Depth and Shape Inference (III)

Introduction to Computational and Biological Vision

CS 202-1-5261

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Shape from Shading
Shape from Shading
Shape from Shading

Shading is more than contours
Shape from Shading

Shading is more than edge contours
Shape from Shading

Inverting the image formation process

Image formation = “Shading from shape” (and light sources)
Shape from Shading

Image formation
Shape from Shading

Polar representation of directions

\[ L(\phi_r, \theta_r) \]

\[ E(\phi_i, \theta_i) \]

\( \phi \) - Azimuth angle

\( \theta \) - Zenith angle
Shape from Shading

The Bidirectional Reflectance Distribution Function (BRDF)

\[ f_\lambda (\phi_i, \theta_i; \phi_r, \theta_r) = \frac{L_\lambda (\phi_r, \theta_r)}{E_\lambda (\phi_i, \theta_i)} \]

Helmholtz’s reciprocity

\[ f (\phi_i, \theta_i; \phi_r, \theta_r) = f (\phi_r, \theta_r; \phi_i, \theta_i) \]

Isotropic materials:

\[ f (\phi_i, \theta_i; \phi_r, \theta_r) = f (\phi_i - \phi_r, \theta_i, \theta_r) \]
**Shape from Shading**

**Total surface reflection**

\[
L(\phi_r, \theta_r) = \int_{\omega} f(\phi_i, \theta_i; \phi_r, \theta_r) \cdot E(\phi_i, \theta_i) \cdot \cos \theta \, d\omega
\]

\[
L(\phi_r, \theta_r)
\]

\[
\delta \theta_i
\]

\[
\delta \omega
\]

\[
\delta \phi_i
\]

\[
\sin \theta_i \delta \theta_i \delta \phi_i
\]
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Total surface reflection

\[ L(\phi_r, \theta_r) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} f(\phi_i, \theta_i; \phi_r, \theta_r) \cdot E(\phi_i, \theta_i) \cdot \sin \theta_i \cdot \cos \theta_i \cdot \delta \theta_i \delta \phi_i \]
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Lambertian (perfectly diffused) surfaces

\[ f_L(\phi_i, \theta_i; \phi_r, \theta_r) = \text{const} = \bar{f} = \rho \frac{1}{\pi} \]
Shape from Shading

Mirrored (perfectly secular) surfaces

\[ f_s(\phi_i, \theta_i; \phi_r, \theta_r) = \frac{\delta(\theta_r - \theta_i) \delta(\phi_r - \phi_i - \pi)}{\sin \theta_i \cos \theta_i} \]
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Mixed surfaces

\[ f(\phi, \theta_i; \phi_r, \theta_r) = \alpha \cdot f_L(\phi_i, \theta_i; \phi_r, \theta_r) + (1 - \alpha) f_S(\phi_i, \theta_i; \phi_r, \theta_r) \]
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The fundamental radiometric relationship

\[ I = L \cdot \frac{\pi}{4} \left( \frac{d}{f} \right)^2 \cdot \cos^4 \alpha \Rightarrow I \propto L \]
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Point light source from direction \((\phi_L, \theta_L)\)

\[
E(\phi_i, \theta_i) = E \cdot \frac{\delta(\theta_L - \theta_i) \cdot \delta(\phi_L - \phi_i)}{\sin \theta_L}
\]

\[
\int_{-\pi}^{\pi} \int_{0}^{\pi/2} E(\phi_i, \theta_i) \cdot \sin \theta_i \cdot \delta \theta_i \delta \phi_i = E
\]

\[
E
\]

\[
\vec{N}
\]

\[
\vec{L}
\]
Shape from Shading

Surface brightness – appearance in the Lambertian case and point light source

\[ f_L(\phi_i, \theta_i; \phi_r, \theta_r) = \rho \frac{1}{\pi} \]

