Detection and Characterization of Damage in Beams via Chaotic Excitation

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August 14, 2009

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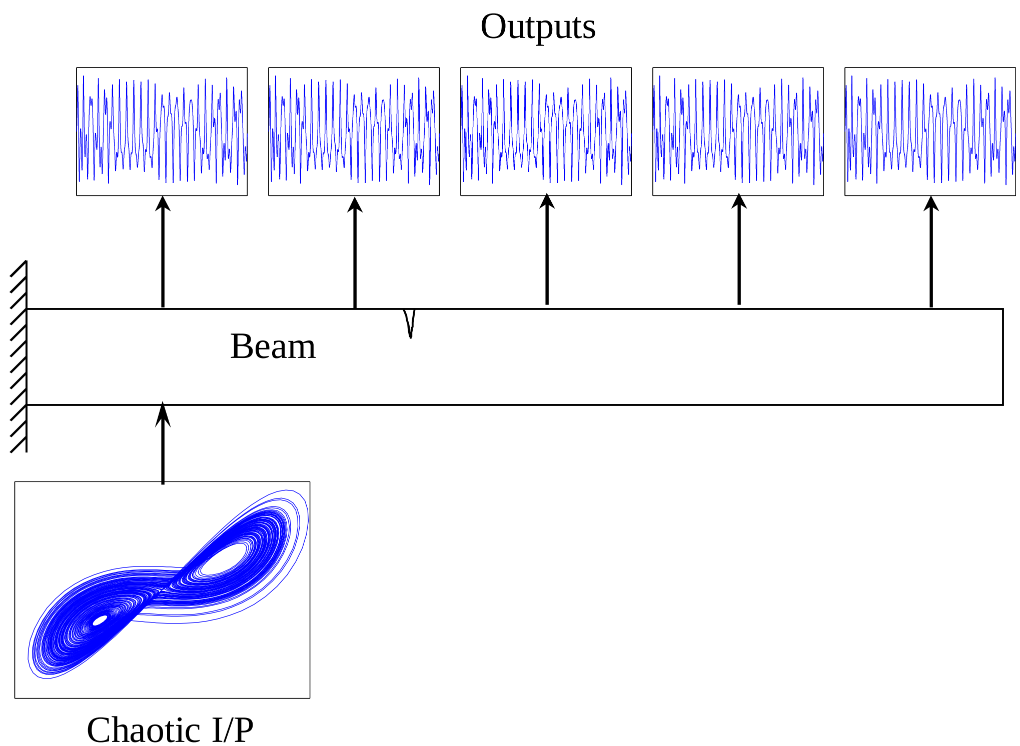
Problem Definition

