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Controlling chaos in Colpitts oscillator

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Abstract

In this paper, an adaptive backstepping design is proposed to synchronize and control the Colpitts oscillator. The proposed control approach enables stabilization of chaotic motion to a steady state as well as synchronization by recursively interlacing the choice of a Lyapunov function with the design of feedback control in a systematical way. Numerical simulations verify the effectiveness of the approach.

1. Introduction

The classical Colpitts oscillator was originally designed to generate periodic waveforms. However, with special circuit parameters it can be used to generate noise-like broadband signals [1–5]. Since the chaotic oscillation of the Colpitts oscillator was reported first by Kennedy [1], there is extensive numerical and experimental evidence of chaotic behavior [1,3–5]. The first experiment on chaos in the Colpitts oscillator was reported at relatively low fundamental frequency, i.e., several kilohertz [1]. Chaotic oscillations have been observed experimentally also in the HF range at the fundamental frequency $f = 25$ MHz using the 2N2222A type BJT with the threshold frequency of $f_T = 300$ MHz [3]. Later, chaos has been demonstrated numerically in the UHF range as well as at $f = 500$ MHz [6], also experimentally at $f = 950$ MHz (Russian 2T938A-2 type BJT with $f = 5$ GHz) [7]. The fundamental frequency can be tuned from several kilohertz to several gigahertz, i.e., to the microwave range.

In the present paper we propose a novel controller to synchronize and control the chaotic Colpitts oscillator, based on the backstepping design method. The proposed approach is a powerful and systematic technique that recursively interlaces the choice of a Lyapunov function with the design of feedback control. Besides, the control scheme can also be used to suppress chaos in a systematic way. Finally, the effectiveness and feasibility of the proposed control technique are numerically verified.

This paper is organized as follows. In Section 2, a brief description of the chaotic behavior in the Colpitts oscillator with an ordinary circuit model is introduced. Section 3 discusses the design of a backstepping controller for synchronization between the chaotic oscillators as well as suppression of chaotic motion. Numerical simulations are carried out to confirm the validity of the proposed approach in Section 4. Finally, some conclusions are given.

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2. Colpitts oscillator and its chaotic behavior

The classical circuit diagram (Fig. 1) of the Colpitts oscillator contains a bipolar junction transistor (BJT) as the gain element and a resonant network consisting of an inductor and a pair of capacitors. Dynamics for the schematic in Fig. 1 can be described by the following differential equations [1,6,8]:

$$\begin{aligned} \frac{dx}{dt} &= y - aF(z), \\ \frac{dy}{dt} &= c - x - by - z, \\ \frac{dz}{dt} &= y - dz, \end{aligned} \tag{1}$$

where $x=\frac{V_{C1}}{V^*}$, $y=\frac{\rho I_L}{V^*}$, $z=\frac{V_{C2}}{V^*}$, $t=\sqrt{LC_1}$, $\rho=\sqrt{\frac{L}{C_1}}a=\frac{\rho}{r}$, $b=R/\rho$, $c=V_0/V^*$, $d=\rho/R_e$, $e=\frac{R_2}{R_1+R_2}c$, $F(z)=\begin{cases} e-1-z, & z<e-1, \\ 0, & z\geqslant e-1, \end{cases}$.

Here we adopt "piecewise-linear" of i–v curves of a BJT. More details on physical parameters and their meanings will

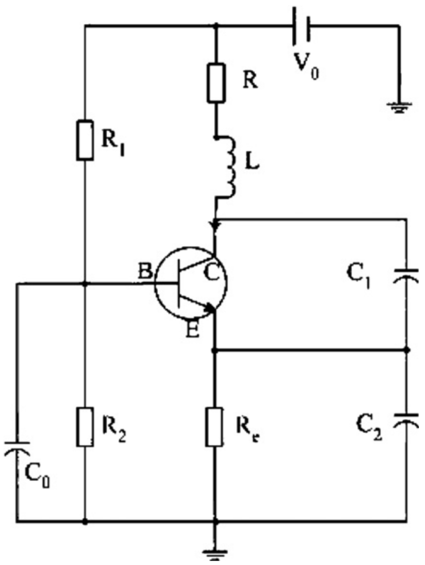

Fig. 1. Circuit diagram of the chaotic Colpitts oscillator.