\[ E(\phi_i, \theta_i) = \frac{\delta(\theta_L - \theta_i)\delta(\phi_L - \phi_i)}{\sin \theta_L} \]

\[ I(x, y) \propto L(\phi_r, \theta_r) = \int_{-\pi}^{\pi/2} \int_{0}^{\pi/2} f(\phi_i, \theta_i; \phi_r, \theta_r) \cdot E(\phi_i, \theta_i) \cdot \sin \theta_i \cdot \cos \theta_i \cdot \delta \theta_i \delta \phi_i \]

\[ L = \rho \frac{1}{\pi} E \cos \theta_L \propto \rho(\hat{N} \cdot \hat{L}) \]
**Shape from Shading**

Shape description – Tangent plane and normal vectors

\[
\vec{r}_x = \left(1, 0, \frac{\partial H}{\partial x}\right) \quad \vec{r}_y = \left(0, 1, \frac{\partial H}{\partial y}\right)
\]

\[
\vec{N} = \vec{r}_x \times \vec{r}_y = (-p, -q, 1)
\]

\[
\hat{N} = \frac{\vec{N}}{\|\vec{N}\|} = \frac{(-p, -q, 1)}{\sqrt{p^2 + q^2 + 1}}
\]
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Shading on Lambertian surface – General point source

\[ I = \rho (\hat{\mathbf{N}} \cdot \hat{\mathbf{L}}) = \rho \frac{-p \cdot L_x - q \cdot L_y + L_z}{\sqrt{p^2 + q^2 + 1} \sqrt{L_x^2 + L_y^2 + L_z^2}} = \rho \frac{p \cdot p_L + q \cdot q_L + 1}{\sqrt{p^2 + q^2 + 1} \sqrt{p_L^2 + q_L^2 + 1}} \]
**Shape from Shading**

Shading on Lambertian surface – Overhead point source

\[ I(x, y) = \rho(\hat{N} \cdot [0,0,1]) = \rho \frac{1}{\sqrt{p^2 + q^2 + 1}} = R(p, q) \]
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The Reflectance Map – Lambertian surface from overhead source position

\[ R(p, q) = \frac{1}{\sqrt{p^2 + q^2 + 1}} \]
Shape from Shading

The Reflectance Map – Lambertian surface from general source position

\[ R(p, q) = \frac{p \cdot p_L + q \cdot q_L + 1}{\sqrt{p^2 + q^2} + 1}\sqrt{p_L^2 + q_L^2} + 1 \]

Gradient point of maximum brightness
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The Reflectance Map – typical real surfaces

\[ R(p, q) \]
Shape from Shading

Surface orientation from shading

\[ I(x, y) = R(p, q) \]
Shape from Shading

Photometric stereo
Shape from Shading

Photometric stereo

\[ I_1(x, y) = R_1(p, q) \]
\[ I_2(x, y) = R_2(p, q) \]
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The SFS problem (special case)

Given \( I(x,y) \) of an (orthographic) projection of \( H(x,y) \), and the reflectance map \( R(p,q) \), find \( H(x,y) \) everywhere.

\[
I(x, y) = R(p, q) = R\left( \frac{\partial}{\partial x} H(x, y), \frac{\partial}{\partial y} H(x, y) \right)
\]
**Shape from Shading**

Shape recovery via characteristic strips

\[ I(x, y) = R(p(x, y), q(x, y)) \]

\[ p(x, y) = \frac{\partial}{\partial x} H(x, y) \]

\[ q(x, y) = \frac{\partial}{\partial y} H(x, y) \]

\[ H(x + \delta x, y + \delta y) \cong H(x, y) + p \delta x + q \delta y \]

\[ p(x + \delta x, y + \delta y) \cong p(x, y) + \frac{\partial p}{\partial x} \delta x + \frac{\partial p}{\partial y} \delta y \]

\[ q(x + \delta x, y + \delta y) \cong q(x, y) + \frac{\partial q}{\partial x} \delta x + \frac{\partial q}{\partial y} \delta y \]
Shape from Shading

