ABSTRACT

In this paper the workspace optimization of translational 3-UPU parallel platforms with prismatic and universal joint constraints is performed. The workspace is parameterized using two design parameters, which are the prismatic joint stroke and the difference between the distances of the center to one of the corners of the triangular plate of the base and of the moving platform. For a large range of values for these design parameters the workspace of the corresponding 3-UPU platforms is calculated. This workspace is called in this paper the constrained workspace to distinguish it from the unconstrained workspace where no joint constraints are considered. For the constrained workspace of each design three performance indices are calculated: a) the workspace volume; b) the average of the inverse of the condition number and c) a Global Condition Index which is a combination of the other two performance indices. Plots of each performance index as a function of the two design parameters are generated and optimal values for these design parameters are determined. Finally, for the optimal design, it is shown that by introducing limits to the angles of the universal joints axis, the isotropy of the parallel platform is improved.

1 INTRODUCTION

In recent years, modular robots were increasingly proposed as means to develop reconfigurable and self-repairable robotic systems [1]. To perform impromptu custom tasks, increase the payload to weight ratio, and, in cases of emergency, self-repair, future inter-planetary robots and manipulation system need to incorporate modularity and self-reconfiguration capabilities. Modular robots utilize many autonomous units, or modules, that can be reconfigured into a vast number of designs. Ideally, the modules will be homogeneous, small, and self-contained. The robot can change from one configuration to another by manual reassembly, or by itself. Self-reconfiguring robots adapt to a new environment or function by changing shape. Modules must interact with one another and cooperate in order to realize self-reconfiguration. Also, modular robots can repair themselves by removing and replacing failed modules. Since one self-reconfigurable modular robot can provide the functionality of many traditional mechanisms, they will be especially suited for space and planetary exploration, where payload mass must be kept minimum. Because they promise self-repairability and virtually limitless functionality, future self-reconfigurable modular robots are expected to be cheaper and more useful than current robot mechanisms in space missions.

In this project we investigate the use of 3-legged parallel platforms as joint modules of reconfigurable robots. Parallel platforms are currently being used in many applications as multi-degree of freedom systems with high rigidity, high payload to weight ratio, high precision and low inertia [2, 3]. These properties are also desired characteristics for the joint modules of reconfigurable robots. Six-legged, six degrees-of-freedom (DOF) parallel platforms have been used as joint modules of reconfigurable robots in [4]. However, the high number of DOF and of actively controlled joints per module increases complexity and cost. In addition, a purely 3 degree-of freedom translational or spherical motion would require activation of all six module legs which means increase in energy consumption. Lately, a special type of 3-legged parallel platforms has received a lot of attention because of its simple design and its pure 3-DOF translational motion with constant orientation. This platform could be combined with 3-legged, 3 DOF parallel platforms modules with purely spherical motion so that they form hybrid kinematic chains with decoupled translation and orientation [5]. Figure 1 shows an example of such a hybrid system. In this example a 2-arm reconfigurable robotic system is formed from a sequence of 3 DOF translational and orientation parallel platform modules.

The 3-DOF translational parallel platform that is studied in this paper was first proposed by Tsai [6] and was later on generalized by Di Gregorio and Parenti-
Castelli [7]. It is made out of two equilateral triangular plates connected at the corners with three identical legs, as it is shown in Figure 2. The legs are linked to the plates with universal joints. Prismatic actuators control the length of the legs. Due to the existence of universal and prismatic joint at each one of the legs, it is also called 3-UPU platform. By properly orienting the axes of universal joints, the moving plate can have only translational motion. The direct and inverse kinematics of this platform have been studied in [6, 7, 8]. Its static and singularity analyses have been performed in [9, 13].