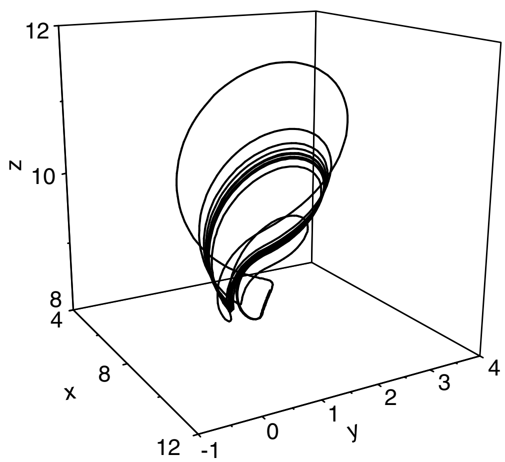

Fig. 2. Phase portrait of the chaotic Colpitts oscillator.

not be given here; we refer the interested reader to [8]. The circuit parameters are as follows: $C_1 = C_2 = 470$ nF, $C_0 = 47\ \mu$F, $R = 36\ \Omega$, $R_e = 510\ \Omega$, $R_1 = R_2 = 3$ kΩ, $V_0 = 15$ V. The typical parameters of the Colpitts oscillator are considered, that is, $a = 30$, $b = 0.8$, $c = 20$, $d = 0.08$, $e = 10$. The phase portrait is illustrated in Fig. 2, showing a clear chaotic oscillation.

3. Backstepping design for the controller

In this section, we will expect to design an appropriate controller to drive the chaotic Colpitts oscillator to a desired state (including an equilibrium point or a synchronized chaotic state). In fact, it is very useful for actual engineering.

The backstepping strategy [9,10] is a step-by-step design approach and consists of a recursive procedure interlacing the choice of a Lyapunov function with the design of a virtual control at each step. At the last step, the true control is obtained. According to the control theory [9,10], the controlled chaotic system can be written in the form

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= y - aF(z) + u,\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= c - x - by - z,\\ \frac{\mathrm{d}z}{\mathrm{d}t} &= y - dz,\end{aligned} \tag{2}$$

where u is a controller to be designed later. For brevity, $\mathbf{X} = (x, y, z)^{\mathrm{T}}$ denotes the state vector. Let the target orbit of the Colpitts oscillator $\mathbf{X}_t = (x_t, y_t, z_t)^{\mathrm{T}}$, which is a solution of the system (1), namely, it satisfies

$$\begin{aligned}\frac{\mathrm{d}x_t}{\mathrm{d}t} &= y_t - aF(z_t),\\ \frac{\mathrm{d}y_t}{\mathrm{d}t} &= c - x_t - by_t - z_t,\\ \frac{\mathrm{d}z_t}{\mathrm{d}t} &= y_t - \mathrm{d}z_t.\end{aligned} \tag{3}$$

Thus, the problem of controlling chaos under investigation here is to select an appropriate control function u such that the trajectory of the controlled system (2) asymptotically approaches the target orbit $\mathbf{X}_t$, i.e., $\lim_{t\to\infty}|\mathbf{X}_t - \mathbf{X}| = 0$.

Subtracting Eq. (2) from Eq. (3), we get the error dynamics as follows:

$$\begin{aligned}\frac{\mathrm{d}e_1}{\mathrm{d}t} &= e_2 - aF(z_t) + aF(z_t + e_3) - u,\\ \frac{\mathrm{d}e_2}{\mathrm{d}t} &= -e_1 - be_2 - e_3,\\ \frac{\mathrm{d}e_3}{\mathrm{d}t} &= e_2 - de_3,\end{aligned} \tag{4}$$

where the error vector: $\mathbf{e} = (e_1, e_2, e_3)^{\mathrm{T}} = (x_t - x, y_t - y, z_t - z)^{\mathrm{T}}$.

