1 Introduction

Nonlinear system identification problems have led to the creation of a variety of characterization methodologies. The use of chaotic response for identification is one of them. Chaotic response of dynamical systems has fascinated researchers for many years. However, the use of chaotic time series for system identification is relatively new and only a few investigations have been reported.

Yuan and Feeny [1] use the technique of combining the periodic orbit extraction and harmonic balance (HB) scheme for the parametric identification of chaotic systems. Though a chaotic time series is aperiodic, there exist several unstable periodic orbits embedded in the chaotic attractor. This technique has been previously applied to the identification of an experimental magneto-elastic oscillator [2]. Auerbach et al. [3] suggested a method for extracting periodic orbits from chaotic time series. A method of obtaining periodic orbits from an experimental chaotic data is given by Lathrop and Kostelich [4]. They used a reconstructed attractor from a scalar time series and obtained a histogram of attractor points for different recurrence times at equal time steps. Nichols and Virgin [5] have used chaotic excitation as the system input for the identification of linear systems. Pecora and Carrol [6] have studied the idea of driving systems with chaotic signals. They focus on drive decompositions and stability issues.

From the literature review, one basic question that arises is whether a chaotic signal can be used as an excitation source for nonlinear system identification. In this work, the feasibility of this is demonstrated for two specific single-degree-of-freedom (SDOF) nonlinear systems—a system with a quadratic nonlinearity and a Duffing-type nonlinearity. The chaotic response of a known system is scaled and is used as the force for the excitation of these nonlinear systems. The system response is assumed to be “chaotic,” with many periodic orbits similar to the input signal. Nearly periodic orbits (both displacement and velocity) are extracted from the system response using certain criteria and following this, a Fourier series identification method (FSIM) is employed for the parametric identification of the system. This method is shown to produce good results for the systems considered.

This section is followed by an overview of the FSIM. Section 3 briefly summarizes the identification procedure using chaotic response available in the literature, following which the new proposition of using a chaotic excitation is developed. Section 4 gives a detailed illustration of the proposed scheme for the identification of a system with quadratic damping and a system with a Duffing-type nonlinearity. The final section summarizes the work and discusses some of the implementation issues that need to be explored further.

2 Fourier Series Identification Method

This section will review the use of FSIM as outlined by Narayanan et al. [7]. This method is similar to the HB method given by Yasuda et al. [8]. Consider vibratory systems governed by nonlinear ordinary differential equations and subjected to harmonic force excitation. Further, it is assumed that the response of the system is periodic. It is assumed that the simulation/experimental data of the excitation force \( f(t) \) and the displacement response \( x(t) \) over a steady state period is available for analysis. In the FSIM, the periodic response of the system is expressed in terms of a truncated Fourier series with terms having frequencies, which are integer multiples/submultiples of the excitation frequency. The coefficients of the harmonic terms are obtained such that the resulting time series matches with the original one to the required degree of accuracy. As the system is nonlinear, a closed form solution is not available, in general. Instead, the Fourier series solution obtained for the response can be used for system analysis as well as system identification.

In the identification scheme, the Fourier series is substituted into the governing equations of motion, yielding algebraic equations in terms of system parameters. Imposing the condition that this equation should be satisfied at all sample points in the time domain, yields a system of linear algebraic equations. This is solved using a pseudo-inversion technique to obtain the unknown system parameters.

2.1 Identification Using FSIM. To illustrate the procedure, the harmonically forced oscillator with quadratic damping nonlinearity given by the following equation is considered:

\[
\ddot{x} + c \dot{x} + kx = f(t)
\]  

where \( m, c, \) and \( k \) are the mass, quadratic damping coefficient, and linear stiffness, respectively. They are the parameters of the system to be determined from the known input-response data. For identification purpose, the oscillator is assumed to be excited with
known harmonic/periodic excitation \( f(t) \). Assume that the response \( x(t) \) of the vibrating system is known and is periodic with a fundamental period \( T = 2\pi/\Omega \), where \( \Omega \) is the fundamental frequency of excitation. Expressing the response in a truncated \( M \) term harmonic Fourier series one gets \( x(t) \) as

\[
x(t) = a_0 + \sum_{j=1}^{M} \left\{ a_j \cos(j\Omega t) + b_j \sin(j\Omega t) \right\}
\]