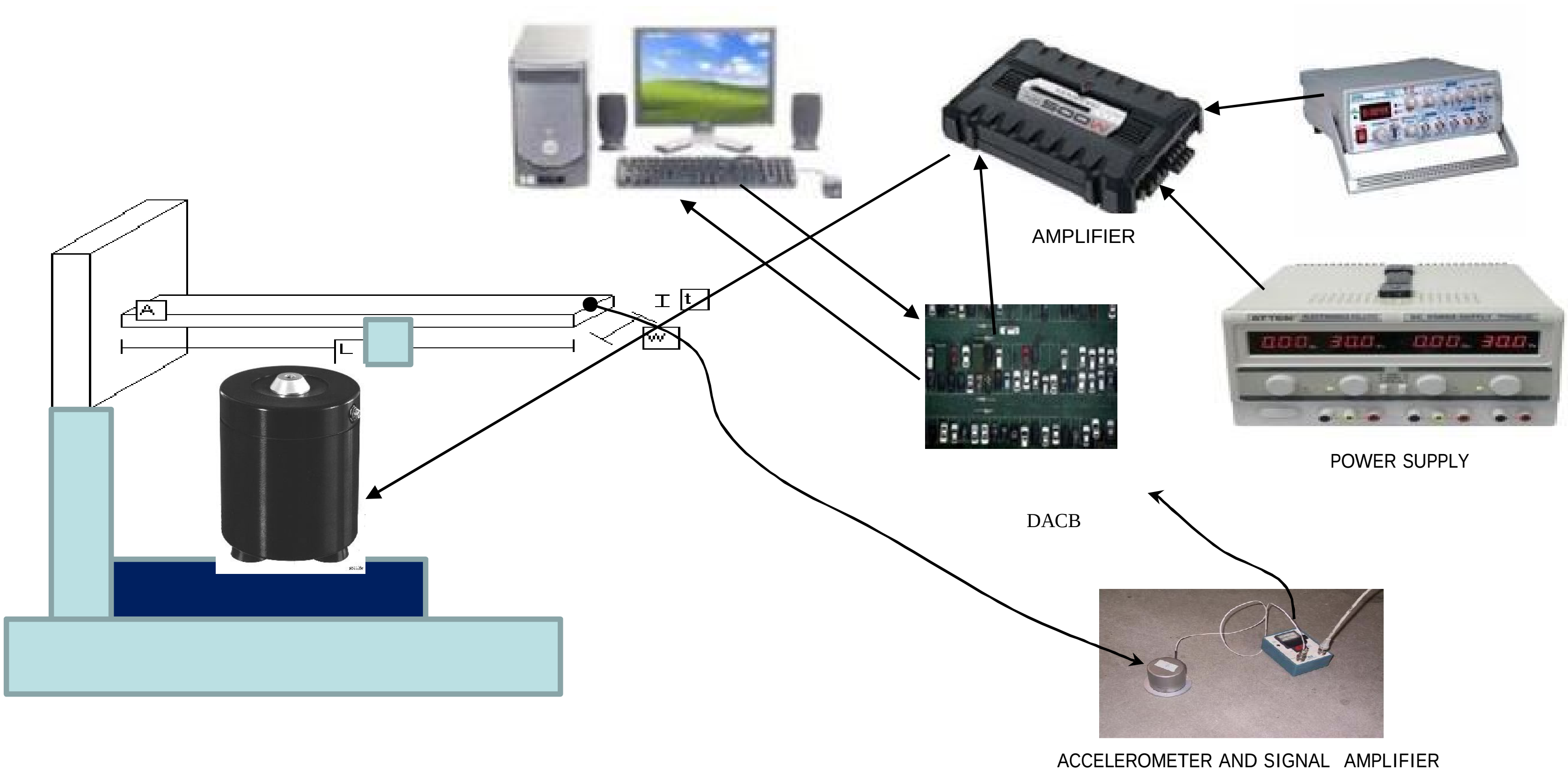


- Apply chaotic excitation
- Record O/P response
- Detect crack/damage magnitude
- Location of the crack/damage in the beam

Software and Tools

- Matlab and Simulink
- QuaRC and DACB
- Shaker
- Power amplifier
- Accelerometer and signal amplifier
- Function generator
- Power supply
- Test specimen
- Connecting cables etc.

Experiment Set-up



Chaotic Input Signal-1

Duffing equation

$$\ddot{x} + c\dot{x} - k_1 x + k_2 x^3 = F\cos(\omega t)$$

State space form

$$\dot{y}_1 = y_2$$

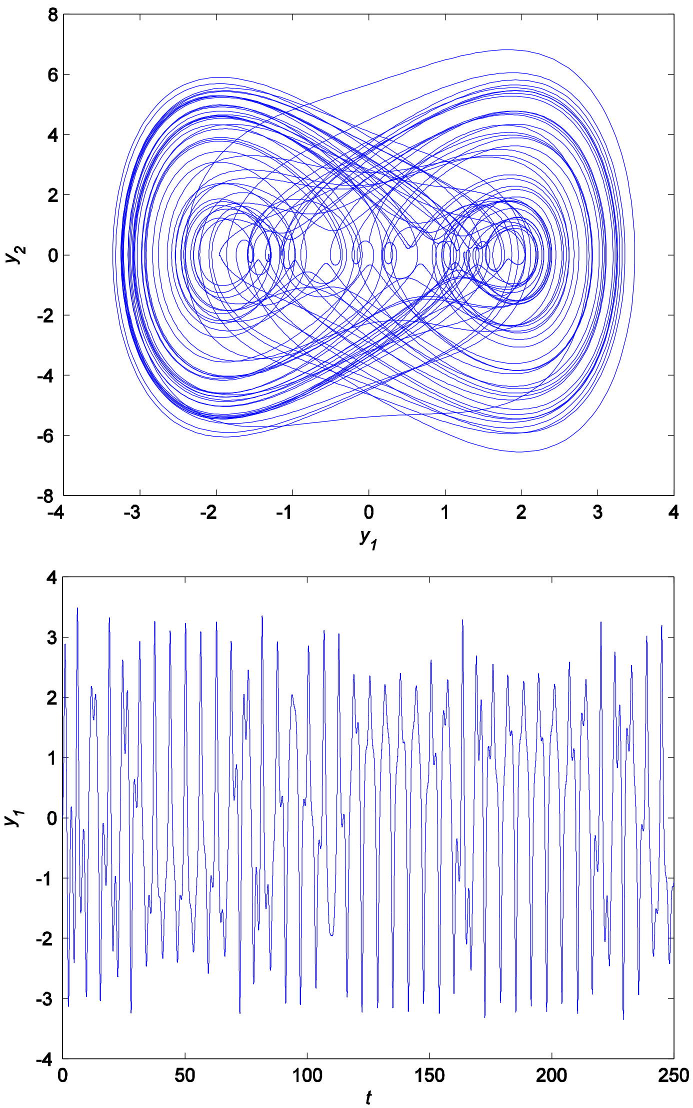
$$\dot{y}_2 = F\cos(y_3) + k_1 y_1 - k_2 y_1^3 - c y_2$$