Now the objective is to find a control law u for stabilizing the error variables of system (4) at the origin. As long as the control input can stabilize the system, the error vector $\mathbf{e}$ converges to zero as time goes to infinity. This implies that the trajectory of the controlled system (2) asymptotically approaches the target orbit (2). Now we begin to design the controller based on the backstepping design method, as follows:

Step 1: Let $w_1 = e_3$, then we can obtain its derivative

$$\dot{w}_1 = \dot{e}_3 = -\mathrm{d}w_1 + e_2, \tag{5}$$

where $e_2 = \alpha_1(w_1)$ is regarded as a virtual controller. Choose Lyapunov function $V_1 = \frac{1}{2}w_1^2$. The derivative of V_1 is as $\dot{V} = w_1(-\mathrm{d}w_1 + \alpha_1(w_1))$. If we choose $\alpha_1(w_1) = 0$, $\dot{V}_1 = -\mathrm{d}w_1^2$ is negative definite. Hence the w_1-subsystem (5) is asymptotically stable. Since the virtual control function $\alpha_1(w_1)$ is estimative when e_2 is considered as a controller, the error between e_2 and $\alpha_1(w_1)$ is $w_2 = e_2 - \alpha_1(w_1)$. We can obtain the following (w_1, w_2)-subsystem

$$\begin{aligned}\dot{w}_1 &= -\mathrm{d}w_1 + w_2,\\ \dot{w}_2 &= -e_1 - bw_2 - w_1.\end{aligned} \tag{6}$$

Consider $e_1 = \alpha_2(w_1, w_2)$ as a controller in system (6) to make system (6) asymptotically stable.

Step 2: In order to stabilize the (w_1, w_2)-subsystem (6), we can choose a Lyapunov function $V_2 = V_1 + \frac{1}{2}w_2^2$. Its derivative is $\dot{V}_2 = -\mathrm{d}w_1^2 - bw_2^2 - w_2\alpha_2(w_1, w_2)$. If $\alpha_2(w_1, w_2) = 0$, then $\dot{V}_2 = -\mathrm{d}w_1^2 - bw_2^2 < 0$ makes the (w_1, w_2)-subsystem (6) asymptotically stable.Similarly, assume that $w_3 = e_1 - \alpha_2(w_1, w_2)$, so $\dot{w}_2 = -w_3 - bw_2 - w_1$. Study the full dimension (w_1, w_2, w_3) system

$$\begin{aligned}\dot{w}_1 &= -\mathrm{d}w_1 + w_2,\\ \dot{w}_2 &= -w_3 - bw_2 - w_1,\\ \dot{w}_3 &= w_2 - aF(z_t) + aF(z_t + w_1) - u.\end{aligned} \tag{7}$$

Step 3: Choose Lyapunov function $V_3 = V_2 + \frac{1}{2}w_3^2$ to make the (w_1, w_2, w_3) system (7) asymptotically stable. The derivative of V_3 is expressed as $\dot{V}_3 = -\mathrm{d}w_1^2 - bw_2^2 + w_3[-aF(z_t) + aF(z_t + w_1) - u]$. Let $u = -aF(z_t) + aF(z_t + w_1) + w_3$, and $\dot{V}_3$ can be described as $\dot{V}_3 = -\mathrm{d}w_1^2 - bw_2^2 - w_3^2 < 0$, which makes the (w_1, w_2, w_3) system (7) asymptotically stable. Finally, the full (w_1, w_2, w_3) system is

$$\begin{aligned}\dot{w}_1 &= -\mathrm{d}w_1 + w_2,\\ \dot{w}_2 &= -w_3 - bw_2 - w_1,\\ \dot{w}_3 &= w_2 - w_3.\end{aligned} \tag{8}$$

In the (w_1, w_2, w_3) coordinates, the equilibrium $(0, 0, 0)$ is global asymptotically stable. In view of $w_1 = e_3$, $w_2 = e_2$, and $w_3 = e_1$, equilibrium of Eq. (4) is still $(0, 0, 0)$ and has not been changed. So following above procedure we can conclude that equilibrium $(0, 0, 0)$ of system (4) is asymptotically stable. In other words, the trajectory of the controlled system (2) asymptotically approaches the target.

