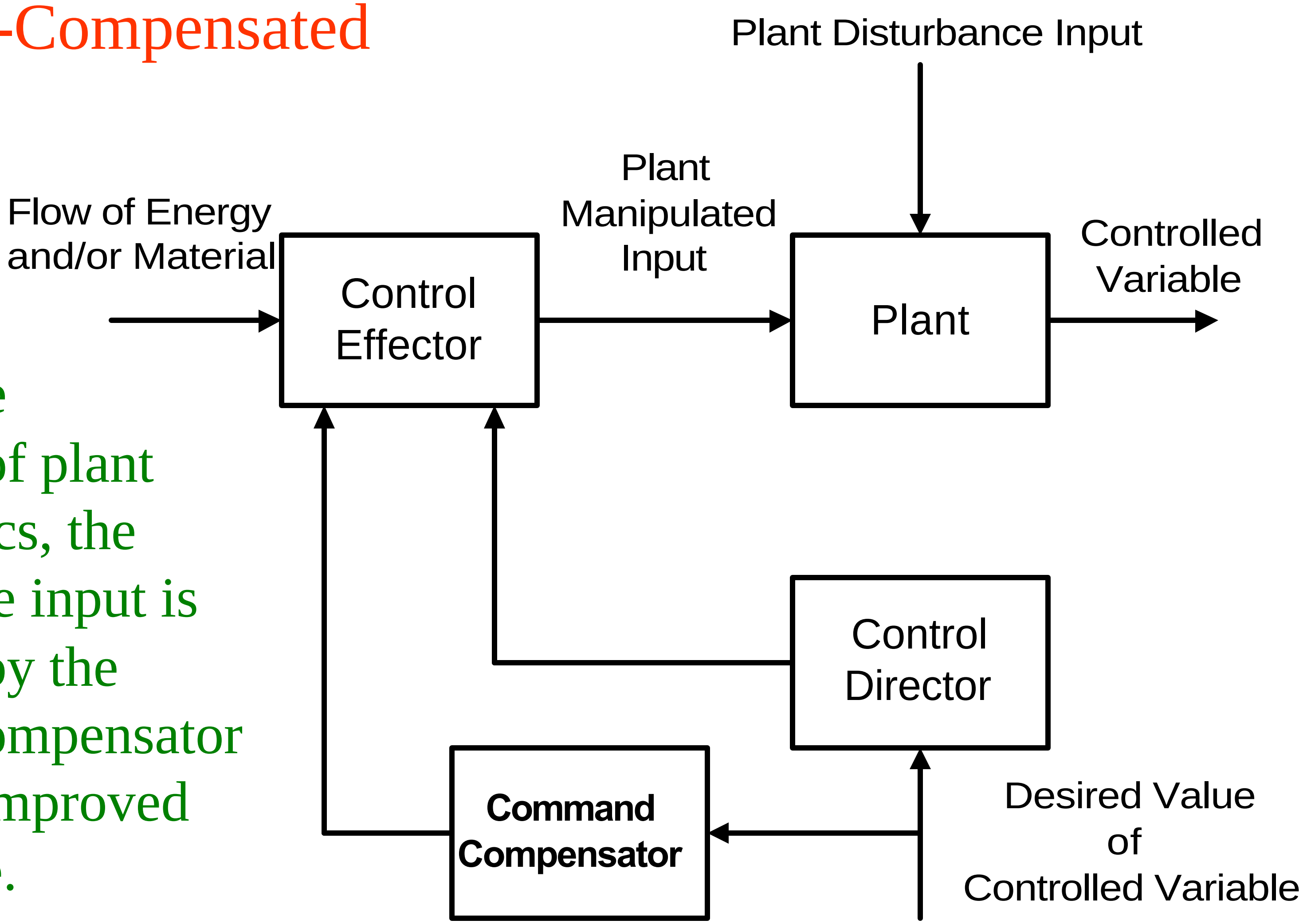
- disturbances are not too great
- changes in the desire value are not too severe
- performance specifications are not too stringent

Open-Loop Input-Compensated Feedforward Control: Disturbance-Compensated



- Measure the disturbance
- Estimate the effect of the disturbance on the controlled variable and compensate for it

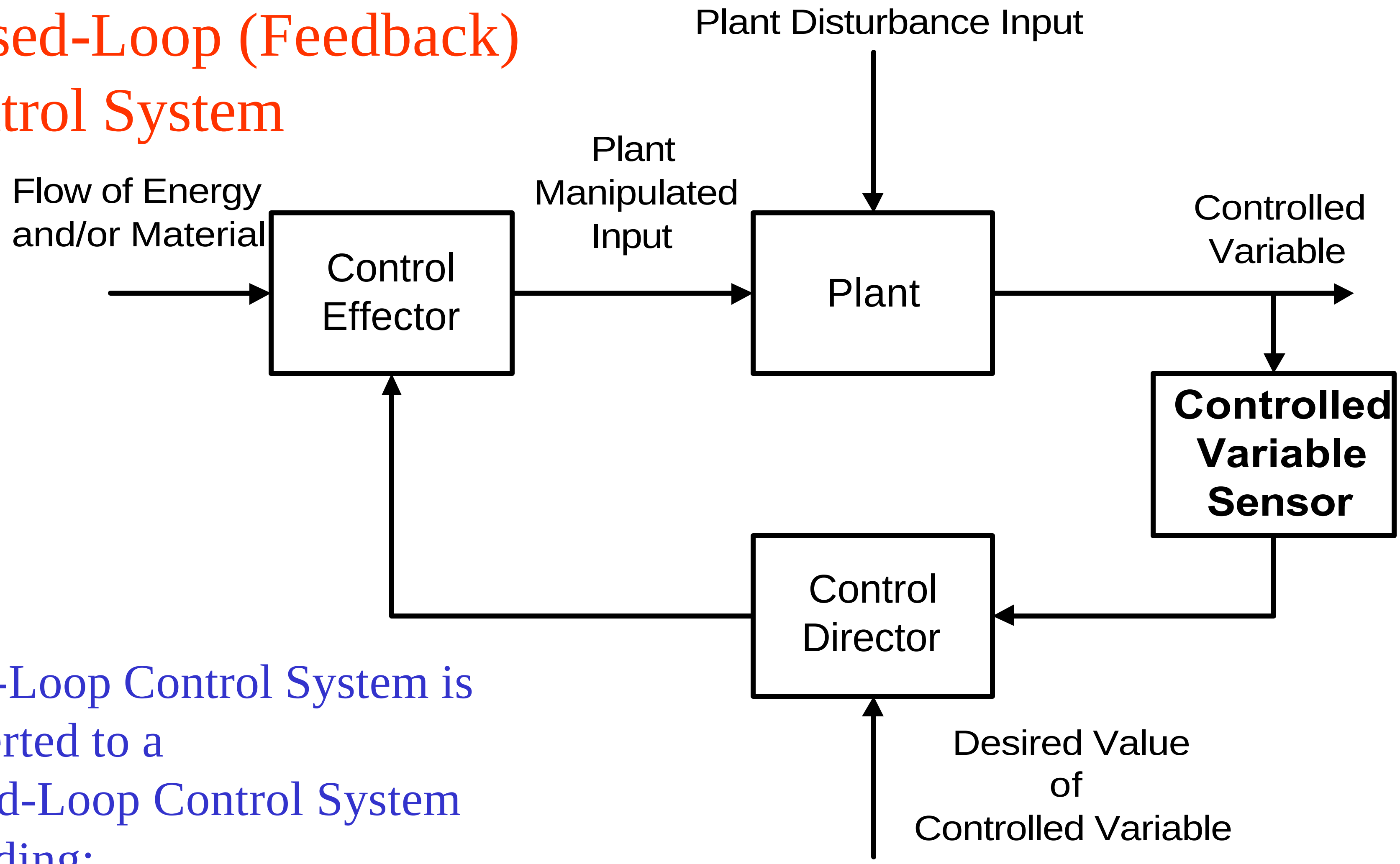
Open-Loop Input-Compensated Feedforward Control: Command-Compensated



