Adaptive control and synchronization of chaotic systems consisting of Van der Pol oscillators coupled to linear oscillators

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Abstract

This paper deals with the problem of control and synchronization of coupled second-order oscillators showing a chaotic behavior. A classical feedback controller is first used to stabilize the system at its equilibrium. An adaptive observer is then designed to synchronize the states of the master and slave oscillators using a single scalar signal corresponding to an observable state variable of the driving oscillator. An interesting feature of the proposed approach is that it can be used for chaos control as well as synchronization purposes. Numerical simulations results confirming the analytical predictions are shown and pspice simulations are also performed to confirm the efficiency of the proposed control scheme. © 2005 Elsevier Ltd. All rights reserved.

1. Introduction

Deterministic chaos has been thoroughly investigated in the last three decades since it was found that many real-world physical systems could behave chaotically [1–3]. It has been found that chaos may be useful in many fields [4–7]. However, it has also appeared that on another hand, chaos may be undesirable in some cases where regular oscillations are needed, like metal cutting processes [8], power electronics [9–11] and so on.

In recent years, great efforts have been devoted to controlling and synchronizing chaotic oscillators [12–21]. Several strategies to control chaos have been proposed and investigated with the objective of stabilizing equilibrium points or periodic orbits embedded in chaotic attractors [2,22–24]. The interest in synchronization lies on the potential applications in areas such as biological oscillators, animal gaits, and secure communication [4–7,2]. Moreover, it has been shown that deterministic chaos is not the only source of irregular oscillations since stable periodic systems may

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also oscillate irregularly when subject to small random noise. Thus robust synchronization is needed particularly in secure communication systems in order to counteract the effects of external perturbations on the process [6,17,25,26].

A wide variety of approaches have been proposed for the synchronization of chaotic systems among which the linear and nonlinear feedback [5,26,27], the time-delay feedback [28], the adaptive control [12,16,17,19,26,29,30], the impulsive control [31], the backstepping control [32–34] and so on. Several studies have also considered the cases where the oscillators models were partially or totally unknown [17,18,22,34]. The techniques used generally lead to acceptable performance. Nevertheless, these methods in general have a drawback, because their physical implementation would appear to be a very hard task. Another problem related to the design of synchronized chaotic oscillators is the number of observable state variables needed from the drive oscillator to build the response system. For most of the publications found in the literature of chaos control and synchronization, more than one state or the full state of the drive is assumed to be available, though this may not always be possible in practice.

Recently we presented a new electronic circuit modelling the dynamics of a Van der Pol (VDP) oscillator coupled to a linear oscillator [14] for which a nonlinear observer based on the method of Grassi and Mascolo [35] was designed, with the assumption that at least two states of the master system were measurable. In fact, it finally appeared that one of the state variables of the drive oscillator used in designing our observer was the time derivative of the other. The question that arose was if it was possible using only one transmitted state variable to construct a linear feedback that synchronizes the drive and the response oscillators, with no prior knowledge of the parameters of the drive system. In this work we show that using a robust linear feedback, it is possible to design an observer that satisfies the conditions posed. The effectiveness of the proposed control scheme will be shown using a physical construction that will be simulated on pspice. On another hand, the adaptive controller used in constructing the system observer can also help in stabilizing the model to its equilibrium, or in tracking any reference trajectory.

The rest of the work is organized as follows. In Section 2 we first give a brief presentation of the model and we investigate the stability of the equilibrium points using the Routh–Hurwitz criteria. In Section 3 we use a classical feedback controller to stabilize the oscillator to its equilibrium state. Section 4 is devoted to the adaptive synchronization of two identical systems consisting of the Van der Pol oscillator coupled to a linear oscillator (further called VDPL system). We also show how the control law should help in stabilizing a VDPL system to its equilibrium. In Section 5 we perform pspice simulations of the adaptive synchronization process. We conclude in the last section.