$$\dot{y}_3 = \omega$$

Parameters[1] used

$$c = 0.05, \quad k_1 = 0, \quad k_2 = 1, \quad F = 7.5, \quad \omega = 1$$

$$y_1(0) = 0, \quad y_2(0) = 0.4, \quad y_3(0) = 0$$



Chaotic Input Signal-2

- Original duffing signal has very low effective frequency content
- The structure was practically stationery to the signal
- To overcome this problem and also to preserve the signal characteristics, original duffing equations are scaled as,

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = \frac{F}{\alpha^2}\cos(y_3) + \frac{k_1}{\alpha^2}y_1 - \frac{k_2}{\alpha^2}y_1^3 - \frac{c}{\alpha}y_2$$

$$\dot{y}_3 = \frac{\omega}{\alpha}$$

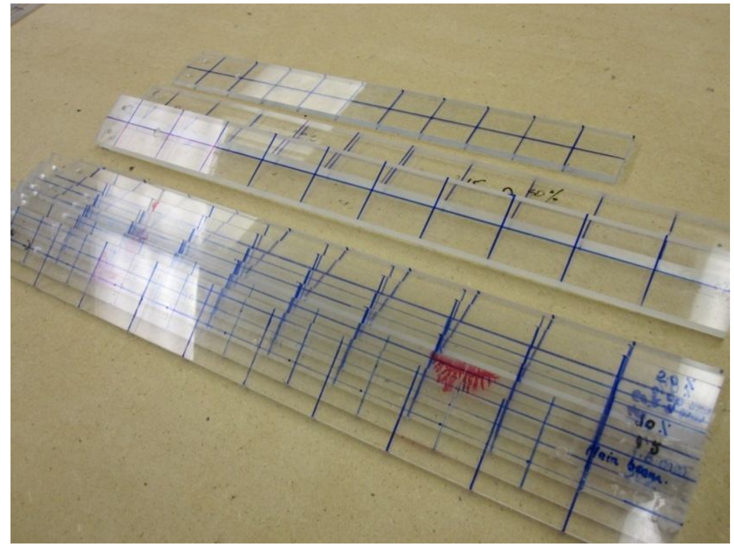
where α is scaling parameter

$\alpha = 0.25$ in our case

Test Specimen

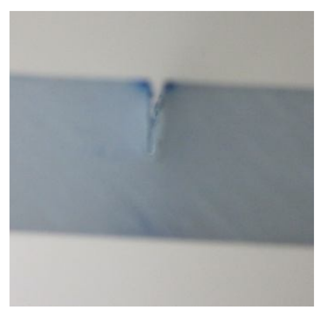
General Properties

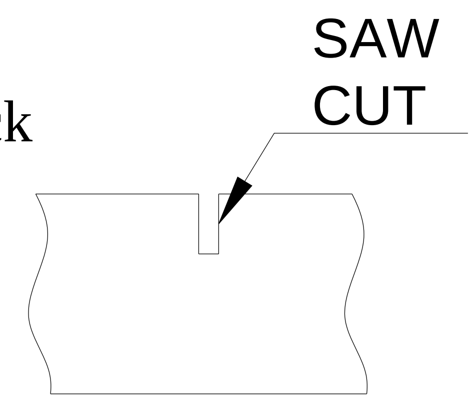
Material: Plexiglass
Width: 50 mm
Thickness: 5 mm
Young’s modulus: 3.3e9 Pa
Density: 1190 Kg/m3
Poisson’s ratio: 0.35



Emulating Crack

- Beams samples are cut on their top surface
- A thin saw cut was used to emulate the crack