![Overall View](image1)

![Close-Up View](image2)

**FIGURE 1:** Two-Arm Reconfigurable Robot with 3-DOF Parallel Platform Modules.

![The 3-UPU Parallel Platform](image3)

**FIGURE 2:** The 3-UPU Parallel Platform.

The workspace of parallel platforms has been studied thoroughly [3, 10]. For the 3-UPU platform, the isotropy and workspace analyses have been performed in [8]. In that work, no constraints were assumed for the leg prismatic and universal joints, which means that full rotation of the universal joints and full extension of the prismatic joints, were considered. The workspace optimization was performed using one design parameter, which is the difference between the distances from the center to the corner of the triangular plate for the base and the moving platform. Using a Monte Carlo method and varying the normalized lengths of the legs from 0 to 1 an optimum design is obtained at the value of 0.37 for the design parameter. However, from the practical point of view, universal joints have angular limitations that will reduce the workspace of the 3-UPU platform. Also, if these platforms are used as modules for a reconfigurable robot, they must be stacked, one on the top of the other, severely limiting their prismatic actuator stroke. From a design standpoint, universal and prismatic joint constraints should be included to produce a viable reconfigurable robot joint module.

In this paper the workspace optimization of translational 3-UPU parallel platforms with prismatic and universal joint constraints is performed. The workspace is parameterized using two design parameters, which are the prismatic joint stroke and the difference between the distances of the center to one of the corners of the triangular plate of the base and of the moving platform. For a large range of values for these design parameters the workspace of the corresponding 3-UPU platforms is calculated. This workspace is called in this paper the constrained workspace to distinguish it from the unconstrained workspace where no joint constraints are considered. For the constrained workspace of each design three performance indices are calculated: a) the workspace volume; b) the average of the inverse of the condition number and c) a Global Condition Index which is a combination of the other two performance indices. Plots of each performance index as a function of the two design parameters are generated and optimal solutions for these design parameters are determined. Finally, for the optimal design, it is shown that by introducing limits to the angles of the universal joints axis, the isotropy of the parallel platform is improved.

## 2 MATHEMATICAL TOOLS

In this section we present the mathematical / kinematic tools that were used to formulate the workspace optimization problem. These tools include the direct and inverse kinematics, the platforms Jacobian matrix, its condition number and the Global Condition Index. For the direct and inverse kinematics and for the calculation of the Jacobian matrix we used the solution methodology proposed in [2, 6]. For the Global Condition Index we used the definition introduced in [11] and used in [8].

The 3-UPU parallel platform consists of a base plate, a moving plate and three identical limbs (see Figure 3). The plates have equilateral triangular shapes of different
sizes. The limbs are connected to the plates with 2-DOF universal joints. A linear actuator controls the leg length, and forms a prismatic joint. Each universal joint is treated as two revolute joints with axes perpendicular to each other and intersecting at a point. In Figure 3, \( u_{ij} \), \( u_{i2} \), and \( u_{ij} \) are the unit vectors along the axes of the universal joints and \( u_{ij} \) is the unit vector along the prismatic joint axis for leg \( i \). Tsi showed that by having \( u_{ij} \) parallel to \( u_{ij} \) and \( u_{ij} \) parallel to \( u_{ij} \) for each leg, then the moving plate would achieve pure translation in three dimensions [6].

**FIGURE 3:** Joint Axes in the 3-UPU Platform.

As seen in Figure 3, the position vectors of points \( A_i \) and \( B_i \) with respect to frames \( A \) and \( B \) respectively, can be written as:

\[
\begin{align*}
\mathbf{a}_i &= [a_{ix}, a_{iy}, 0]^T, \\
\mathbf{b}_i &= [b_{ix}, b_{iy}, 0]^T
\end{align*}
\]

(1)

Frames \( A \) and \( B \) are defined at the base and moving triangle respectively. Their origin is placed at the corresponding triangle's center. Their z-axis is perpendicular to each triangle's plane and their x and y-axes are in the triangle's plane. Parameter \( q_i \) denotes the length of leg \( i \). Subscript \( i \) denotes one of the legs and can take the values 1, 2, or 3. Vector \( \mathbf{c}_i \) is defined as the difference of \( \mathbf{a}_i \) and \( \mathbf{b}_i \). Letters in bold represent three-dimensional vectors. Superscript \( A \) or \( B \) on the left of a vector denotes the reference frame where its coordinates are calculated. Superscript \( T \) on the right of a vector denotes the transpose of a vector. Since vectors \( \mathbf{a}_i \) and \( \mathbf{b}_i \) are constants, vector \( \mathbf{c}_i \) is a constant too. Vector \( \mathbf{e} \) determines the position of the center of the moving plate in the base coordinate system:

\[
\mathbf{e} = [e_x, e_y, e_z]^T
\]

