Multi-View Geometry
(Ch7 New book. Ch 10/11 old book)

Guido Gerig
CS-GY 6643, Spring 2017
gerig@nyu.edu

Credits: M. Shah, UCF CAP5415, lecture 23
Visual cues

- Shading
Visual cues

- Shading
- Texture

*The Visual Cliff*, by William Vandivert, 1960
Visual cues

- Shading
- Texture
- Focus

From *The Art of Photography*, Canon
Visual cues

- Shading
- Texture
- Focus
- Motion
Visual cues

• Shading
• Texture
• Focus
• Motion

• Shape From X
  - $X = (\text{shading, texture, focus, motion, rotation, ...})$
Visual cues

Shadows
Visual cues

- Shading
- Texture
- Focus
- Motion
- Shape From X
  - \((X = \text{shading, texture, focus, motion, rotation, ...})\)
- Stereo (disparity, multi-view)
http://www.well.com/~jimg/stereo/stereo_list.html
Stereo photography and stereo viewers

Take two pictures of the same subject from two slightly different viewpoints and display so that each eye sees only one of the images.

Invented by Sir Charles Wheatstone, 1838

Image courtesy of fisher-price.com
Human stereopsis: disparity

**Disparity** occurs when eyes fixate on one object; others appear at different visual angles.

*From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology*

Adapted from David Forsyth, UC Berkeley
Human stereopsis: disparity

Disparity: \( d = r - l = D - F \).

Adapted from M. Pollefeys
Example: Stereo to Depth Map
Pinhole Camera Model

\[ x = -f \frac{X}{Z} \]
Basic Stereo Derivations

Derive expression for $Z$ as a function of $x_1$, $x_2$, $f$ and $B$
Basic Stereo Derivations

\[ x_1 = -f \frac{X}{Z} \quad x_2 = -f \frac{X + B}{Z} = x_1 - f \frac{B}{Z} \]

\[ Z = \frac{fB}{x_1 - x_2} \]
Basic Stereo Derivations

Define the disparity: \( d = x_1 - x_2 \)

\[
Z = \frac{fB}{d}
\]
Standard stereo geometry

Disparity $d$:
$$d = |u' - u|$$
Standard stereo geometry

Observations on disparity:
- $d$ shows large differences at small distances
- $d$ gets very small on large distances
Stereo Correspondence

- Search over disparity to find correspondences
- Range of disparities to search over can change dramatically within a single image pair.
Standard stereo geometry: Changes of $\Delta Z$ with $\Delta d$

\[ \frac{B}{Z} = \frac{d}{f} \]

\[ d = \frac{Bf}{Z} \]

\[ Z = f(d) \]

\[ f'(d) = \frac{dZ}{dd} = -\frac{Bf}{d^2} \]

Observations:
- at small $d$ (far), $\Delta d$ corresponds to large $\Delta Z$
- at large $d$ (close), $\Delta d$ corresponds to small $\Delta Z$
- important for analysis of precision/resolution
Why is disparity important?

Given dense disparity map, we can calculate a depth/distance/range map.

Goal: 3D from Stereo via Disparity Map

\[(x', y') = (x + D(x, y), y)\]
Example: Stereo to Depth Map
Random dot stereograms

• Julesz 1960: Do we identify local brightness patterns before fusion (monocular process) or after (binocular)?

• To test: pair of synthetic images obtained by randomly spraying black dots on white objects
Random dot stereograms
Random dot stereograms
Random dot stereograms
A Cooperative Model  (Marr and Poggio, 1976)
A random dot stereogram. These two images are derived from a single array of randomly placed squares by laterally displacing a region of them as described in the text. When they are viewed with crossed disparity (by crossing the eyes) so that the right eye’s view of the left image is combined with the left eye’s view of the right image, a square will be perceived to float above the page. (See pages 210–211 for instructions on fusing stereograms.)
Random dot stereograms

- When viewed monocularly, they appear random; when viewed stereoscopically, see 3d structure.

- Conclusion: human binocular fusion not directly associated with the physical retinas; must involve the central nervous system.

- Imaginary* “cyclopean retina” that combines the left and right image stimuli as a single unit.

*This was because it was as though we have a cyclopean eye inside our brains that can see cyclopean stimuli hidden to each of our actual eyes.
Autostereograms

Exploit disparity as depth cue using single image

(Single image random dot stereogram, Single image stereogram)

Images from magiceye.com
Autostereograms

Images from magiceye.com
Optical flow
Where do pixels move?
Optical flow

Where do pixels move?
Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923