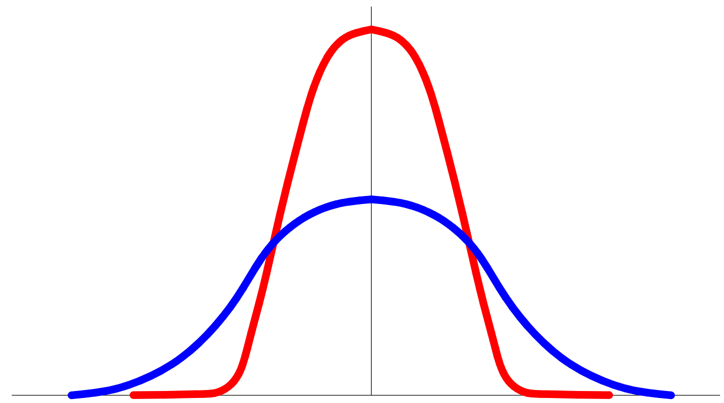




Statistical Characteristics

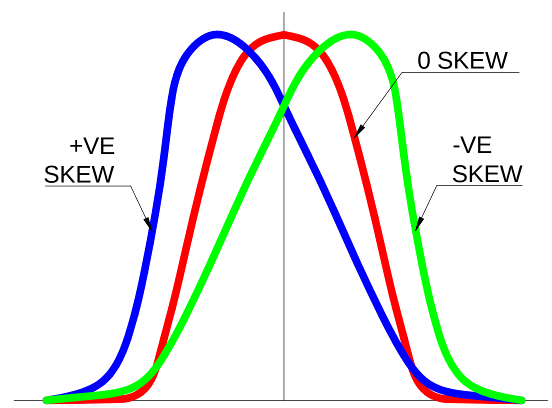
(i) Standard Deviation

$$\sigma = \left(\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2 \right)^{\frac{1}{2}}$$



(ii) Skewness

$$\text{skewness} = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^3}{(n-1)\sigma^3}$$



Chaotic Characteristics-1

a) Lyapunov Exponent (LE)

+Ve value of LE indicates that data is chaotic. It is calculated as,

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln \frac{|\delta Z_i|}{|\delta Z_0|}$$

λ = Lyapunov exponent

δZ_0 = Initial separartion

b) Correlation Dimension (CD)

Measure of the dimensionality of the space occupied by a set of points. Given as,

$$CD = \lim_{x\to\infty} \frac{\log C(r)}{\log(r)}$$

$C(r)$ = Correlation sum

r = radius selected for calculation

Chaotic Characteristics-2

c) Wave Fractal Dimension (FD)

Fractal dimension applicable for waveforms

Always lies between 1 and 2 and calculated as,

$$FD = \frac{\log(n)}{\log(n) + \log(\frac{d}{L})}$$

$d =$ diameter estimate

$=$ max dist(1,i)