(2)

For the direct kinematics the leg lengths \( q_i \) and the triangular plates' geometry are known and the 3 coordinates of vector \( \mathbf{e} \) are determined from the following 3 scalar equations:

\[
e^2 - 2e \cdot \mathbf{c}_i + c_i^2 = q_i^2, \quad \text{for } i=1, 2, 3.
\]

(3)

For the inverse kinematics vector \( \mathbf{e} \) and the triangular plates' geometry are known and the limb lengths, \( q_i \) are calculated from the following 3 equations:

\[
q_i = \pm \sqrt{(e_x - c_{ix})^2 + (e_y - c_{iy})^2 + e_z^2},
\]

(4)

The manipulator Jacobian matrix \( J \) relates the end-effector velocities \( \dot{x} \) to the actuated joint velocities \( \dot{\theta} \):

\[
\dot{\theta} = J^T \ddot{x}
\]

(5)

For the 3-UPU platform \( J \) is a 3x3 matrix and is given by:

\[
J = \begin{bmatrix}
\mathbf{u}_{1,3}^T \\
\mathbf{u}_{2,3}^T \\
\mathbf{u}_{3,5}^T
\end{bmatrix}
\]

(6)

The condition number of the Jacobian matrix is defined by Equation (7)

\[
k = \|J\| \|J^{-1}\|
\]

(7)

where \( \| \cdot \| \) denotes the norm 2 of a matrix [12].

Robot manipulators and their configurations where \( k \) is equal to 1 are called isotropic. In these configurations the system is able to develop same amount of forces and velocities in all end-effector directions. For high values of \( k \), there are end-effector directions where the manipulator can develop much higher forces or velocities than in other directions. In many applications, this is not a desirable system property because the system looses its homogeneity in force and velocity development. Configurations in which \( k \) has an infinite value are singular configurations. In these configurations there are directions in space where the end-effector can either not move or not apply forces. Isotropy is a very local manipulator property because it changes from configuration to configuration. A manipulator can have a good isotropy in one configuration and a bad one in another. In many robot applications isotropy is an important property, and needs to be taken into account during the robot's design phase. Defining a global isotropy index that will be able to characterize the system isotropy in the whole workspace is an important but difficult problem to solve. The problem is that a robot system will always have singular or bad isotropy configurations. By taking a type of an average of the condition number is only a rough indication of the quality of the system global isotropy and says nothing about the magnitude and number of bad isotropy configurations. Nevertheless, in this work we will use global isotropy indices based on the average of the condition number, as they are being used in the current literature.

In this work, three performance indices will be used to characterize a robotic system's workspace:

a) The volume of the workspace. It is calculated multiplying the initial volume with the number of points inside workspace and dividing by the number of selected points. Obviously, using this performance index, optimal designs correspond to maximum workspace volume.

b) The average of the inverse of the condition number. The inverse of the condition number characterizes the system's isotropy. The average of the inverse of the condi-
tion number is calculated by summing the inverse of the condition number in every point in the workspace and then dividing it with the number of points considered in the workspace. Optimal designs correspond to values of the average condition number close to 1.

c) The Global Condition Index proposed in [11] and used in [8]. This index, defined in Equation (8), is the ratio of the integral of the inverse condition numbers calculated in the whole workspace, divided by the volume of the workspace. In reality it is calculated as the product of the sum of the inverse condition number with the workspace volume divided by the number of points in the workspace. Therefore, it is like a combination of the first two performance indices:

\[ \eta = \frac{A}{B} \text{, where } A = \int_{W} \left( \frac{1}{k} \right) dW \text{, and } B = \int_{W} dW \]  

(8)

where \( k \) is the condition number at a particular point in the workspace \( W \). \( B \) is the total volume of the workspace. The Global Condition Index is bounded as:

\[ 0 \leq \eta < 1 \]  

(9)

Isotropic systems correspond to value of \( \eta \) equal to 1 and systems with bad isotropy correspond to values of \( \eta \) approaching zero.