Based on the knowledge of plant characteristics, the desired value input is augmented by the command compensator to produce improved performance.

- Open-loop systems without disturbance or command compensation are generally the simplest, cheapest, and most reliable control schemes. These should be considered first for any control task.
- If specifications cannot be met, disturbance and/or command compensation should be considered next.
- When conscientious implementation of open-loop techniques by a knowledgeable designer fails to yield a workable solution, the more powerful feedback methods should be considered.

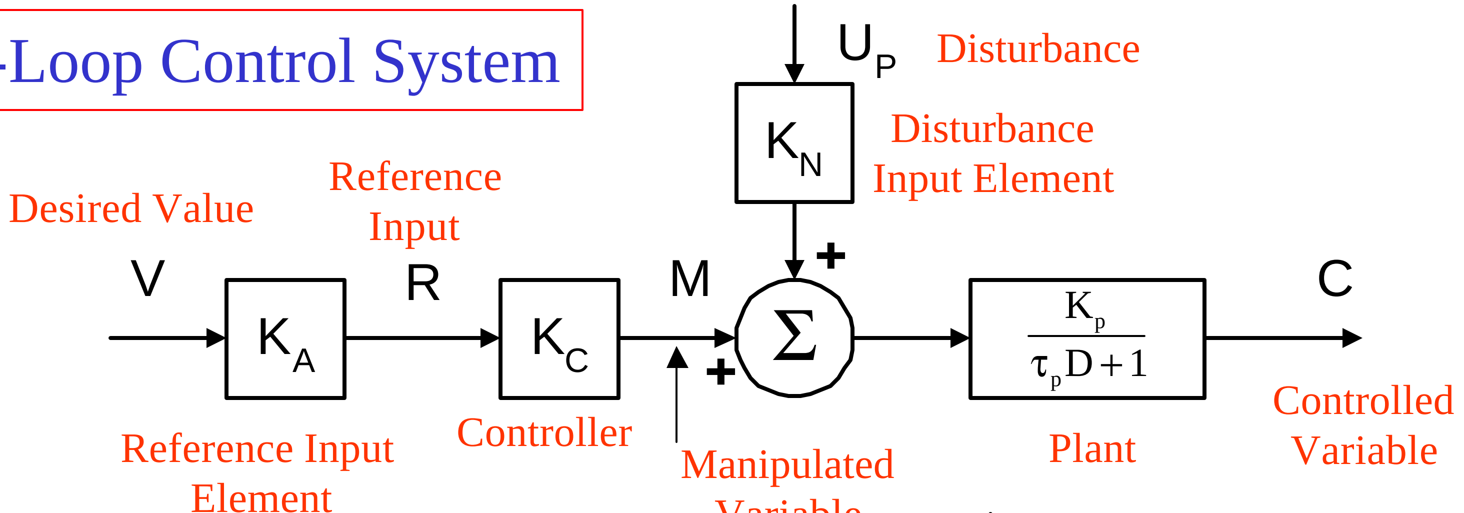
Closed-Loop (Feedback) Control System



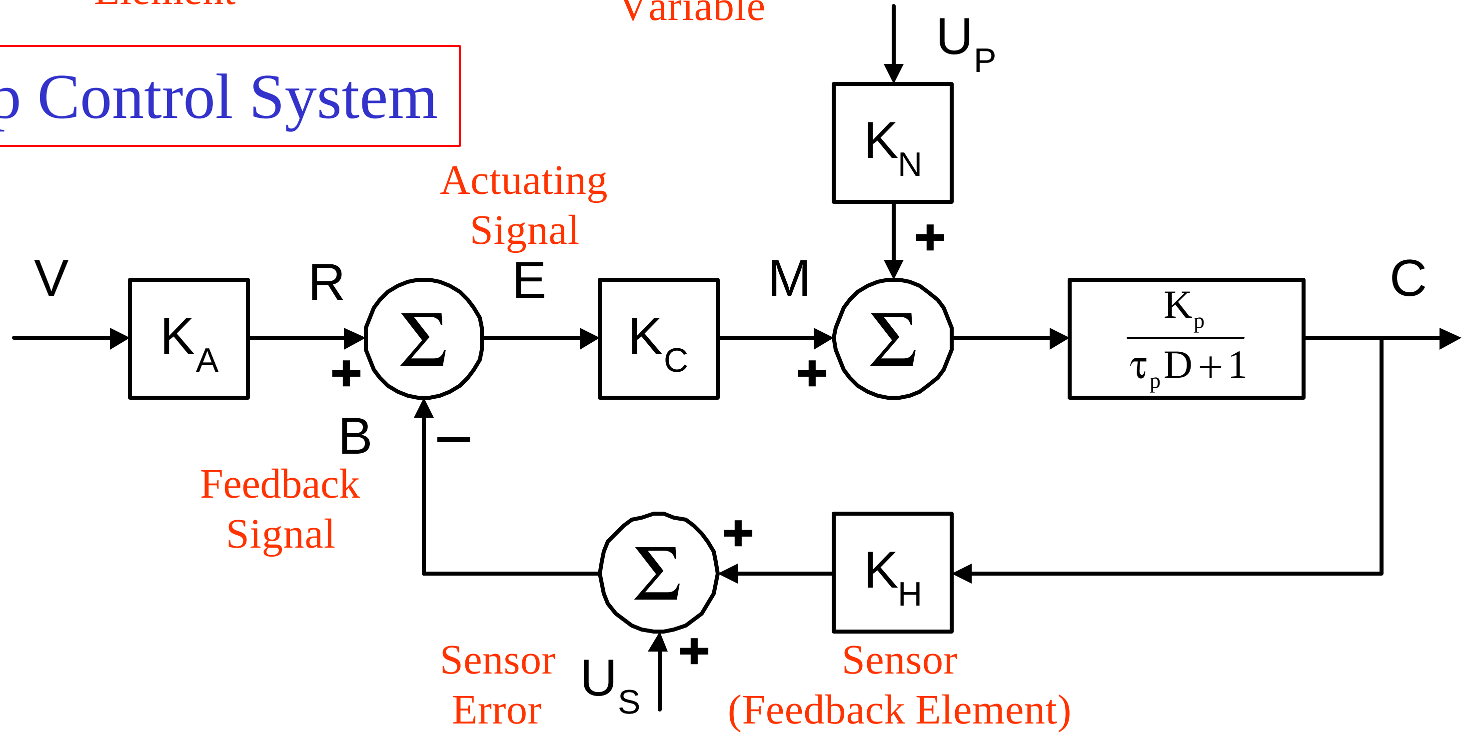
Open-Loop Control System is converted to a Closed-Loop Control System by adding:

- measurement of the controlled variable
- comparison of the measured and desired values of the controlled variable

Open-Loop Control System



Closed-Loop Control System



Basic Benefits of Feedback Control

- Cause the controlled variable to accurately follow the desired variable.
- Greatly reduce the effect on the controlled variable of all external disturbances in the forward path. It is ineffective in reducing the effect of disturbances in the feedback path (e.g., those associated with the sensor), and disturbances outside the loop (e.g., those associated with the reference input element).
- Are tolerant of variations (due to wear, aging, environmental effects, etc.) in hardware parameters of components in the forward path, but not those in the feedback path (e.g., sensor) or outside the loop (e.g., reference input element).
- Can give a closed-loop response speed much greater than that of the components from which they are constructed.

Instability in Feedback Control Systems

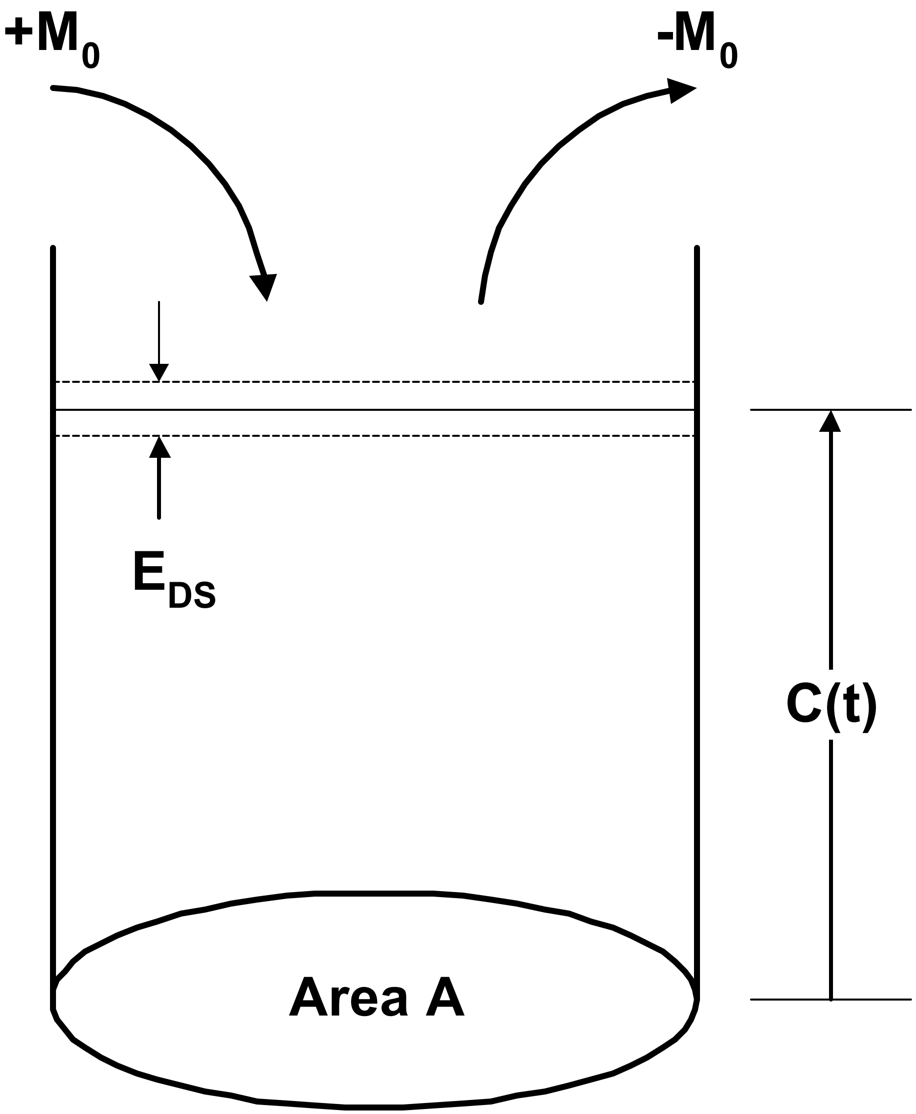
- All feedback systems can become unstable if improperly designed.
- In all real-world components there is some kind of lagging behavior between the input and output, characterized by τ's and ω_n's.
- Instantaneous response is impossible in the real world!
- Instability in a feedback control system results from an *improper balance between the strength of the corrective action and the system dynamic lags*.

Consider the following example:

Liquid level *C* in a tank is manipulated by controlling the volume inflow rate *M* by means of a 3-position on/off controller.

Transfer function *K/D* between *M* and *C* represents conservation of volume between inflow rate and liquid level.