(2)

Substituting this periodic solution in Eq. (1), one obtains

\[
\begin{align*}
-m\Omega^2 & \sum_{j=1}^{M} j^2 [a_j \cos(j\Omega t) + b_j \sin(j\Omega t)] + c\Omega^2 \sum_{j=1}^{M} [j - a_j \sin(j\Omega t) + b_j \cos(j\Omega t)] \\
+ b_j \cos(j\Omega t) \left( \sum_{j=1}^{M} j^2 - \sum_{j=1}^{M} a_j \sin(j\Omega t) + b_j \cos(j\Omega t) \right) \\
+ k \left\{ a_0 + \sum_{j=1}^{M} [a_j \cos(j\Omega t) + b_j \sin(j\Omega t)] \right\} \\
= F \cos(\Omega t)
\end{align*}
\]

Equation (3) can be written compactly as

\[
mp_1(t) + cp_2(t) + kp_3(t) = p_4(t)
\]

(4)

where

\[
\begin{align*}
p_1(t) &= -\Omega^2 \sum_{j=1}^{M} j^2 [a_j \cos(j\Omega t) + b_j \sin(j\Omega t)] \\
p_2(t) &= \Omega^2 \sum_{j=1}^{M} [j - a_j \sin(j\Omega t) + b_j \cos(j\Omega t)] \\
p_3(t) &= \sum_{j=1}^{M} [j - a_j \sin(j\Omega t) + b_j \cos(j\Omega t)] \\
p_4(t) &= x(t)
\end{align*}
\]

(5)

where for uniformity of notation, \( p_3(t) \) is used for \( x(t) \). For a set of \( N \) discrete, equally spaced time samples in one excitation time period \( T \) the matrix form of the above equation is given by

\[
\begin{bmatrix}
p_1(0) & p_2(0) & p_3(0) \\
p_1(\Delta t) & p_2(\Delta t) & p_3(\Delta t) \\
\vdots & \vdots & \vdots \\
p_1((N-1)\Delta t) & p_2((N-1)\Delta t) & p_3((N-1)\Delta t)
\end{bmatrix}
\begin{bmatrix}
m \\
c \\
k
\end{bmatrix}
\]

(6)

Equation (6) can be written compactly as \([G]r = f\), where \( r \) is the parameter set, \( [r] = [m \ c \ k]^T \), with the superscript \( T \) indicates a transpose. The identified parameter set \( [r] \) is obtained as:

\[
[r] = [G]^+ f
\]

(7)

where \([G]^+\) is the pseudo-inverse of \([G]\). The pseudo-inverse is equivalent to the minimization of \( \|G[r] - f\|_2 \). The formulation similar to the above can be applied to other types of systems having smooth nonlinearities.

2.2 Direct Determination of Error in Parameters. If the identification is carried out using the simulated data as mentioned earlier, the parameters of the original and the identified system are available for error estimation. Error measures based on difference in parameter values for the quadratic damping oscillator can be defined as

\[
m_c = (m - m_c)/(c - c) \text{ and } k_c = (k - k_c)/k
\]

(8)

where subscript \( \text{c} \) represents the identified parameters, \( m_c, c_c \), and \( k_c \) are the normalized errors in the identification of mass, quadratic damping, and linear stiffness, respectively. The total parameter error \( E_p \) is computed as

\[
E_p = \sqrt{[m_c^2 + c_c^2 + k_c^2]}/n_p
\]

(9)

where \( n_p \) is the number of parameters, which is three in this case. One of the most commonly used error norms, in system identification, is the root mean square (RMS) error in the response, which is defined as

\[
E_r = \sqrt{1/ \int_0^T (x_i(t) - x(t))^2 dt}
\]

(10)

where \( x_i(t) \) is the response based on the identified parameters. This error will not enable us to judge the robustness of the proposed algorithm as the pseudo-inverse used in the identification leads to a least square error minimization of algebraic sum of the forces in the governing equation. In an experimental scenario, only a measurable norm like \( E_r \) is available for checking the correctness of the result. However, in the development of this algorithm, \( E_p \) can be estimated since the study is done with a known input, output as well as system parameters. This can be used to check the quality of the identification algorithm itself.

