THE REFLECTANCE MAP AND SHAPE-FROM-SHADING

REFLECTANCE MODELS

LAMBERTIAN MODEL

\[ E = L \rho \cos \theta \]

albedo

PHONG MODEL

\[ E = L (a \cos \theta + b \cos^n \alpha) \]

Diffuse albedo

Specular albedo

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a=0.3, b=0.7, n=2

a=0.7, b=0.3, n=0.5
REFLECTANCE MODELS

• Description of how light energy incident on an object is transferred from the object to the camera sensor
REFLECTANCE MAP IS A VIEWER-CENTERED REPRESENTATION OF REFLECTANCE

\[(f_x, f_y, -1) = (0,1,f_x) \times (1,0,f_y)\]

Surface Orientation

\[z = f(x,y)\]

Depth

IMAGE PLANE

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REFLECTANCE MAP IS A VIEWER-CENTERED REPRESENTATION OF REFLECTANCE

\[(f_x, f_y, -1) = (p, q, -1)\]

p, q comprise a **gradient** or **gradient space** representation for local surface orientation.

Reflectance map expresses the reflectance of a material directly in terms of viewer-centered representation of local surface orientation.
LAMBERTIAN REFLECTANCE MAP

LAMBERTIAN MODEL

\[ E = L \rho \cos \theta \]

\[ \cos \theta = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2} \sqrt{1 + p_s^2 + q_s^2}} \]
LAMBERTIAN REFLECTANCE MAP

\[ E = L\rho \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2} \sqrt{1 + p_s^2 + q_s^2}} \]

Grouping \( L \) and \( \rho \) as a constant, local surface orientations that produce equivalent intensities under the Lambertian reflectance map are quadratic conic section contours in gradient space.

\[ I = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2} \sqrt{1 + p_s^2 + q_s^2}} \]
LAMBERTIAN REFLECTANCE MAP

\[ \mathbf{p}_s = 0 \quad \mathbf{q}_s = 0 \]
LAMBERTIAN REFLECTANCE MAP

\[ p_s=0.7 \quad q_s=0.3 \]
LAMBERTIAN REFLECTANCE MAP

\[ p_s = -2 \quad q_s = -1 \]
PHOTOMETRIC STEREO

Derivation of local surface normal at each pixel creates the derived normal map.
NORMAL MAP vs. DEPTH MAP

IMAGE PLANE

Depth

Surface Orientation
NORMAL MAP vs. DEPTH MAP

• Can determine Depth Map from Normal Map by integrating over gradients p,q across the image.

• Not all Normal Maps have a unique Depth Map. This happens when Depth Map produces different results depending upon image plane direction used to sum over gradients.

• Particularly a problem when there are errors in the Normal Map.
NORMAL MAP VS. DEPTH MAP

• A Normal Map that produces a unique Depth Map independent of image plane direction used to sum over gradients is called integrable.

• Integrability is enforced when the following condition holds:

\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]
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GREEN’S THEOREM

\[
\iint (\frac{\partial p}{\partial y} - \frac{\partial q}{\partial x}) dxdy = \int (pdx + qdy)
\]
NORMAL MAP vs. DEPTH MAP

VIOLATION OF INTEGRABILITY

Penrose Staircase
SHAPE FROM SHADING

From a monocular view with a single distant light source of known incident orientation upon an object with known reflectance map, solve for the normal map.
SHAPE FROM SHADING

• Formulate as solving the Image Irradiance equation for surface orientation variables \( p,q \):

\[
I(x,y) = R(p,q)
\]

• Since this is underconstrained we can’t solve this equation directly

• What do we do ??.
SHAPE FROM SHADING
(Calculus of Variations Approach)

• First Attempt: Minimize error in agreement with Image Irradiance Equation over the region of interest:

\[ \iiint_{object} (I(x, y) - R(p, q))^2 \, dx \, dy \]
SHAPE FROM SHADING
(Calculus of Variations Approach)

• Better Attempt: Regularize the Minimization of error in agreement with Image Irradiance Equation over the region of interest:

\[
\iint_{\text{object}} p_x^2 + p_y^2 + q_x^2 + q_y^2 + \lambda (I(x,y) - R(p,q))^2 \, dx \, dy
\]