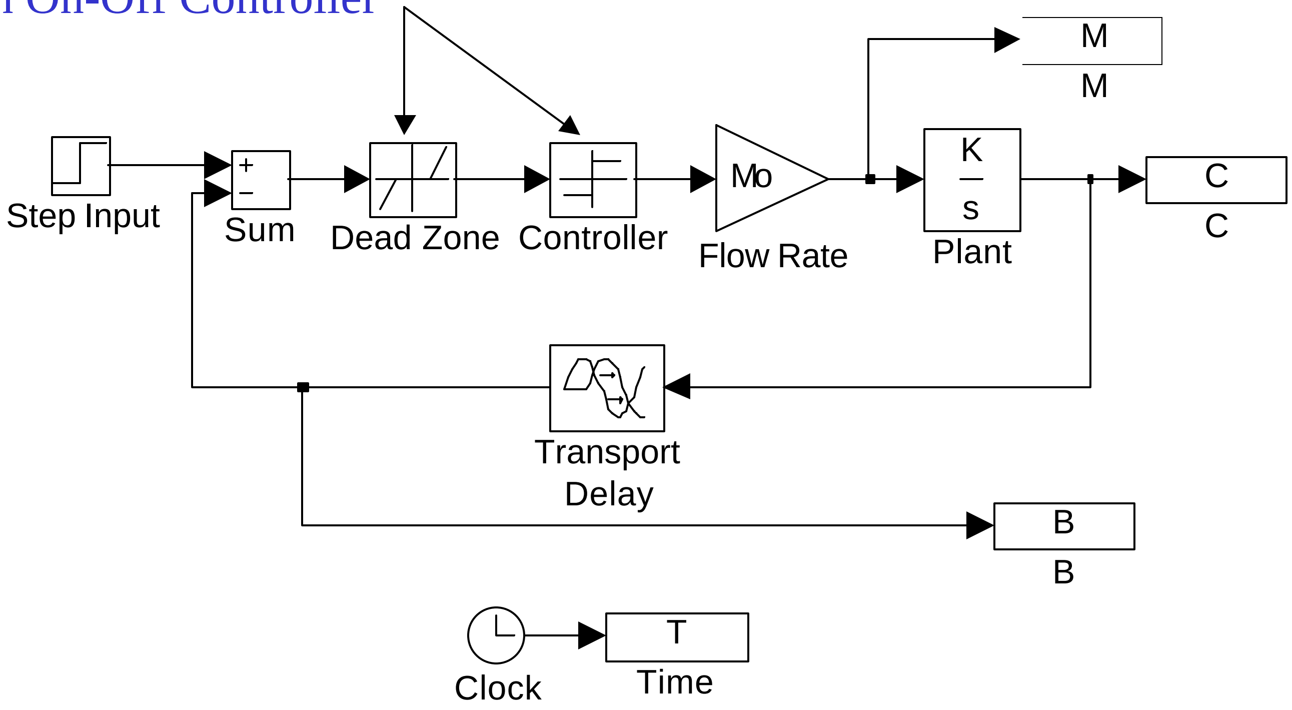
Liquid-level sensor measures *C* perfectly but with a data transmission delay, τ_{DT}.