2. The model and its stability

2.1. The model

The circuit diagram of the chaotic system is shown on Fig. 1(a). The nonlinear resistance (NR) part is characterized by a third-order voltage–current (i–V) characteristic of the form \( i = aV + bV^3 \) \((a < 0, b > 0)\). An electronic circuit for such a resistance can be found in [14]. However, an improved, new and simple equivalent model more suitable for pspice simulations is shown on Fig. 1(b). It consists of two ideal analogue voltage multipliers \( E_1 \) and \( E_2 \) mounted with two ideal voltage-to-current converters \((G_1 \text{ and } G_2)\). A plot of the \( i–V \) characteristic of the circuit is shown in Fig. 1(c).

It can easily be shown [14] that the dynamics of the circuit of Fig. 1(a) is described by the following set of coupled second-order differential equations:

\[
\begin{align*}
\dot{V}_1 + \frac{a}{C_2} \left[ 1 - \left( \sqrt{-\frac{3b}{a}} V_1 \right) ^2 \right] \dot{V}_1 + \frac{1}{L_2 C_2} V_1 + \frac{C_1}{C_2} \dot{V}_2 &= 0 \\
\dot{V}_2 + \frac{R}{L_1} \dot{V}_2 + \frac{1}{L_1 C_1} V_2 - \frac{1}{L_1 C_1} V_1 &= 0
\end{align*}
\]

where the over dot denotes the differentiation with respect to the time \( t \). Setting

\[
\begin{align*}
t &= t \sqrt{L_2 C_2}; \quad x_1 = V_1 \sqrt{-\frac{3b}{a}}; \quad \lambda_1 = -a \sqrt{\frac{L_2}{C_2}}; \quad \lambda_2 = \frac{R}{L_1} \sqrt{L_2 C_2}; \quad \alpha = \frac{C_1}{C_2} \sqrt{-\frac{3b}{a}} \\
\omega_2^2 &= \frac{L_2 C_2}{L_1 C_1}; \quad \lambda = \omega_2^2 \sqrt{-\frac{a}{3b}}; \quad x_3 = V_2
\end{align*}
\]

These are the variables used in the pspice simulations of the adaptive synchronization process. We conclude in the last section.
the system of Eqs. (1) can be rewritten in the following form:

\[
\begin{align*}
\dot{x}_1 - e_1 (1 - x_1^2) x_1 + x_1 + a x_3 &= 0 \\
\dot{x}_3 + e_2 x_3 + \omega^2 x_3 - \lambda x_1 &= 0
\end{align*}
\]

(3)

where the overdot now indicates the differentiation with respect to \(\tau\) (that we rename as \(t\) in the new scale without loss of generality). Obviously the system of Eqs. (3) represent a Van der Pol oscillator \((X_1)\) coupled to the linear oscillator \((X_3)\).

If in addition we set

\[
\dot{x}_1 = x_2; \quad \dot{x}_3 = x_4
\]

the system of Eqs. (3) can now take the following general form:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= e_1 (1 - x_1^2) x_2 - x_1 + x (e_2 x_4 + \omega^2 x_3 - \lambda x_1) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -e_2 x_4 - \omega^2 x_3 + \lambda x_1
\end{align*}
\]

(4)

With the selection of parameters \(e_1 = 3.872; e_2 = 0.000645; \lambda = 9.12; x = 0.547 \) and \(\omega^2 = 5\) the system shows a chaotic behavior characterized by a maximal Lyapunov exponent \(\lambda_{\text{max}} = 0.062\) which confirms occurrence of chaotic oscillations. The plot on Figs. 2(a) and (b) show the graphs \(x_1 - x_2\) and \(x_1 - x_3\) obtained from a direct numerical integration.
of Eqs. (4), while the graphs of Figs. 2(c) and (d) show the corresponding graphs from a pspice simulation of the circuit of Fig. 1(a).
Fig. 2 (continued)
2.2. Stability of the equilibrium points

It is easily shown that system (4) has only one equilibrium at the origin (0, 0, 0, 0) where the Jacobian matrix of system (4) has the following characteristic equation:

\[ \theta^4 + (\varepsilon_2 - \varepsilon_1)\theta^3 + (1 + \omega_2^2 - \varepsilon_1\varepsilon_2 + \lambda x)\theta^2 + \varepsilon_2 - \varepsilon_1\omega_2^2\theta + \omega_2^2 = 0 \]  
(5)

For the parameters provided above, the first Routh–Hurwitz determinant (which is \(\varepsilon_2 - \varepsilon_1\)) is negative. Hence the origin is an unstable equilibrium.