2.3 Implementation. If the periodic response of a system is given, an accurate signal decomposition can be done by performing a discrete Fourier transform (DFT) on the response data. In practice, a fast Fourier transform (FFT) is used. The FFT coefficients, which are complex, may be converted into equivalent real valued Fourier series coefficients. Following the Fourier series fit, the system identification by pseudo-inverse as mentioned above can be done.

To generate data for the study, the periodic response of a known system to a harmonic excitation is obtained by numerical integration in MATLAB. The above data is transformed to the frequency domain, to obtain the Fourier series solution to the problem. Using the above input-output data again, the FSIM is made use of to identify the parameters of the system. The number of sampling points \( N \) is chosen to ensure that the frequency range of interest is covered; in this paper it is chosen to be 128. The equally spaced sampling time interval is \( \Delta t = T/N \). The number of harmonics chosen is usually data dependent. To keep out relatively insignificant Fourier coefficient terms, a fairly low approximation error, of the order of \( 10^{-6} \), has been used. The effect of changing the approximation error, on the number of significant Fourier series terms, has not been investigated in this paper.

3 Identification Using Chaotic Excitation: A New Proposal

In this work, a new method of identification is proposed in which the system is excited by a force, which is derived from the chaotic response of a known system. The basic proposition of this kind has been reported earlier (Narayanan et al. [7]) and the idea is reviewed in this section and a more extensive study of the same is conducted in this paper. Consider the case of a periodic input, with the response of the nonlinear system being chaotic. Nearly periodic orbits embedded in the chaotic attractor can be extracted. The HB method of system identification can be applied on the any one of the extracted periodic orbits [1], where the periodic input force has to be phase-matched with that of the extracted output signal. Instead of a frequency domain method, a time domain based FSIM can also be used. In contrast to the periodic response, simultaneous measurement of both the displacement and velocity...
responses are required for obtaining the periodic orbits in the case of chaotic response. Thus, the FSIM explained in Sec. 2 can be used for parametric identification. However, the disadvantage of the identification using chaotic response is its limited application to cases where the system response is chaotic. It is difficult to predetermine a harmonic or a periodic excitation, which will yield a chaotic response. However, a chaotic excitation usually yields chaotic response irrespective of the type of system including a linear system.

A block diagram of the identification procedure is shown in Fig. 1. A harmonic force input \( f_x(t) \) is applied to a known system \( G_n \). The excitation and the system parameters of \( G_n \) are chosen such that the response \( x_0(t) \) is chaotic. The chaotic displacement of the system is scaled (multiplied) by a suitable factor \( x_c \) and it is denoted by \( f_x(t) \). The \( f_x(t) \) is given as the force excitation for system \( S \) to be identified, which gives a response, \( x(t) \).

It can be verified that the response of the system \( S \) is chaotic. The nearly periodic orbits contained in the state space of system \( S \) are extracted using the recurrence property of the chaotic attractor. The chaotic data set is scanned to locate the occurrence of periodic orbits satisfying the following condition:

\[
\|y_{1,k+1} - y_1\| \leq e_1 \quad \|y_{2,k+1} - y_2\| \leq e_2 \tag{11}
\]

where \( y_1 \) and \( y_2 \) refer to the state variables of system \( S \) \((y_i \text{ is same as } x \text{ and } y_2 \text{ is } x_i)\), \( i \) is any sample point in the orbit, \( k \) is the number of sample points in a period, \( e_1 \) and \( e_2 \) are chosen tolerances or gaps in the periodic orbit for the respective state variables. The tolerances are obtained as a prescribed fraction \( \rho \) of the size of the attractor. Some details on the determination of \( \rho \) are given later in Sec. 4.1. Let \( \delta_1 = |y_{1\text{max}} - y_{1\text{mid}}| \), \( \delta_2 = |y_{2\text{max}} - y_{2\text{min}}| \), where \( \delta_1 \) and \( \delta_2 \) give the size of the attractor. The tolerances are given by \( e_1 = \rho \delta_1 \) and \( e_2 = \rho \delta_2 \).