Tank Liquid-Level Feedback Control System

Tank Liquid-Level Feedback Control System: MatLab / Simulink Block Diagram

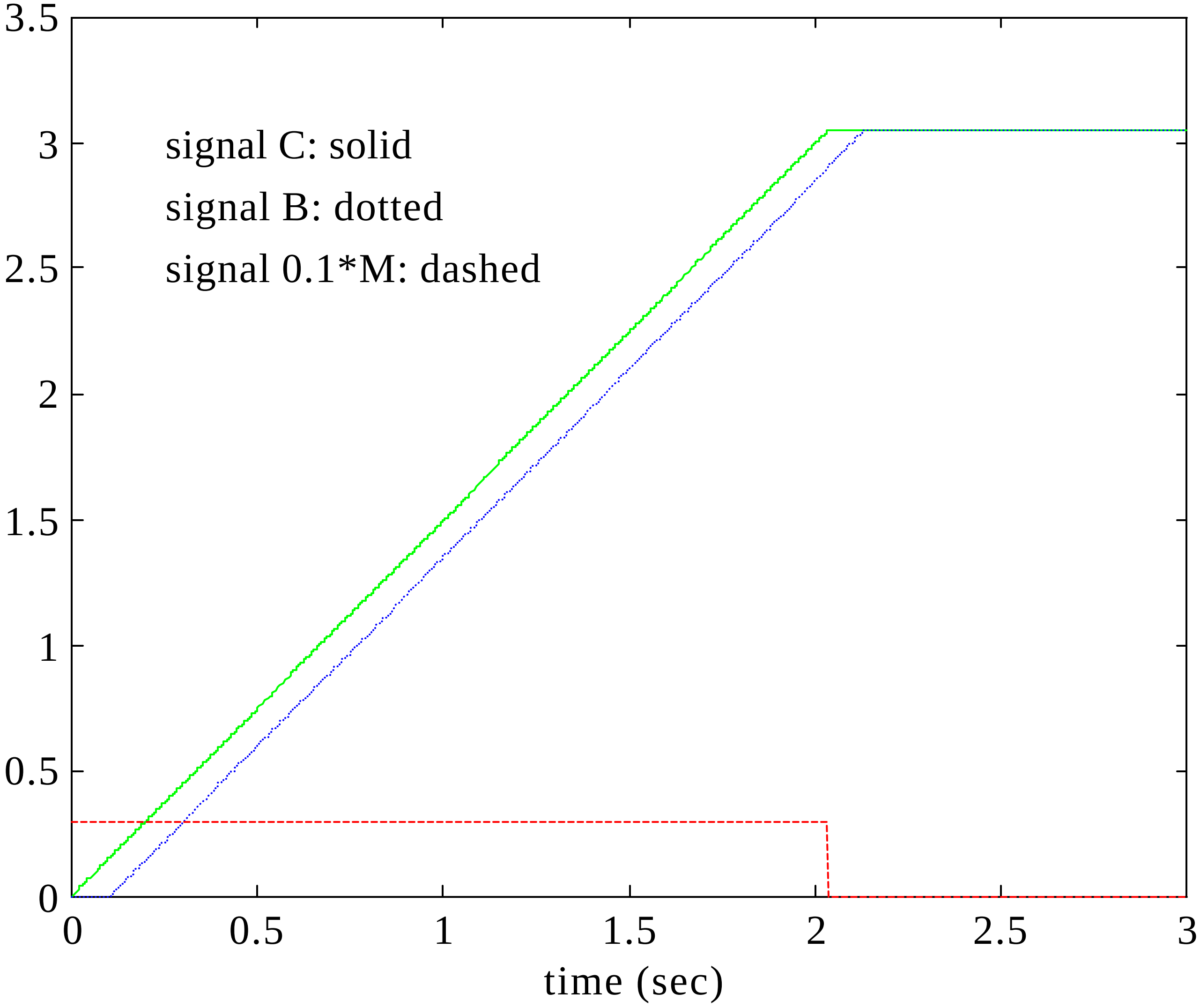
3-Position On-Off Controller



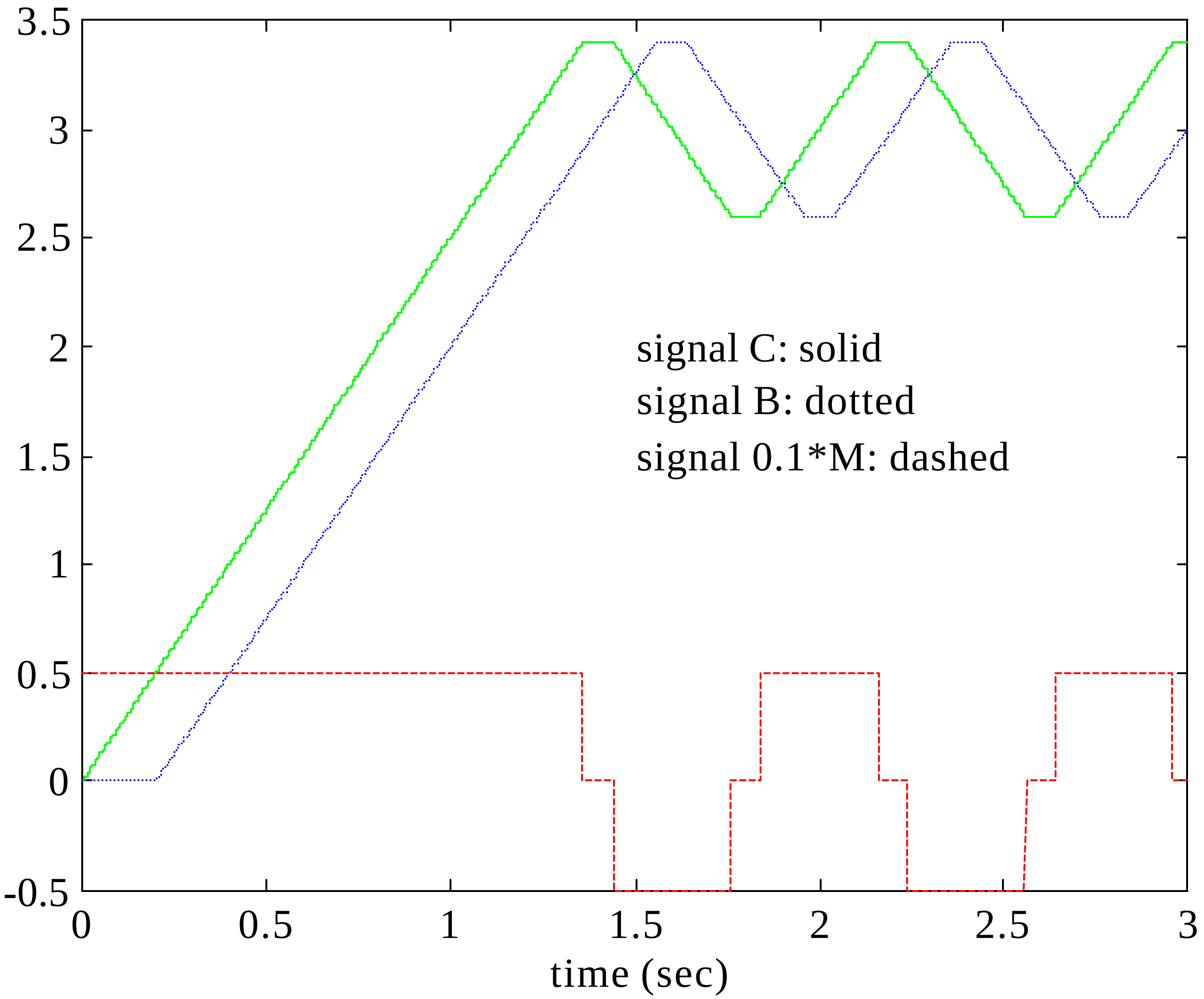
Mo=3, K=0.5, tau_dt=0.1 : stable

Mo=5, K=0.5, tau_dt=0.2 : unstable

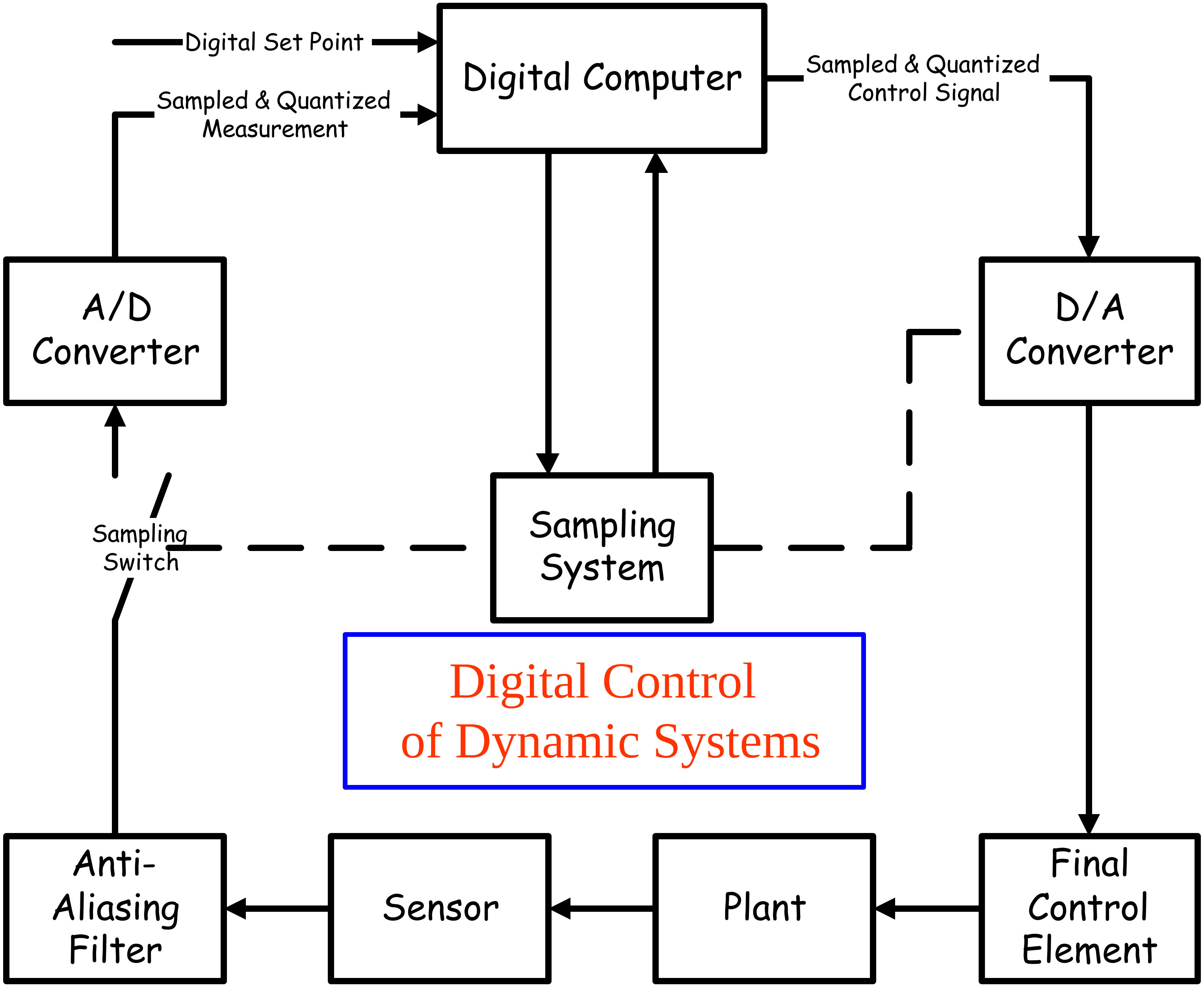
Stable Behavior of the Tank Liquid-Level Feedback Control System



Unstable Behavior of the Tank Liquid-Level Feedback Control System



Digital Control of Dynamic Systems



Advantages of Digital Control

- The current trend toward using dedicated, microprocessor-based, and often decentralized (distributed) digital control systems in industrial applications can be rationalized in terms of the major advantages of digital control:
  - Digital control is less susceptible to noise or parameter variation in instrumentation because data can be represented, generated, transmitted, and processed as binary words, with bits possessing two identifiable states.