3. Stabilization of the equilibrium by means of linear feedback control

In order to suppress chaos and stabilize the system at its equilibrium, we introduce an external control law \(u = (u_1, u_2, u_3, u_4)^T\) where for obvious reasons we choose \(u_1 = 0\) and \(u_2 = -k(x_2 - x_{20})\), \(x_{20}\) being the second coordinate of the equilibrium \((x_{20} = 0)\). The controlled system takes the form:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \varepsilon_1(1 - x_1^2)x_2 - x_1 + \varepsilon_3x_4 + \omega_2^2x_3 - \lambda x_1 - kx_2 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -\varepsilon_2x_4 - \omega_2^2x_3 + \lambda x_1
\end{align*}
\]  
(6)

The system of Eqs. (6) has the equilibrium point \((0, 0, 0, 0)\) and its Jacobian matrix has the characteristic polynomial:

\[ \theta^4 + \varepsilon_2 - \varepsilon_1\theta^3 + (\omega_2^2 + a_1 + \varepsilon_2(k - \varepsilon_1))\theta^2 + (a_1\varepsilon_2 - \lambda a_3 + \omega_2^2(k - \varepsilon_1))\theta + \omega_2^2a_1 - \lambda a_2 = 0 \]  
(7)

where we set \(a_1 = 1 + \lambda x; a_2 = 2\omega_2^2; a_3 = \varepsilon_2\).

If in addition we set:

\[
\begin{align*}
k_1 &= k - \varepsilon_1 \\
m_2 &= \varepsilon_2(k - \varepsilon_1) + \omega_2^2 + a_1 \\
m_3 &= a_1\varepsilon_2 - \lambda a_3 + \omega_2^2(k - \varepsilon_1) \\
m_4 &= \omega_2^2a_1 - \lambda a_2
\end{align*}
\]  
(8)

the Routh–Hurwitz conditions for the stability of the controlled system (6) at its equilibrium are the following:

\[
\begin{align*}
k_1 &> 0 \\
k_1m_2 - m_3 &> 0 \\
k_1(m_2m_3 - k_1m_4) - m_1^2 &> 0 \\
k_1(m_2m_3 - m_4(k_1m_2 - m_3)) - m_1^2m_4 &> 0
\end{align*}
\]  
(9)

The set of inequalities (9) in their actual form cannot be analytically used. However, using the previous numerical values of the system parameters, we vary numerically the control gain \(k\) and check for the domains satisfying all the Routh–Hurwitz conditions. It appears that the controlled system (6) is Hurwitz-stable for \(k > 3.872\), which is quite the same value as the term \(\varepsilon_1 - \varepsilon_2\). This condition was perfectly confirmed using a direct numerical integration of system (6).

4. Adaptive feedback synchronization and control

4.1. Design of the nonlinear observer

In this section we assume that we have two systems to synchronize, consisting each of a VDPL system. Let system (4) be the drive or master oscillator: We can rewrite Eqs. (4) in the general form:

\[ \dot{x} = Ax + Bf(x) \quad x(0) = x_0 \]
\[ y = Cx \]  
(10)
where \( x = (x_1, x_2, x_3, x_4)^T \in \mathbb{R}^4 \) is the system state, \( y \in \mathbb{R} \) the output, and:

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-(1 + ix) & e_1 & z o_2^2 & e_2 x \\
0 & 0 & 0 & 1 \\
\lambda & 0 & -o_2^2 & -e_2
\end{bmatrix}; \quad B = [0 1 0 0]^T; \quad C = [0 1 0 0]; \quad f(x) = -e_4 x_1^2 x_2
\]

As indicated above (see Section 1), only one of the two state outputs of Ref. [14] was to be sent to the observer. Since in the sequel it is shown that the missing state should be reconstructed by the controller, we choose to send the state variable \( x_2 \) because it is preferable to recover the missing state (here \( x_1 \)) from an integration process instead of a derivation. In practice, this helps in avoiding noises.

