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Review of Applications of Whispering-Gallery Mode Resonators in Photonics and Nonlinear Optics

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We review recent advances in the application of dielectric whispering-gallery (WG) resonators in optics and photonics, tracing the growth of the technology from the experiments with freely flying spherical droplets of transparent liquids to integrated on-chip microresonators. Both passive (such as filters) and active (such as lasers) whispering-gallery-mode (WGM)-based devices are discussed. Problems and possible future developments in the field are outlined.

I. Introduction

Optical resonators consisting of two or more mirrors are utilized in all branches of modern linear and nonlinear optics. Practical usage of such resonators is technically restricted, especially when high performance of the devices, i.e., a high quality (Q) factor and high mode stability, is important. Fabrication of good optical mirrors, their alignment, and binding are rather expensive and difficult tasks. Open dielectric resonators based on complete internal reflection become an alternative to the usual optical resonators. Fabrication of the open dielectric resonators is rather simple and inexpensive. The resonators demonstrate high mode stability and high quality factors. They can be easily tuned, stabilized, and integrated into the optical networks.

In this article, we review applications of open dielectric resonators in photonics, leaving the detailed description of the resonators' properties to textbooks [1,2] and reviews [3–8]. We use here a broad meaning of photonics, which includes linear, nonlinear, and quantum optics, optical communications, and optical engineering, as well as other branches of science and technology. Special attention is given to microwave photonics, in which dielectric resonators are used to process microwave signals by optical means.

The review covers applications of the structures called optical monolithic total internal reflection resonators, whispering-gallery resonators, photonic rings, etc. The resonators necessarily are made of transparent optical dielectrics and have monolithic ring resonator design. No incorporated couplers, as in fiber ring resonators, are allowed. The optical modes in such resonators, e.g., morphology-dependent resonances or whispering-gallery modes, can be understood as closed circular beams supported by total internal reflections from boundaries of the resonators.

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The whispering-gallery modes originally were introduced for sound waves propagating close to the cylindrical wall in St. Paul's Cathedral, London [9], where the body of the modes was partially confined due to the suppression of the wave diffraction by the sound reflection from the curved dome walls. The effective volumes and field distributions of those modes depend on the radius of the "resonator" [10].

Modern dielectric optical resonators have cylindrical, spherical, spheroidal/toroidal, ring, and other shapes and topologies with various confining principles. For example, it is not completely fair to call quasi-single-dimension objects, such as microrings, whispering-gallery resonators. In microring resonators, the curvature of the resonator does not play a significant role in formation of the modes' transverse structures. Those objects could be understood as loops made of an optical waveguide. However, for the sake of unification, throughout the review we use the terms "whispering-gallery resonators" (WGRs) and "whispering-gallery modes" (WGMs) to describe those resonators.

Finally, we have to mention that photonic technologies in general—and based on WGRs, in particular—have the potential to significantly improve spacecraft communications systems by increasing performance and reducing size and power requirements, and thus reducing the associated costs. Photonics links are fundamentally efficient because of high-efficiency lasers, waveguides, and detectors, but the advantage of photonics systems extends beyond link efficiency and includes signal processing functions. In particular, several approaches based on photonics are under development for both analog and digital processing of communications signals. These include functions ranging from direct downconversion of analog signals to photonic analog-to-digital (A/D) conversion. These approaches allow for the extension of the photonics system to the entire communications system, including the transmission/receive links and data processing.

A major challenge in implementing such capabilities in spacecraft communications is the development of extremely efficient components that can serve as the building blocks. Our research at JPL has been directed towards addressing this challenge. In particular, we have developed WGR-based ultra-high-Q optical-domain radio-frequency (RF) filters, ultra-high-efficiency modulators, and photonic microwave receivers to enable signal processing functions. Our studies, along with similar work done by other groups, confirm the great potential of the WGR-based photonic devices for planetary exploration applications, where orbiters, landers, and rovers require low power, low mass, and high-efficiency communications. This review's aim is to outline results of those studies and promote the new technologies based on WGRs for space applications.

A. Early WGM Studies

The study of WGMs was started almost a century ago with the work of Lord Rayleigh, who studied propagation of sound over a curved gallery surface [9–11]. In 1909, though, Debye derived equations for the resonant eigenfrequencies of free dielectric and metallic spheres that naturally take into account WGMs [12]. These equations also can be deduced from the theoretical studies by Mie on the scattering of plane electromagnetic waves by spheres [13]. Generalized properties of electromagnetic resonances in dielectric spheres were widely discussed afterwards, with emphasis on the microwave, not optical, modes of the spheres (see, e.g., [14,15]).

The first observations of WGMs in optics can be attributed to solid-state WGM lasers. Laser action was studied in $\mathrm{Sm{:}CaF_2}$ crystalline resonators [16]. The size of the resonators was in the millimeter range. Pulsed laser operation due to complete internal reflection in a ruby ring at room temperature has been observed [17]. Short-lived transient oscillations rather than spikes in the laser output were explained by assuming a WGM Q of 10^8 to 10^9.