Starting from a reference point \((y_{1,j}, y_{2,j})\) in the trajectory, scanning of the points which are ahead in time is carried out, checking each time whether the new point is in the neighborhood of the reference point, based on the tolerance criteria given in Eq. (11). If these criteria are satisfied, then an orbit is captured, its initial and end points completely describe the orbit since the orbit is deterministic if the tolerance criteria are not satisfied, one proceeds to the next point. The process is continued until the end of the data set. Now the procedure is repeated with the next reference point in the forward direction until the end of the orbit. This will yield a large number of periodic orbits, out of which many of them may be close. Some suitable criterion has to be developed to eliminate nearly identical orbits to make the periodic orbit set clearly distinct and this is discussed later in Sec. 4.1.2.

The chaotic orbits are characterized by the recurrence property, i.e., points once visited in the state space are approached again and again very closely, enabling the capturing of periodic orbits. This property is also described in terms of stretching and folding of the chaotic attractor. The periodic orbits are known to be unstable; however for the purpose of identification these orbits are sufficient. A chaotic response is broad banded, similar to a random signal; however it is deterministic. This implies that chaotic attractors are low dimensional when compared to a random signal with similar frequency characteristics. On the other hand, capturing nearly periodic orbits is difficult in the case of random excitation.

If a system is periodically forced, and the response is chaotic, the periods of the response of the extracted periodic orbits occur as integer multiples of the forcing period. In contrast, since the system considered in the present work is chaotically forced, different periods exist which are not integer multiples of the period of harmonic excitation \( f_x(t) \); many of the orbits have periods not equal but close to the integer multiple of the period of \( f_x(t) \). For any selected periodic orbit, the corresponding portion of the chaotic input is chosen for the identification and this will constitute the input-output data to be used in the identification using Eq. (7). Now, the algorithm for parametric identification using chaotic excitation is outlined below:

(i) Use the identification scheme as shown in Fig. 1 and obtain the steady state input-output data \( \{f_x(t); y_1(t), y_2(t)\} \), from the full data set by removing the transient portion.

(ii) Extract nearly periodic orbits from the state space of system \( S \). Pick any one of the extracted periodic orbits and let the sequential discrete sample points of \( x(t) \) in it be \( \{x_i\}_p \).

(iii) Obtain the data set \( \{f_{x_i}\}_p \) from the input \( \{f_x(t)\} \) that corresponds in time with the set \( \{x_i\}_p \). The \( \{(f_{x_i}, x_i)\}_p \) set constitutes a mapping pair, which is related by Eq. (6).

(iv) Carry out the DFT of \( \{x_i\}_p \). The DFT coefficients are converted into an equivalent Fourier coefficients set. These Fourier coefficients are to be used in Eq. (7) for obtaining the unknown parameters.

4 Examples

Two examples are used to show the feasibility of the method. In both the cases the chaos generator is chosen as a system with a Duffing nonlinearity, given by the following equation:

\[
m\ddot{x} + c_1\dot{x} + k_x x + \alpha_x x^3 = f_x(t) = F\cos(\Omega t) \tag{12}
\]

The following parameter values are chosen as given in Yuan and Feeny [1]: \( m = 1, c_1 = 0.2, k_1 = 1, \alpha_t = 1, F = 27 \), and \( \Omega = 1.33 \). This gives a chaotic response and in the simulation, data points are sampled with time steps of size \( dt = 2\pi/NO \). The initial 50 cycles of \( f_x(t) \) are skipped to get the steady state data. The response starting from cycle 51 until cycle 200 is considered for the analysis. The phase plane plot of the system \( G_n \) is shown in Fig. 2(a). The state space variables of system \( S \) is denoted as \( y_1 \) and \( y_2 \) and that of system \( G_n \) is denoted as \( y_3 \) and \( y_4 \) respectively. The corresponding Poincaré section points are denoted by a suffix \( p \) in figures. The Poincaré section of system \( G_n \) with sampling period as the period of \( f_x \) is shown in Fig. 2(b) for 51–2000 cycles of \( f_x(t) \). The length of the orbit used in determining Poincaré section is much longer than that for periodic orbit extraction, as this is necessary for determination of fractal dimension of the chaotic attractor. For identification purpose alone, the period length as in Fig. 2(a), would be sufficient. The correlation dimension \( D_n \) of the fractal set in Fig. 2(b) is determined [9] to be 1.31. The two systems considered with the above response as an excitation are (i) system \( S \) having a quadratic nonlinearity and (ii) system \( S \) having a Duffing-type nonlinearity.