**Assumption 1.** The nonlinear function vector \( f(x) (f(0) = 0) \) satisfies the Lipschitz condition, that is, there always exists a positive scalar \( k_f \) such that

\[
\| f(x) - f(\tilde{x}) \| \leq k_f \| x - \tilde{x} \|
\]

**Remark 1.** For the control design purpose the Lipschitz \( k_f \) constant is normally required to be known. In practice, it is not straightforward to obtain the precise value of \( k_f \). However, it is well known that for real physical systems, the nonlinear functions are always bounded [36]. Moreover quadratic and cubic functions are locally Lipschitz and the Lipschitz constant can be well approximated by the bound on the 2-norm of the Jacobian of the system [37]. The adaptive observer to be designed in the sequel is only guaranteed to be globally convergent in a region of stability depending on \( k_f \).

**Assumption 2.** There exists a matrix \( L \) and two positive definite and symmetrical matrices \( P \) and \( Q \) satisfying the following conditions:

\[
P(A - LC) + (A - LC)^T P = -Q \]
\[
B^T P = C
\]

For system (10) we propose the following adaptive observer:

\[
\dot{x} = Ax + B f(x) + \frac{1}{2} KB(y - C \dot{x})
\]

where \( K \) is the adaptive parameter adjusted as follows:

\[
K = \sigma (y - C \dot{x})^2 \quad (\sigma > 0)
\]

We can now write the error system:

Let \( e = (e_1, e_2, e_3, e_4)^T \) be the error vector, with \( e_1 = x_1 - \dot{x}_1, e_2 = x_2 - \dot{x}_2, e_1 = x_1 - \dot{x}_1, e_3 = x_3 - \dot{x}_3, \) and \( e_4 = x_4 - \dot{x}_4. \) We have:

\[
\dot{e} = Ae + B(f(x) - f(\dot{x})) - \frac{1}{2} KB(y - C \dot{x})
\]

which can be rewritten as

\[
\dot{e} = (A - LC)e + B(f(x) - f(\dot{x})) - \frac{1}{2} KB(y - C \dot{x}) + L Ce
\]

where we have added and subtracted the term \( L Ce. \) We are now ready to state the main result of this paper.

**Theorem.** If the system of Eqs. (10) satisfies Assumption 1, then the error system (15) is asymptotically stable, which means that system (13) asymptotically synchronizes with system (10).

**Proof.** Let’s choose the Lyapunov function

\[
V = e^T Pe + \frac{1}{2\sigma} (K - \dot{K})^2
\]

\[828\]

\[829\]

\[830\]
Moreover, taking the time derivative of (16) along (15) we have:

\[
\dot{V} = e^T (A - LC)^T Pe + B^T (f(x) - f(\hat{x})) Pe - \frac{1}{2} e^T C^T B^T K^T Pe + e^T C^T L^T Pe + e^T P (A - LC)e
\]

\[+ e^T PB (f(x) - f(\hat{x})) \frac{1}{2} e^T PKBCe + e^T PLe + \frac{1}{\sigma} \hat{K} (K - \hat{K}) \]

\[
\dot{V} = e^T [(A - LC)^T P + P (A - LC)] e + 2B^T Pe (f(x) - f(\hat{x})) + 2e^T PLe + \frac{1}{\sigma} \hat{K} (K - \hat{K}) - e^T KPB Ce
\]

(17)

Notice that:

\[
\left( ||B^T Pe|| \frac{k_k - \varepsilon ||e||}{\varepsilon} \right)^2 \geq 0 \Rightarrow 2 ||B^T Pe|| ||PLe|| \leq \frac{1}{\varepsilon} ||B^T Pe||^2 k_k^2 + \varepsilon ||e||^2
\]

(18)