$L =$ total length of curve

$n =$ number of steps in curve, $L / \bar{\alpha}$

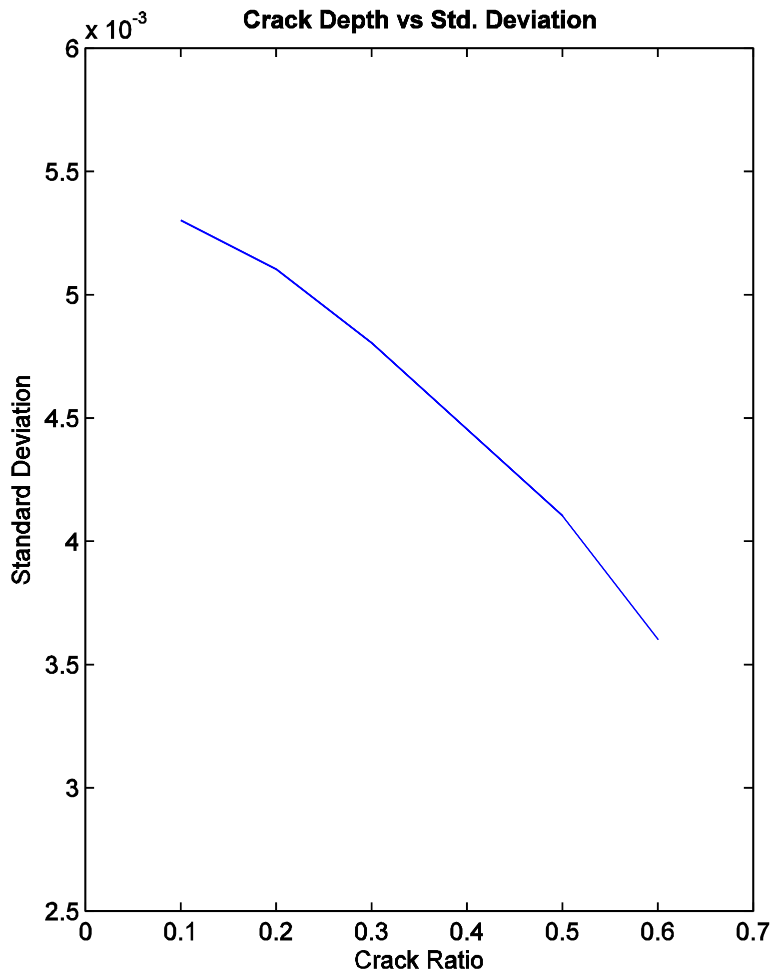
$\bar{\alpha} =$ average step

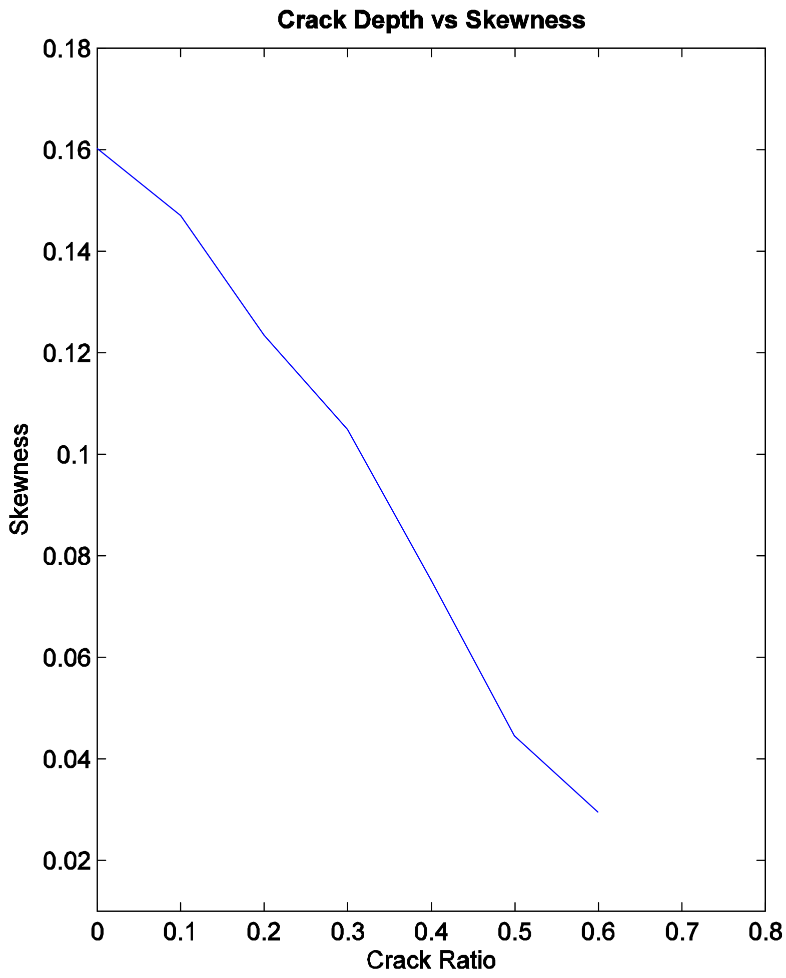
Experiment Notes

- The experiments are performed in various sets based on the crack location, beam length and beam thickness.
- The results that would follow is for the following set,

Sample No.	Length (mm)	Width (mm)	Thk (mm)	Crack Loc. (from support)	Crack Depth (%)
1	500	50	6	100	0
2	500	50	6	100	10
3	500	50	6	100	20
4	500	50	6	100	30
5	500	50	6	100	40
6	500	50	6	100	50
7	500	50	6	100	60