4.1 Quadratic Damping Nonlinearity. Consider a system \( S \) having a quadratic damping nonlinearity given by Eq. (12). The parameters to be identified are \( m, c_1, \text{ and } k_1 \). The first row of Table 1 gives the system parameter values and the \( x_1 \) value used in the simulation. The phase plane of the chaotic response of the system \( S \) for the full range of analysis period is shown in Fig. 3(a), and the Poincaré section in Fig. 3(b). The correlation dimension for the system \( S \) is 1.37.

Now the nearly periodic orbits are extracted from the chaotic attractor shown in Fig. 3(a) using the tolerance criteria given by Eq. (11). For this, some trials are needed to fix the value of the fraction for the tolerance, \( \rho \). The criterion used in the trials is that it should yield sufficiently large number of periodic orbits. A large
value of $\rho$ will lead to an error due to nonperiodicity in the orbit whereas too small a value for $\rho$ may not yield any periodic orbits. Furthermore, for longer lengths of the chaotic orbit (data set), smaller values of $\rho$ will yield a sufficient number of orbits. Thus the choice of $\rho$ depends on the type of the system, excitation as well as the duration of the time series of the chaotic signal available. With these considerations, the value of $\rho$ was taken to be $2 \times 10^{-4}$ for both the examples discussed here.

A large number of periodic orbits with different periods are available in the chaotic data. The period of any orbit is expressed as a multiple of the fundamental period of harmonic excitation. The periodic orbits are arranged in the ascending order of the periods and the orbit period variation is shown in Fig. 4. Out of the several extracted periodic orbits available for identification, any one of them can be picked up for identification.

As an illustration, consider an extracted orbit as shown in Fig. 5. The first and the last time index for this orbit are (2949, 3847) and it has an orbit period given by (3847–2949)/128 = 7.0156 and the orbit closure gap is 0.0845. The time index ranges from the 51 to 200 cycles of steady state, each cycle having $N=128$ points. The time and the time index are reckoned from the beginning of steady state motion. The orbit details are given in the last row of Table 1a and the identified parameters are given in the last row of Table 2. The error $E_p=0.0016$ implies that the identification is accurate. The number of harmonics in the Fourier series fit is equal to 448 and this high value is due to several factors, such as the extremely small truncation error used in the fit, discontinuity in the tangent as well as a small discontinuity of the orbit at the gap (shown as a square in Fig. 5).

### 4.1.1 Effect of Noise

A simplified study was done on the effect of noise on the identification results. The first orbit given in Fig. 2 Generator system response with (a) phase plane (cycles 51–200) and (b) Poincaré section (cycles 51–2000).
Tables 1 and 2 was considered for this. A uniformly distributed random noise, with a specified noise to signal RMS ratio, \( e_n \) was added to every column of the \([G]\) matrix for the above selected orbit and the identification is carried out with the noisy data. The resulting parametric error is denoted as \( E_p \). Figure 6 shows the variation of \( E_p \) over a range of \( e_n \). For small values of noise (\( e_n \leq 0.06 \)), the identification is robust.

4.1.2 Study by Varying Scale Factor \( s_c \). The scale factor \( s_c \) is varied over a range and the influence on the identification result is studied. Five sets of \( s_c \) over a wide range are used and for each case, the periodic orbits are extracted and ordered by period length; Fig. 7(a) shows the results. There are orbits of nearly identical periods in the figure. These extra orbits having nearly identical lengths/periods may be eliminated as they may be nearly close in the state space; thus they will not yield distinct data sets. A method to ensure that the orbits are distinct can be developed using a measure like the geometric center. If any extra orbits having nearly the same period, as well as nearly the same centroids based on a tolerance for the difference in the length of the orbits and the centroidal distance between them, are detected, they can be eliminated. Higher order measures such as second-order moment can also be used. In this study the orbits of nearly identical lengths were eliminated to obtain a set of orbits with monotonically increasing periods and it is shown in Fig. 7(b). The