4. Numerical simulations

In this section, numerical experiments are given to verify the effectiveness of the above control approach. Fourth-order Runge–Kutta method is used to solve the systems of differential Eqs. (2) and (3) with time step size equal 0:001 in all numerical simulations.

Case 1 Now consider synchronization between two identical Colpitts Oscillators. In this case, the initial values are arbitrarily chosen as follows: $\mathbf{X}_t(0) = [8, 2, 3]$ and $\mathbf{X}(0) = [5, 10.5, 8.5]$, respectively, for the system (2) and the system (3). After 20 s, the motion trajectories have entered into the chaotic attractor. From then on we activate the foregoing controller$u$. The simulation results are shown in Fig. 3. Fig. 3 displays the time waveforms of the error state variables before and after the controller is activated. One can see that as the controller is activated, the controlled system eventually

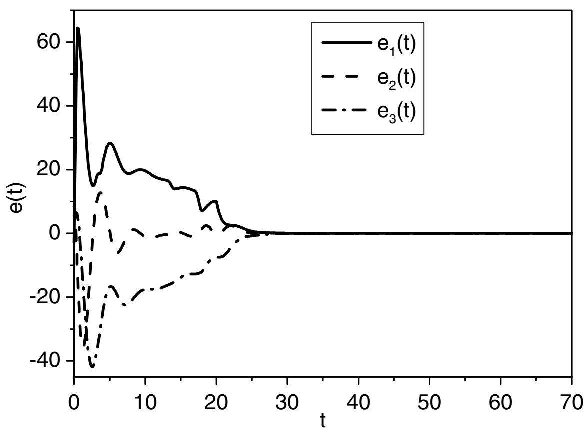

Fig. 3. Temporal evolution of the synchronization error: e_1 (solid), e_2 (dash), and e_3 (dash dot).

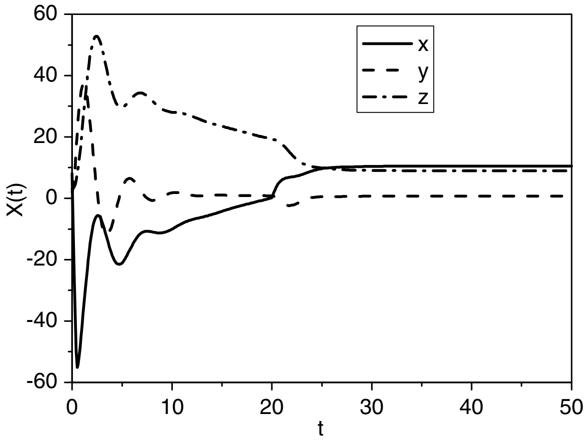

Fig. 4. Controlling chaotic behavior to a stable state. Time waveform of the state vector x (solid), y (dash), and z (dash dot).

tracks the chaotic target orbit. We observe in numerical simulations that after several seconds there is synchronization between two chaotic systems in spite of different initial conditions.

Case 2 (Controlling chaotic behavior to a stable state). It is well known that the equilibrium point of the chaotic system is $\left(c - \frac{a(bd+1)(e-1)}{a+d}, \frac{ad(e-1)}{a+d}, \frac{a(e-1)}{a+d}\right) = (10.45, 0.718, 8.98)$. Our purpose is to design a backstepping controller to make trajectory of the controlled chaotic system asymptotically approach the desired equilibrium point.

Similarly, let the initial conditions be $\mathbf{X}(0) = [5, 10.5, 8.5]$. After 20 s, the motion trajectories have entered into the chaotic attractor. From then on the active controller is activated. The numerical results are illustrated in Fig. 4. As expected, one can observe that the system orbit eventually converges to the desired equilibrium point $(10.45, 0.718, 8.98)$. With the above controller, chaos is suppressed.

5. Conclusions

In this paper, we firstly present the circuit model and its chaotic behavior in Colpitts oscillator. Secondly, based on backstepping design method, synchronization and suppression of chaotic motion are reported by using only one controller. The method here to use is a systematic design approach and consists of a recursive procedure interlacing the choice of a Lyapunov function with the design of a controller. Finally, numerical simulations are provided to show the effectiveness and feasibility of the proposed approach.

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