3 OPTIMIZATION ALGORITHM

Based on the direct kinematics of the 3-UPU parallel platform, shown in Equation (3), the platforms workspace depends on two geometric parameters: the magnitude \( c_i \) of vector \( \mathbf{c} \) and the stroke \( s_i \) of the prismatic joint which is directly related to each leg's length \( q_i \). Due to the fact that the 3-UPU platforms have equilateral triangular base and moving plates, and assuming that all legs are using the same actuator, then \( c_i \) and \( s_i \) have the same value for all legs.

The stroke \( s_i \) of the actuator is the maximum amount of travel for each leg's prismatic actuator. It is expressed with a percentage value of the minimum length of the limb \( q_{\text{min}} \) which in turn is taken such that \( q_{\text{max}} \) is equal to 1. The fact that the leg's maximum length is considered to be equal to 1 means that all lengths are normalized with respect to the leg's maximum length as it is specified from each application's requirements. In this project the range of values of the stroke of the actuators is considered to be between 20% and 87.5%. These values were selected based on the technical specifications of the majority of commercially available prismatic actuators. The minimum value of each leg's length \( q_{\text{min}} \) is equal to \( 1/(1+s_i/100) \). So in each platform's configuration the value of the leg length \( q_i \) should be between \( q_{\text{min}} \) and 1.

The range of values for \( c_i \) is between 0.27 and 0.645. Platforms with values for \( c_i \) close to zero (i.e. platforms where the triangular base and moving plates are almost equal) present extra DOF (self-motions) that could not be controlled from the actuation motion and such designs are not acceptable. Platforms with very large values for \( c_i \) have very small workspace.

The algorithm to calculate the platform's workspace as a function of the design parameters consists of the following steps:

a) The algorithm selects the values of \( c_i \) and \( s_i \) from their acceptable ranges of values. For \( s_i \), 27 values in equal increments are selected between 20% and 87.5%. For \( c_i \), 15 values in equal increments are selected between 0.27 and 0.645.

b) A three-dimensional, rectangular shaped enclosure box is determined that encloses all points that can be reached by the center of the moving plate. Usually, this box is selected larger than the expected platform's workspace to be sure that all points in the platform's workspace are included in the enclosure box. For each set of values of \( c_i \) and \( s_i \), a Monte Carlo method is used to randomly select 1,000,000 points inside the determined enclosure box.

c) For each selected point, the platform's inverse kinematics is solved using Equation (4). In order that the selected enclosure point lies in the platform's unconstrained workspace, the calculated length \( q_i \) should be a real number and lie in the range from 0 to 1. If in addition the determined limb length \( q_i \) is within the acceptable range of motion of the actuator, i.e. between \( q_{\text{min}} \) and 1, then the selected enclosure point is within the actuator-constrained workspace.

d) Using Equation (7), the condition number \( k \) is calculated at each enclosure point found to be within the platform's unconstrained workspace.

e) Once the condition number of all points in the unconstrained workspace has been calculated then the three performance indices are calculated. The workspace volume is determined by multiplying the volume of the enclosure box with the total number of points inside the workspace and dividing it by the total number of points generated. The average of the inverse of the condition number is calculated by dividing the sum of the reciprocal of the condition numbers with the total number of points inside the workspace. The Global Condition Index is determined from Equation (8). The numerator \( A \) is calculated as the product of the sum of the reciprocal of the condition numbers with the volume of the unconstrained workspace. The denominator \( B \) of Equation (8) is the total number of points inside the workspace.

f) For each point within the constrained workspace the angles of the revolute joints (each universal joint is treated as two revolute joints) at the base are determined using inverse kinematic equations. If all of them fall between the allowable angular limits, then the point is within the joint-constrained workspace. The angular limits of the revolute joints were chosen such that the condition number of the platform remains below 10 at every non-singular configuration.
The unconstrained workspace and the calculation of its global condition number verified the results of [6]. However, the emphasis of this work is the identification of the properties of the actuator-constrained workspace and of the joint-constrained workspace and our results are presented in the next section.