**Table 2 Quadratic damping: identified parameters**

<table>
<thead>
<tr>
<th>Orbit index</th>
<th>( m )</th>
<th>( c )</th>
<th>( k )</th>
<th>( E_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0008</td>
<td>0.0501</td>
<td>1.0037</td>
<td>0.0027</td>
</tr>
<tr>
<td>3</td>
<td>1.9976</td>
<td>0.0500</td>
<td>0.9987</td>
<td>0.0016</td>
</tr>
</tbody>
</table>

**Table 3 Selected orbit periods; the first column gives orbit number in the reduced set (Fig. 7(b)); the other columns give corresponding original orbit number (Fig. 7(a)) with the period of the orbit in brackets**

<table>
<thead>
<tr>
<th>Case</th>
<th>( s_c = 0.1 )</th>
<th>( s_c = 100.1 )</th>
<th>( s_c = 200.1 )</th>
<th>( s_c = 300.1 )</th>
<th>( s_c = 400.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1.6797)</td>
<td>(1.1719)</td>
<td>(0.9609)</td>
<td>(2.0313)</td>
<td>(0.8359)</td>
</tr>
<tr>
<td>2</td>
<td>3(3.6250)</td>
<td>2(4.1328)</td>
<td>2(3.7891)</td>
<td>2(2.9922)</td>
<td>2(0.9688)</td>
</tr>
<tr>
<td>3</td>
<td>5(7.6719)</td>
<td>3(5.0078)</td>
<td>3(5.1641)</td>
<td>3(5.5469)</td>
<td>3(1.9531)</td>
</tr>
<tr>
<td>4</td>
<td>7(9.4603)</td>
<td>4(5.9844)</td>
<td>4(5.7344)</td>
<td>8(3.9922)</td>
<td>5(2.3672)</td>
</tr>
<tr>
<td>5</td>
<td>8(11.5313)</td>
<td>5(7)</td>
<td>6(6.8828)</td>
<td>10(5.1563)</td>
<td>6(2.9922)</td>
</tr>
</tbody>
</table>
first five orbits in each of the reduced sets are used to study the effect of \( s_c \) on the parametric error. Table 3 shows the list of orbits for the five values of \( s_c \) in the same order as in Fig. 7(a). The orbit numbers 1–5 are chosen from the reduced set shown in Fig. 7(b). The corresponding original orbit number, as in Fig. 7(a), along with the period of the orbit (in brackets) is given in the table. The variation of \( E_p \) with orbit number (period), for different values of \( s_c \), is shown in Fig. 8. Except for \( s_c=0.1 \) the identification of the parameter values is very good. This is because of the nonlinear effects being weak for low excitation amplitudes.

4.2 System With Duffing-type Nonlinearity. Now a system with a Duffing-type nonlinearity with system parameters given in Table 4, is considered. The generator system \( G_n \) is the same as considered earlier, with \( s_c=1 \). The response is obtained and the correlation dimension is calculated to be 1.75. Figure 9(a) shows a periodic orbit picked up for identification, while Fig. 9(b) is the corresponding excitation orbit. The details are given in Tables 4 and 5. Next, an orbit having 10 periods is extracted and the results are given in the last rows of Tables 4 and 5. The identification of the system parameters is good in both cases.

A study of identification with noise is carried out as explained in Sec. 4.1.1. The orbit number 10 with first and last time indices \( 8735, 9247 \) having a period 4 was chosen for identification with noise addition. In this case the identification is robust only for really small values of noise to signal ratios, i.e., \( e_n<0.01 \). The reasons for this are not clear and needs to be investigated further.

4.2.1 Identification With Noise and Effect of Variation of \( s_c \). As in the case of the quadratic nonlinearity system, the reduced sets after the elimination of nearly identical periods are obtained using the centroid criterion outlined in Sec. 4.1.2. Table 6 gives the list of orbits used for identification for each of the \( s_c \) values. The variation of \( E_p \) with orbit number, for the set of \( s_c \) values, is shown in Fig. 10. It can be seen that for \( 2.1\leq s_c \leq 6.1 \) the identification is quite successful. Again for low values of \( s_c \) the results are poor. Too low a value of \( s_c \) may not sufficiently excite the nonlinear term, whereas an \( s_c \) that is too large can result in a force generated by the cubic term which may even bury the presence of other terms, resulting in the poor inversion of the \( [G] \) matrix in Eq. (7).