- Very high accuracy and speed are possible through digital processing. Hardware implementation is usually faster than software implementation.
- Digital control can handle repetitive tasks extremely well, through programming.
- Complex control laws and signal conditioning methods that might be impractical to implement using analog devices can be programmed.
- High reliability can be achieved by minimizing analog hardware components and through decentralization using dedicated microprocessors for various control tasks.

- Large amounts of data can be stored using compact, high-density data storage methods.
- Data can be stored or maintained for very long periods of time without drift and without being affected by adverse environmental conditions.
- Fast data transmission is possible over long distances without introducing dynamic delays, as in analog systems.
- Digital control has easy and fast data retrieval capabilities.
- Digital processing uses low operational voltages (e.g., 0 - 12 V DC).
- Digital control has low overall cost.

Digital Signals are:

- discrete in time
- quantized in amplitude

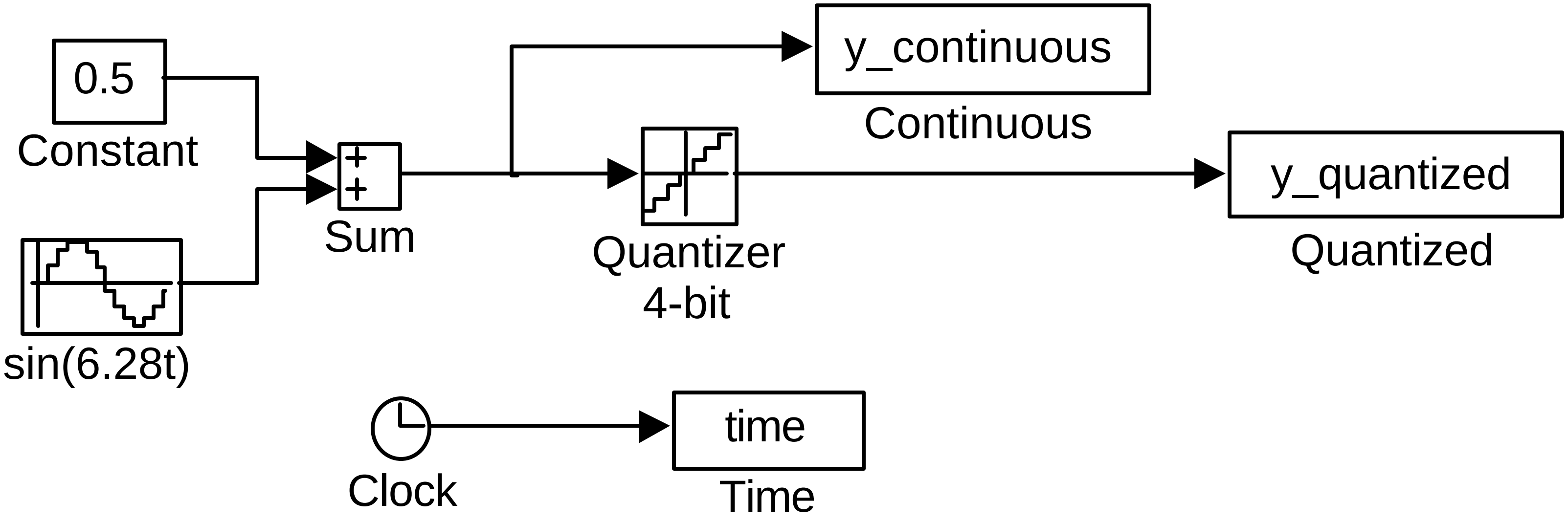
You must understand the effects of:

- sample period
- quantization size

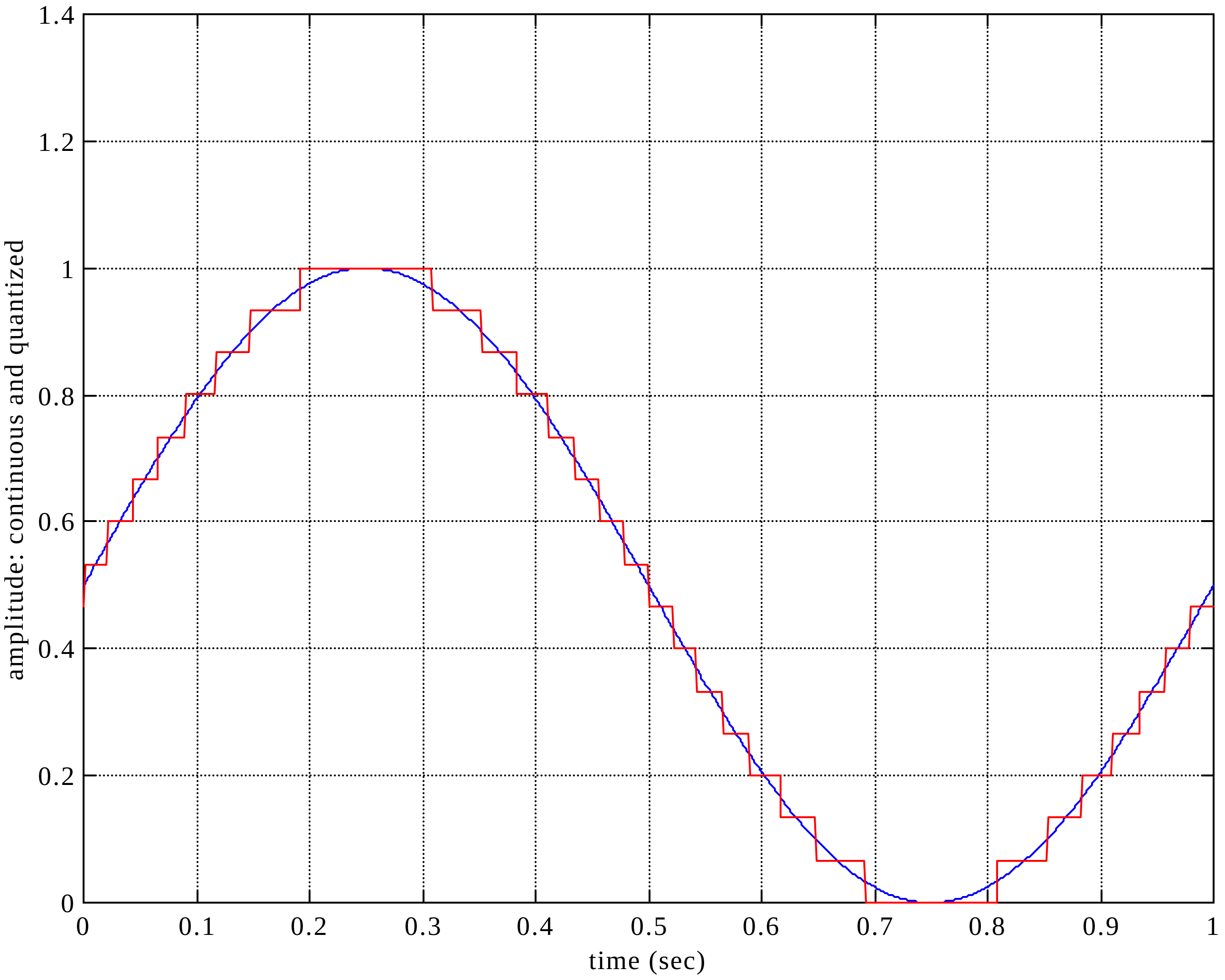
	Discrete in Time	**Continuous in Time**
Discrete in Amplitude	D-D	D-C
Continuous in Amplitude	C-D	C-C