Also,

\[
2 ||B^T Pe|| ||PLe|| \leq \frac{1}{\mu} ||B^T Pe||^2 + \mu ||PLe||^2
\]

(19a)

Moreover,

\[
||PLe||^2 = ||PLL^T P|| ||e||^2 \leq \lambda_{\max}(PLL^T P)||e||^2
\]

(19b)

where \( \varepsilon, \mu \) are positive scalars and \( \lambda_{\max}(X) \) is the maximum eigenvalue of the matrix \( X \). Using (12), (18) and (19), we may now write the following inequality:

\[
\dot{V} \leq -(\lambda_{\min}(Q) - \varepsilon - \mu \lambda_{\max}(PLL^T P)) ||e||^2 + \frac{1}{\sigma} (K - \hat{K}) \tilde{K} + \left( \frac{k_k^2}{\varepsilon} + K - \hat{K} - \frac{1}{\mu} \right) ||B^T P||^2 ||e||^2
\]

(20)

where we have used the fact that \( \lambda_{\min}(Q) \leq ||Q|| \leq \lambda_{\max}(Q) \).

Now, we may rewrite inequality (20) in the form:

\[
\dot{V} \leq -(\lambda_{\min}(Q) - \varepsilon - \mu \lambda_{\max}(PLL^T P)) ||e||^2 + K \left( \frac{\dot{K}}{\sigma} - ||B^T P||^2 ||e||^2 \right) + \hat{K} \left( \frac{\dot{K}}{\sigma} + \frac{1}{\mu} + \frac{k_k^2}{\varepsilon} \right) ||B^T P||^2 ||e||^2
\]

(21)

thus, by letting

\[
\hat{K} = \frac{1}{\mu} + \frac{k_k^2}{\varepsilon}
\]

(22)

we have

\[
\dot{V} \leq -(\lambda_{\min}(Q) - \varepsilon - \mu \lambda_{\max}(PLL^T P)) ||e||^2 + (K - \hat{K}) \left( \frac{\dot{K}}{\sigma} - ||B^T P||^2 ||e||^2 \right)
\]

(23)

By choosing \( \hat{K} = \sigma ||B^T P||^2 = \sigma ||Ce||^2 = \sigma (x_2 - \hat{x}_2)^2 \) the inequality (23) becomes:

\[
\dot{V} \leq -(\lambda_{\min}(Q) - \varepsilon - \mu \lambda_{\max}(PLL^T P)) ||e||^2
\]

(24)

Since \( \lambda_{\min}(Q) > 0 \), it suffices to choose \( \varepsilon \) and \( \mu \) small enough to turn condition (24) into \( \dot{V} \leq 0 \), which means that system (15) is Lyapunov stable, whence \( e \in L_\infty \). Therefore, using (15) and (16) we have \( V(t) \in L_\infty \) and \( \dot{e} \in L_\infty \). Moreover, integrating (24) with setting \( \psi_0 = (\lambda_{\min}(Q) - \varepsilon - \mu \lambda_{\max}(PLL^T P)) \), we obtain:

\[
V(t) \leq V(0) - \int_0^t \psi_0 ||e(\tau)||^2 d\tau
\]

and thus:

\[
\int_0^t ||e(\tau)||^2 d\tau \leq \frac{V(0) - V(t)}{\psi_0}
\]

(25)

Since \( V(0) \) is finite, we deduce that \( e \in L_2 \). Hence, since \( \dot{e} \in L_\infty \), by using Barbalat’s lemma [36] we obtain that \( \lim_{t \to \infty} ||e(t)|| = 0 \). This concludes the proof. \( \square \)

The numerical simulations were carried using a fourth-order Runge–Kutta algorithm with a time step of 0.001 with the numerical parameters of Section 2 and \( \sigma = 1 \). The time evolution of the adaptive gain \( K(t) \) is shown on Fig. 3(a), while the graphs of the synchronization errors are provided on Fig. 3(b) and (c).
Fig. 3. (a) Time evolution of the adaptive control parameter. (b) Time evolution of the error $e_1 = x_1 - \hat{x}_1$. (c) Time evolution of the error $e_3 = x_3 - \hat{x}_3$. 
Fig. 3 (continued)

Fig. 4. Adaptive feedback stabilization of the system state to the equilibrium (0,0,0,0): $X_1(t)$ in full line; $X_3(t)$ in dashed lines.