In both examples, there are a large number of nearly periodic orbits, of different periods, in the extracted set, available for identification. A suitable choice of the orbits that yield minimum error is a problem that needs to be addressed. Intuitively one can state that very lengthy orbits may not yield good results due to the occurrence of redundant data in the sets. Moreover, long period

Table 5 Identified parameters for the system with Duffing-type nonlinearity

<table>
<thead>
<tr>
<th>Orbit number</th>
<th>( m )</th>
<th>( c )</th>
<th>( k )</th>
<th>( \alpha )</th>
<th>( E_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9999</td>
<td>0.0199</td>
<td>0.9994</td>
<td>0.0500</td>
<td>0.0015</td>
</tr>
<tr>
<td>20</td>
<td>0.9995</td>
<td>0.0200</td>
<td>0.9994</td>
<td>0.0500</td>
<td>7.61 \times 10^{-4}</td>
</tr>
</tbody>
</table>

Table 4 Orbit details of a system with Duffing-type nonlinearity

\(|r|: m=1, c=0.02, k=1; \alpha=0.05; s_c=1\)

<table>
<thead>
<tr>
<th>Orbit number</th>
<th>From</th>
<th>To</th>
<th>Period</th>
<th>Fundamental frequency</th>
<th>Harmonics</th>
<th>Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16379</td>
<td>16506</td>
<td>0.9922</td>
<td>1.3405</td>
<td>62</td>
<td>0.0011</td>
</tr>
<tr>
<td>20</td>
<td>11614</td>
<td>12894</td>
<td>10</td>
<td>0.1330</td>
<td>639</td>
<td>0.0024</td>
</tr>
</tbody>
</table>
orbits will need large number of harmonics for representing the signal and more time for finding the Fourier coefficients and a discrete Fourier transform (DFT) may be required as the length is unlikely to be a power of 2.

5 Conclusions

A new technique for nonlinear system identification is proposed in which a chaotic signal is used for system excitation. A system with quadratic damping and a Duffing oscillator are used for demonstration purposes. In conjunction with the FSIM, this proposition opens up an alternative paradigm for handling nonlinear system identification problems. There may be potential for better identification if a chaotic signal is used as the excitation signal for identification of nonlinear systems in more general cases. The exploratory nature of the chaotic signal is useful in revealing the system nature and this may be a good choice of excitation, even for a nonparametric identification.

It was indicated that simultaneous measurements of displacement and velocities (apart from the excitation force) are necessary for generating the data for identification. In the case of identification using a harmonic/periodic input with periodic response, only a displacement measurement will be sufficient. The need for an additional measurement is somewhat of a limitation of the proposal for experimental implementation. This can be overcome by measuring, say, velocity and the numerically integrating the result to obtain the displacement. For long records the integration errors are likely to become significant. However, if this extra measurement can be done or evaluated, the resulting orbits provide a rich data set which is not available from a normal periodic response. A Duffing oscillator was arbitrarily chosen as the generator for excitation. Instead any suitable chaotic signal can be directly used as an input. The scale factor $s_c$ has to be properly selected and will require some trials in the simulation. Increasing the scale factor $s_c$ amounts to imparting higher levels of excitation (this is analogous to changing the amplitude of excitation of a harmonic force, keeping the frequency unchanged) and in a nonlinear system, this will call for larger participation of various nonlinear terms/forces induced by different parameters of the system; this helps in the identification. However, the relative values of these forces induced by the parameters also affect the accuracy of identification. For instance, in a Duffing oscillator, a value of $s_c$ that is too low, may not sufficiently excite the nonlinear term, whereas an $s_c$ that is too large, can result in a force generated by the cubic term which may even bury the presence of other terms, resulting in the poor inversion of the $[G]$ matrix in Eq. (7). This needs to be investigated further.

### References