4 RESULTS

Figure 4 shows the volume of the constrained workspace as a function of the actuator stroke $s_i$ and of the design parameter $c_i$. Dark red points indicate a large value of the volume while dark blue indicate a small value. From the plot it can be seen that the volume of the constrained workspace increases as the stroke increases and avoid points with high condition number. Figures 7 and 8 present the workspace of the parallel platform for $c_i$ equal to 0.27 and stroke equal to 85% without and with angular constraints, respectively. These values of stroke and of $c_i$ are the optimal ones based on the Global Condition Index shown in Figure 6. The color of the points represents the value of the condition number. The dark blue points have small condition numbers and they are generally found toward the center of the workspace. The dark red points have high condition numbers and they are generally found toward the bottom edges of the workspace. It can clearly be seen that by imposing stricter constraints on the range of motion of the universal joints, the workspace isotropy is improved while the workspace volume is slightly de-

Figure 5 shows the average value of the inverse of the condition number as a function of the stroke and of $c_i$. It can be seen that isotropy is affected a lot by $c_i$. Isotropic designs correspond to platforms with large values of $c_i$. In these platforms the moving plate is much smaller than the base. Such platforms do not present self motions (i.e. extra uncontrollable DOF) as those platforms with very small $c_i$'s do. Hence, these designs are away from singular designs and hence they present good isotropic behavior. Generally, the stroke does not affect isotropy as seen from Figure 5. However, for large values of $c_i$'s, where in general isotropy is very good, the best designs correspond to strokes between 30% and 40%.

Figure 6 shows the Global Condition Index as a function of the design parameters. Overall, the Global Condition Index increases as stroke increases. So if good isotropy, within the whole workspace, is desired combined with large workspace volume, then actuators with high strokes should be used. For these large stroke values the Global Condition Index shows a maximum for values of the design parameter $c_i$ around 0.55 and around 0.27. These can be considered as optimal design values for $c_i$ if the Global Condition Index is used as the design criterion.

The role of angular constraints for universal joints is to include the practical limits of the universal joints and to avoid points with high condition number. Figures 7 and 8 present the workspace of the parallel platform for $c_i$ equal to 0.27 and stroke equal to 85% without and with angular constraints, respectively. These values of stroke and of $c_i$ are the optimal ones based on the Global Condition Index shown in Figure 6. The color of the points represents the value of the condition number. The dark blue points have small condition numbers and they are generally found toward the center of the workspace. The dark red points have high condition numbers and they are generally found toward the bottom edges of the workspace. It can clearly be seen that by imposing stricter constraints on the range of motion of the universal joints, the workspace isotropy is improved while the workspace volume is slightly de-

5 CONCLUSIONS

In this paper the workspace optimization of translational 3-UPU parallel platforms with prismatic and universal joint constraints is performed. The 3-UPU platforms will be used as joint modules in reconfigurable robots and the quality of their constrained workspace is an important design feature. The workspace is parameterized using two design parameters, which are the prismatic joint stroke and the difference between the distances of the
center to one of the corners of the triangular plate of the base and of the moving platform. Three workspace performance indices are calculated as a function of the design parameters and optimal solutions for these design parameters are determined. These performance indices quantify the workspace volume, the system isotropy and a combination of the last two properties.

It is shown that for different performance indexes different parameters correspond to the best design. The limits that the actual actuators and joints impose determine limitations in the workspace of parallel platforms but they can be used in the same time to exclude points with bad isotropy from the platform’s workspace.

FIGURE 7: Workspace Without Angular Constraints - 
\( ci=0.27 \) and stroke=85%.

FIGURE 8: Workspace with Angular Constraints - 
\( ci=0.27 \) and stroke=85%.

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7 REFERENCES