Shape recovery via characteristic strips

\[ I(x, y) = R(p(x, y), q(x, y)) \]

\[
\frac{\partial}{\partial x} I(x, y) = \frac{\partial R(p, q)}{\partial p} \frac{\partial p(x, y)}{\partial x} + \frac{\partial R(p, q)}{\partial q} \frac{\partial q(x, y)}{\partial x}
\]

\[
\frac{\partial}{\partial y} I(x, y) = \frac{\partial R(p, q)}{\partial p} \frac{\partial p(x, y)}{\partial y} + \frac{\partial R(p, q)}{\partial q} \frac{\partial q(x, y)}{\partial y}
\]
**Shape from Shading**

Shape recovery via characteristic strips

\[ I(x, y) = R(p(x, y), q(x, y)) \]

\[
\frac{\partial}{\partial x} I(x, y) = \frac{\partial R(p, q)}{\partial p} \frac{\partial p(x, y)}{\partial x} + \frac{\partial R(p, q)}{\partial q} \frac{\partial p(x, y)}{\partial y}
\]

\[
\frac{\partial}{\partial y} I(x, y) = \frac{\partial R(p, q)}{\partial p} \frac{\partial q(x, y)}{\partial x} + \frac{\partial R(p, q)}{\partial q} \frac{\partial q(x, y)}{\partial y}
\]
Shape from Shading

Shape recovery via characteristic strips

\[ I(x, y) = R(p(x, y), q(x, y)) \]

\[ \frac{\partial}{\partial x} I(x, y) = \nabla R \cdot \nabla p \]

\[ \frac{\partial}{\partial y} I(x, y) = \nabla R \cdot \nabla q \]

\[ \delta H \cong p \delta x + q \delta y \]

\[ \delta p \cong \frac{\partial p}{\partial x} \delta x + \frac{\partial p}{\partial y} \delta y = \nabla p \cdot (\delta x, \delta y) \]

\[ \delta q \cong \frac{\partial q}{\partial x} \delta x + \frac{\partial q}{\partial y} \delta y = \nabla q \cdot (\delta x, \delta y) \]

A smart choice

\[ \delta x = \frac{\partial R(p, q)}{\partial p} \delta s \]

\[ \delta y = \frac{\partial R(p, q)}{\partial q} \delta s \]

\[ \delta p \cong \frac{\partial}{\partial x} I(x, y) \cdot \delta s \]

\[ \delta q \cong \frac{\partial}{\partial y} I(x, y) \cdot \delta s \]
**Shape from Shading**

Shape recovery via characteristic strips

\[ \delta x = R_p \delta s \quad \Rightarrow \quad \dot{x} = R_p \]

\[ \delta y = R_q \delta s \quad \Rightarrow \quad \dot{y} = R_q \]

\[ \delta H = (pR_p + qR_q) \delta s \]

\[ \delta p = I_x \delta s \quad \Rightarrow \quad \dot{p} = I_x \]

\[ \delta q = I_y \delta s \quad \Rightarrow \quad \dot{q} = I_y \]
Shape from Shading

Shape recovery via characteristic strips

Shape from Shading via Characteristic Curves

Given

- $I(x,y)$ of an (orthographic) projection of unknown $H(x,y)$
- The reflectance map $R(p,q)$
- Initial data $x_0, y_0$, $H(x_0, y_0)$, $p(x_0, y_0)$, $q(x_0, y_0)$

Develop a curve solution on $H(x,y)$ by taking small steps of size $\delta s$ via the system

$$\delta x = R_p \delta s$$

$$\delta y = R_q \delta s$$

$$\delta H = (p R_p + q R_q) \delta s$$

$$\delta p = I_x \delta s$$

$$\delta q = I_y \delta s$$
Shape from Shading

Shape recovery via characteristic strips
Shape from Shading

Shape recovery via characteristic strips