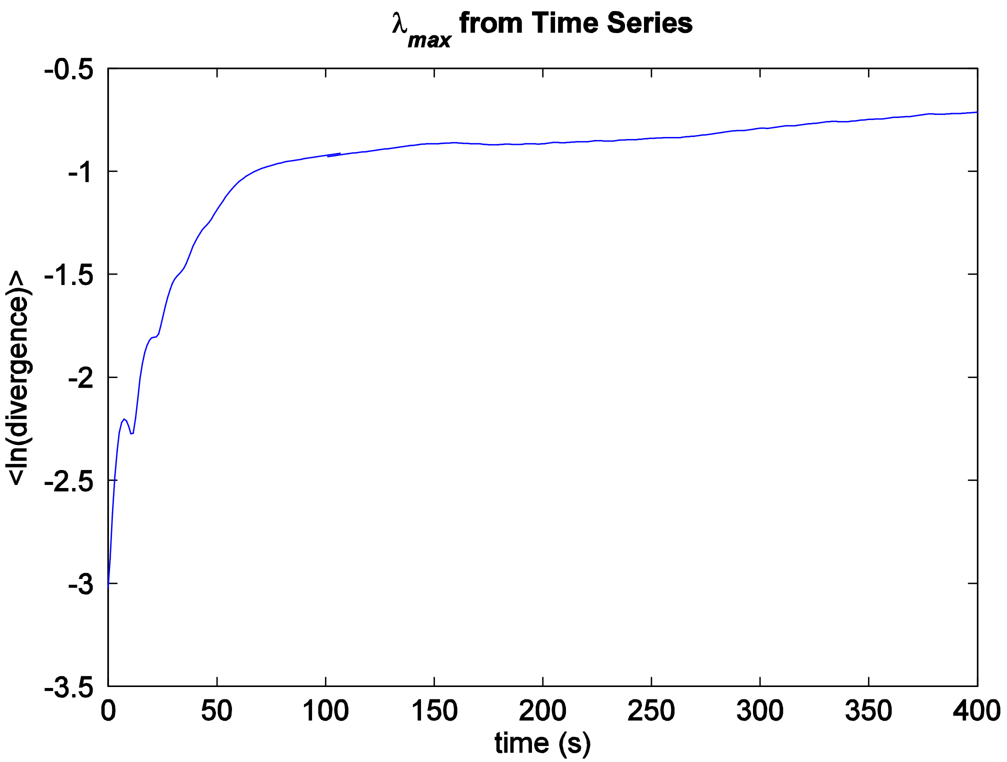
Results-1
Standard Deviation and Skewness





Results-2
Lyapunov Exponent (LE)

- LE has to be calculated before performing any chaotic characterization of time series.
- Slope of the extended region in the graph is LE.
- This analysis was performed for each time series data recorded.

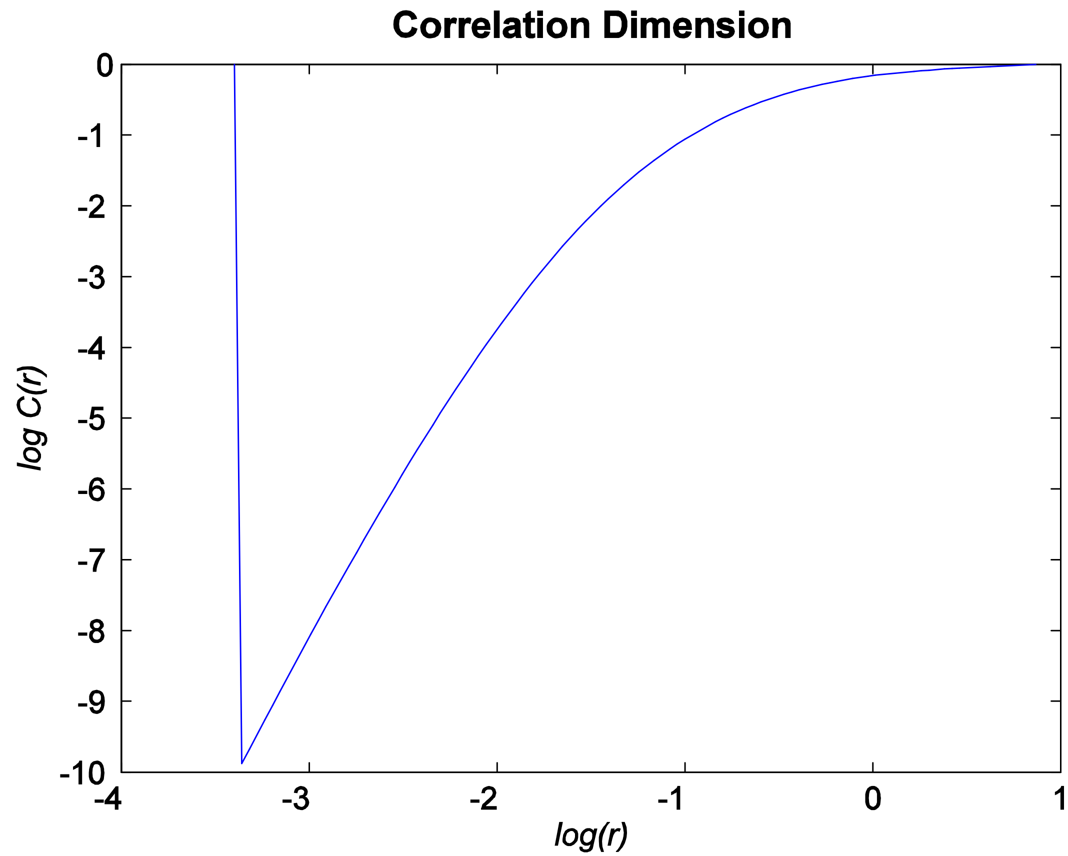


Graph corresponds to Sample no. 1.