- In a real sense, the problems of analysis and design of digital control systems are concerned with taking into account the effects of the sampling period, T, and the quantization size, q.
- If both T and q are extremely small (i.e., sampling frequency 50 or more times the system bandwidth with a 16-bit word size), digital signals are nearly continuous, and continuous methods of analysis and design can be used.
- It is most important to understand the *effects of all sample rates*, fast and slow, and the *effects of quantization* for large and small word sizes.

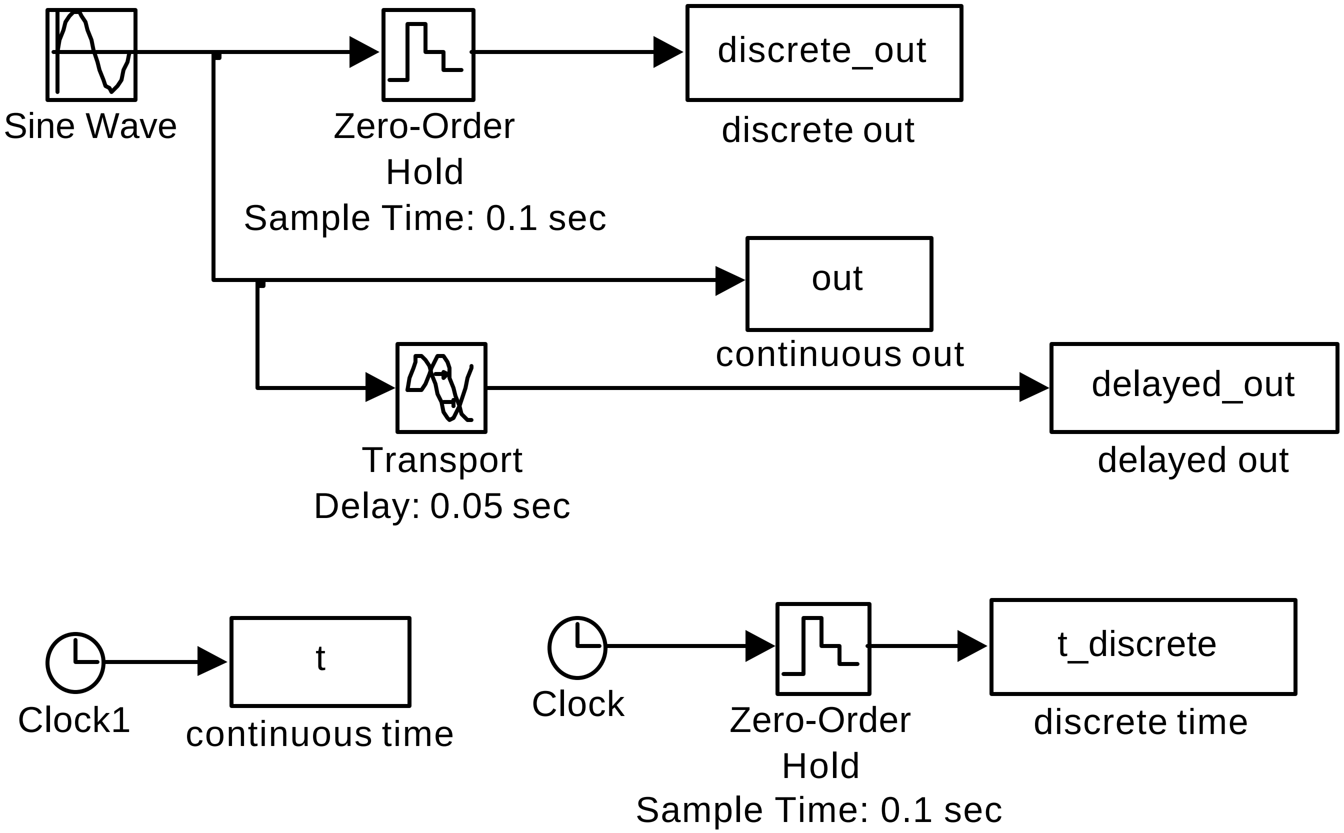
MatLab / Simulink Block Diagram: Demonstration of Quantization