Fig. 4. Adaptive feedback stabilization of the system state to the equilibrium (0,0,0,0): $X_1(t)$ in full line; $X_3(t)$ in dashed lines.
4.2. Stabilizing the unstable equilibrium of the chaotic VDPL oscillator

In Section 3, we used the classical feedback control scheme to depict conditions of stability of the unstable equilibrium of a VDPL system. It can be seen that the method is not accurate, since it involves poles placement techniques and needs information on the system parameters. In addition it leads to a local stability of the closed-loop system. The idea of converting the stabilization problem of unstable equilibrium to a tracking problem has already been suggested elsewhere [20]. The most interesting feature of the method proposed here is that it requires no prior knowledge of the system parameters: in fact, as shown in the sequel, the system will be automatically driven to its equilibrium by the adaptive controller.

Suppose that the dynamics of the system to be stabilized is described by Eqs. (13) and (14), where \( y \) is now a reference trajectory to be tracked, or a coordinate of the system equilibrium. It is obvious that the adaptive controller will steer the system to the reference state provided that the above assumptions are satisfied. In particular, let the reference coordinate \( y \) be any coordinate of the equilibrium (the origin, with \( y = 0 \)). Then, cancelling the variable \( y \) from Eqs. (13) and (14), a numerical integration of these equations shows that the whole system state asymptotically evolve to the origin as shown on Fig. 4.

Fig. 5. (a) The practical implementation of the control strategy. (b) Simplified pspice implementation of the adaptive synchronization strategy.
5. Pspice simulations

Our objective in this section is to implement a practical set-up for the adaptive synchronization strategy presented above, and to perform pspice simulations to verify the practical feasibility of the strategy.

Consider the response circuit on Fig. 4 where a voltage-controlled current source (VCCS), \( I_c(t) \) has been connected in parallel with the capacitor \( C_2 \). If the controlling current depends adaptively on the error voltage \( \frac{\tilde{V}_1(t)}{V_1(t)} \) as

\[
I_c(t) = k_2(t)G_i(\tilde{V}_1(t) - V_1(t))
\]

where \( k_2(t) = G_i(i(t) - \tilde{i}(t))^2 \), (with \( k_2(t) \) standing for the gain \( \frac{1}{2}K(t) \) of Eqs. (13))

\[
G_i = -\sqrt{\frac{C_2}{L_2}}, \quad G_k = \frac{3b\sigma G_i}{aC_2}
\]

where \( \sigma \) is a positive scalar gain factor, then it is straightforward to show that the dynamics of the controlled system of Fig. 5(b) is described by a set of differential equations similar to systems (10) and (13). One should note that the error voltage \( \tilde{V}_1(t) - V_1(t) \) used by the controller is reconstructed from the error current \( i(t) - \tilde{i}(t) \) through the voltage substractor \( E_3 \) and the voltage integrator (INTEG1). This allows us to effectively use only one transmitted signal to achieve the full control process. The current-controlled voltage sources \( H_1 \) and \( H_2 \) with unit gain provide voltages images of the currents flowing through the capacitors \( C_2 \) and \( C_{2S} \). We provide on Fig. 6 the pspice simulation results for the time variations of the synchronization error \( \tilde{V}_1(t) - V_1(t) \) with \( G_k = -1.16 \times 10^{10} \).

6. Conclusion

In this paper, the control and synchronization problem of an electronic circuit consisting of a Van der Pol oscillator coupled to a linear oscillator has been investigated. Using Lyapunov design methods, an adaptive observer was designed to synchronize with our model using a single scalar transmitted state of the system. Also, it was shown that
the linear state feedback controller could asymptotically stabilize the closed-loop system. Numerical and pspice simulations were conducted to show the effectiveness and the practical feasibility of the control strategy. Applications to secure communications are under investigation.

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