$$\lambda = 0.4581$$

Results-3
Correlation Dimension (CD)

- CD is calculated using recorded time series.
- In the figure below, slope of the linear portion corresponds to CD

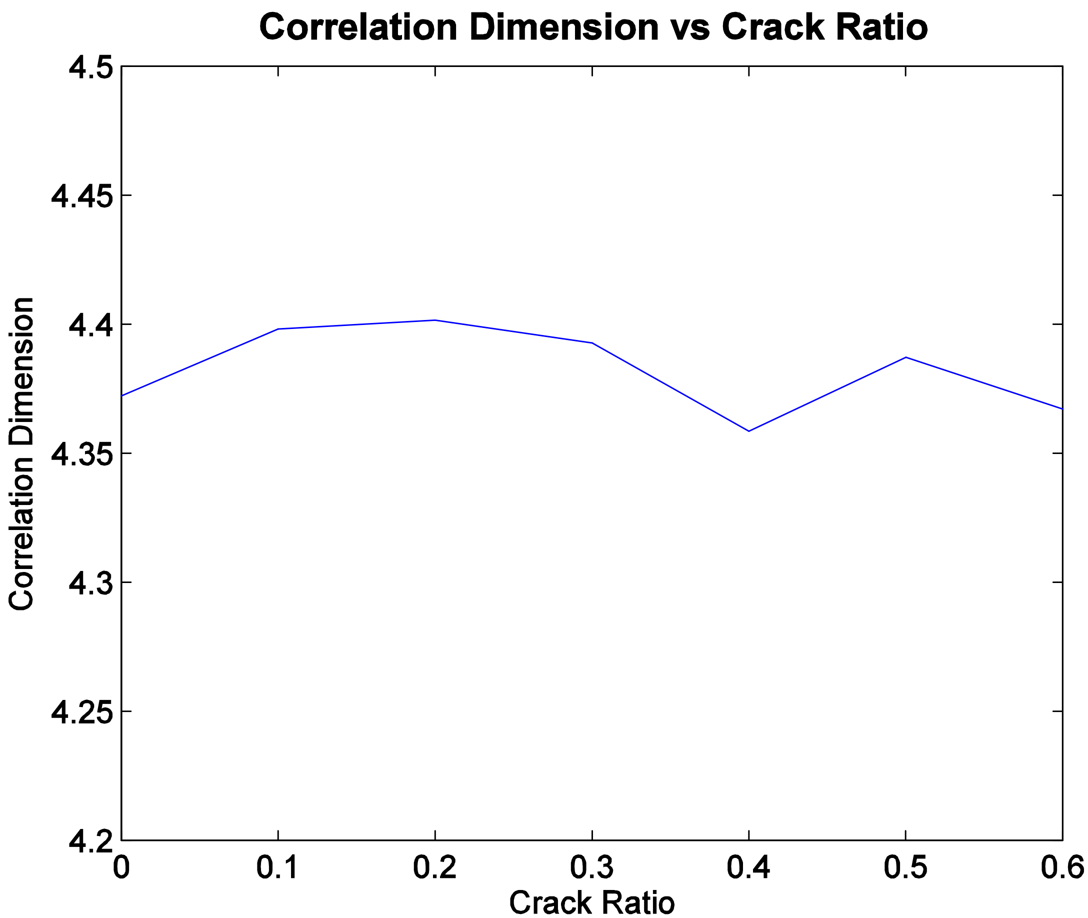


Graph corresponds to Sample no. 1.

$CD = 4.3916$

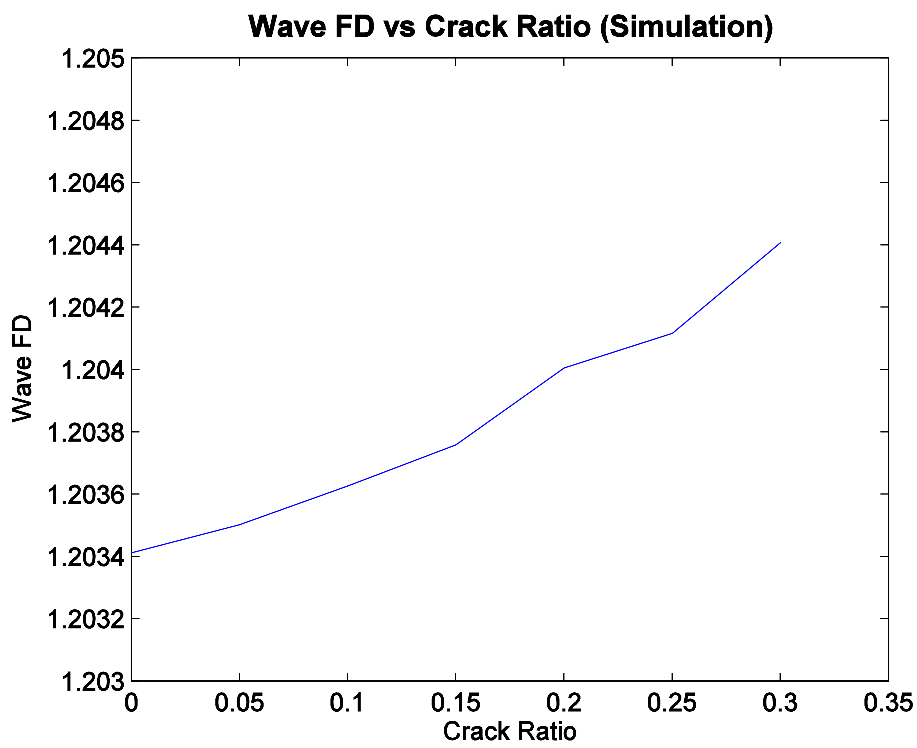
Results-4
Correlation Dimension (CD)