Simulation of Continuous and Quantized Signal

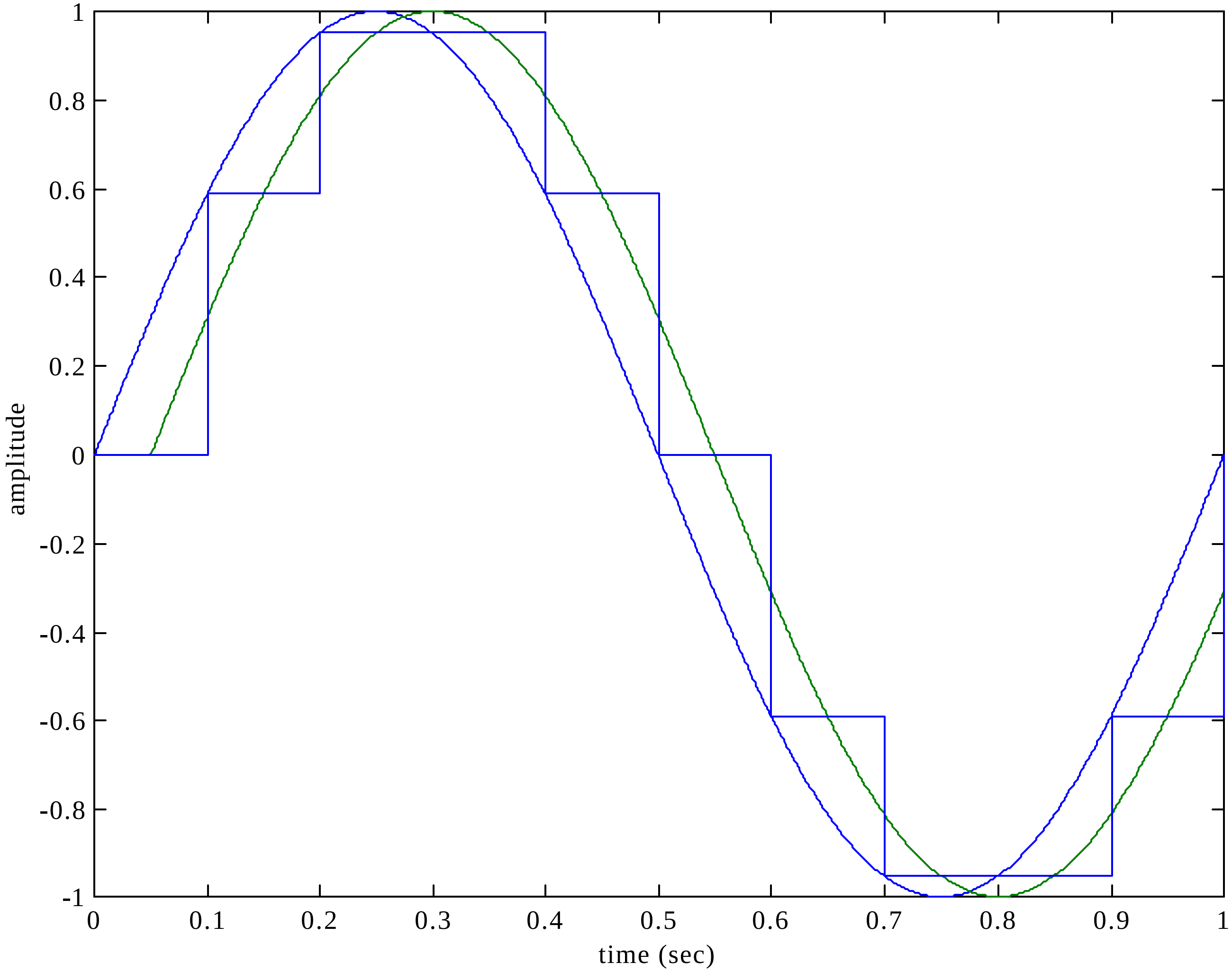


MatLab / Simulink Block Diagram: Demonstration of D/A Conversion



It is worthy to note that the *single most important impact* of implementing a control system digitally is the delay associated with the D/A converter, i.e., T/2.

Continuous Output and D/A Output

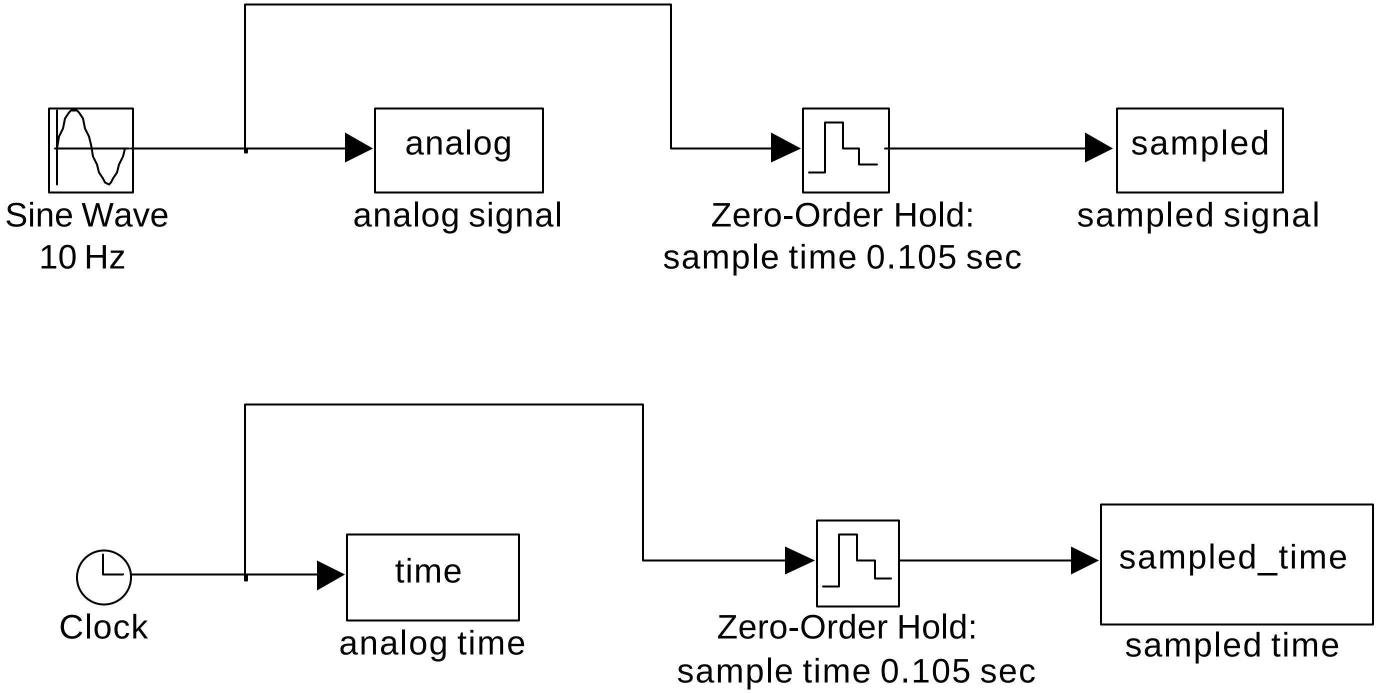


Aliasing

- The analog feedback signal coming from the sensor contains useful information related to controllable disturbances (relatively low frequency), but also may often include higher frequency "noise" due to uncontrollable disturbances (too fast for control system correction), measurement noise, and stray electrical pickup. Such noise signals cause difficulties in analog systems and low-pass filtering is often needed to allow good control performance.

- In digital systems, a phenomenon called *aliasing* introduces some new aspects to the area of noise problems. If a signal containing high frequencies is sampled too infrequently, the output signal of the sampler contains low-frequency ("aliased") components not present in the signal before sampling. If we base our control actions on these false low-frequency components, they will, of course, result in poor control. The theoretical absolute minimum sampling rate to prevent aliasing is 2 samples per cycle; however, in practice, rates of about 10 are more commonly used. A high-frequency signal, inadequately sampled, can produce a reconstructed function of a much lower frequency, which can not be distinguished from that produced by adequate sampling of a low-frequency function.

MatLab / Simulink Block Diagram: Demonstration of Aliasing



Sine Wave
10 Hz

analog

analog signal

Zero-Order Hold:
sample time 0.105 sec

sampled

sampled signal

Clock

time

analog time

Zero-Order Hold:
sample time 0.105 sec

sampled_time

sampled time

Simulation of Continuous and Sampled Signal