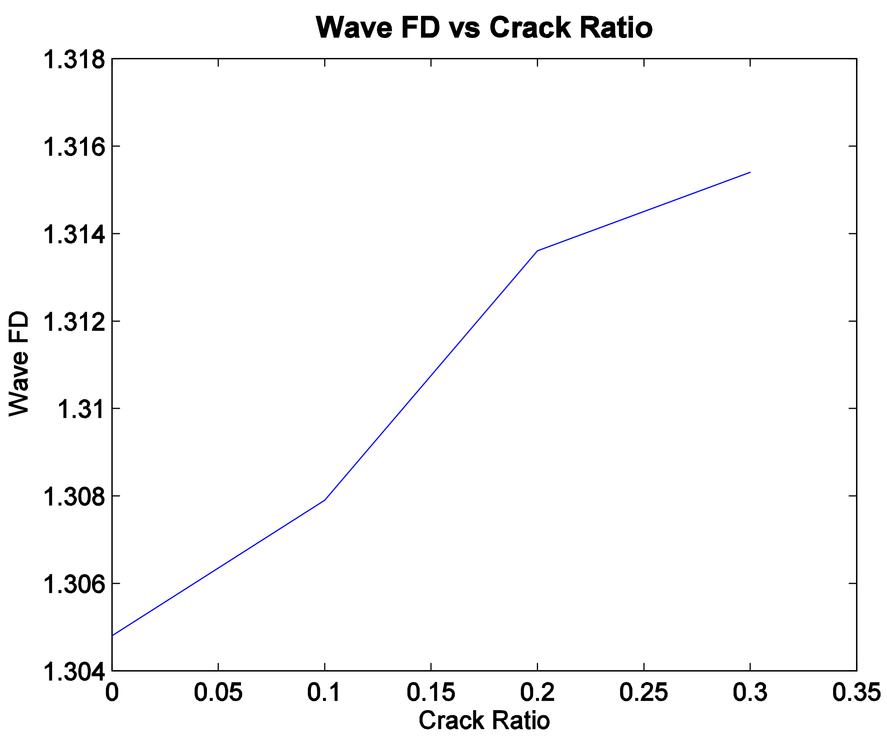
- CD is not a good measure of crack severity



Results-5
Wave Fractal Dimension (FD)

- Wave Fractal dimension was calculated by simulation and also recorded from the experiment.
- Results show proportionate increase





Conclusion

- The experiments for crack detection in beam structure are performed and results are analyzed
- A proportionate increase in SD and Skewness of recorded data with respect to increasing crack depth was observed
- +Ve Lyapunov Exponents were obtained indication that beam vibration is in chaotic mode due to excitation
- Correlation dimension was found to be unreliable measure of crack detection
- Wave fractal dimension of time series was found to be proportionately increasing with crack depth indicating its potential use to detect crack

Future Work

- Major part missing here was crack location in the beam. This at present time requires lot of data collection and would be done in future.
- Several other chaotic characteristics from literature can be thought to be applied to study their behavior.

Acknowledgement

I wish to thank my supervisors, Dr. Virkram Kapila, at NYU–POLY Institute, and Mr. Chandresh Dubey for their support and guidance throughout this Project. I would also like to extend my sincere thanks to Mr. Alex Betti, who is in charge of the machine shop, for his time and efforts during the experimental testing and his help in creating specimens for this research. Finally, I wish to thank my partners, engineering students whose patience, support and encouragement helped me during the six weeks we spent in these on going projects.

Experiment Set-up