In liquid resonators, WGMs were first observed by elastic light scattering from spherical dielectric particles [18,19]. It was recognized that WGMs could help in measurements of spherical particle size, shape, refractive index, and temperature [20,21]. WGMs were used to determine the diameter of the optical fiber [22]. The strong influence of WGMs on fluorescence and Raman scattering was recognized in [23–25] and [26–28], respectively. Laser action in a droplet was first studied in [29,30].

B. Coupling to High-Q Optical WGMs as the Turning Point in the Technology Development

The problem of efficient and robust coupling to WGMs is, in our opinion, the most important problem for practical applications of WGRs. The WGR-based devices became commercially available [31] only recently, because of a well-developed waveguide coupling technique,.

Free-beam optics does not allow efficient coupling with ultra-high-Q WGMs in large enough resonators. Really, such a coupling relies upon radiative exchange of a WGM with external space. This radiative exchange is extremely small. Radiative coupling of a microsphere and a beam of light may be described using generalized Lorentz–Mie scattering theory [32–35] or modified ray theory, described in [36] and confirmed in [37].

Let us roughly estimate an efficiency of coupling of a pumping light beam interacting with an ideal sphere. The beam cross-section radius is comparable to the sphere radius a (total cross section $\sim 2\pi a^2$). The scattering cross-sectional mode area is $\sim a\lambda$ [36]. Therefore, only $\sim\lambda/(2\pi a)$ of the total power interacts with the sphere.

To estimate the ratio of the mode-coupling quality factor, Q_c, and total quality factor, Q, we assume that the radiative losses and optical pumping are of the same origin and are proportional to the interaction surface area. The radiative emission occurs from all sphere surfaces (area equal to $4\pi a^2$). The optical pumping is going through a surface belt with thickness $\sim\lambda$ (area equal to $2\pi a\lambda$). Therefore, the ratio of the coupling and total quality factors is proportional to the sphere radius $Q_c/Q \sim a/\lambda$.

The coupling efficiency depends on both the ratio between the amount of radiative power of the pump beam penetrating into the mode scattering area and the total optical pump power, and the ratio of the coupling and total quality factors. Using the estimations, we conclude that the coupling efficiency is less than $(\lambda/a)^2$.

In reality, the radiative coupling to a WGR is much less than estimated above. It is simply negligible in the vast majority of experiments. It is enough to mention that the theory of open resonators yields an abnormally high radiative emission Q-factor for lossless microspheres. It must be of the order of 10^{73} at $\lambda = 600$ nm for a droplet of water with a 50-μm radius [6]. In reality, Q-factor is more than 60 orders of magnitude less because of light scattering and absorption. If one tries to make a free beam to interact with a high-Q WGM, the efficiency of the coupling is proportional to the ratio of the tiny radiative losses and “technical” losses that determine the real value of Q. This results in virtually zero coupling efficiency.

Again, the quality factor of a liquid dielectric microcavity is determined not by the radiative tunneling emission, but by the surface and volume scattering and absorption [38,39,41–44]. Hence, WGMs with comparably high Q’s were excited primarily due to surface and volume scattering effects in experiments on resonator-enhanced droplet spectroscopy.

Efficient controllable coupling, both to and from a WGR, is the critical requirement for feasibility of practical applications of the resonators. Parasitic coupling loss in this process is quantified by defining a coupling “ideality” as the ratio of power coupled to a desired mode divided by power coupled or lost to all modes (including the desired mode). Ideally, the resonator should be critically coupled to the fibers. The condition of critical coupling is a fundamental property of waveguides coupled to resonators. It refers to the condition in which internal resonator loss and waveguide coupling loss are equal for a matched resonator–waveguide system, at which point the resulting transmission at the output of the waveguide goes to zero on resonance.

Prism couplers with frustrated total internal reflection are among the oldest methods to couple light and WGMs. Efficient coupling with the prism requires the optimum shape of the gap between the waveguide and prism to obtain perfect coupling [45–47]. Prism–waveguide coupling efficiency of more than 90 percent was demonstrated in [48,49]. Prism coupling to WGMs was studied and demonstrated in [39,40,50–53]. The best efficiency of prism coupling to WGMs reported to date is ∼80 percent [40]. Prism couplers typically are not ideal because of the large number of possible output modes.

At present, in addition to the well-known prism couplers, WGR coupling devices include (1) side-polished fiber couplers [54–56] having limited efficiency owing to residual phase mismatch, (2) fiber tapers [57–60] (almost ∼100 percent coupling achieved), (3) "pigtailing" technique-based couplers utilizing angle-polished fiber tips in which the core-guided wave undergoes total internal reflection [61,62], and (4) couplers for binding WGRs and semiconductor lasers [63,64]. In addition, planar waveguides are used to couple to ring and disk WGRs [65,66]. Strip-line pedestal antiresonant reflecting waveguides were proposed for robust coupling to microsphere resonators and microphotonic circuits [67].

The operating principle of all these devices is based on providing efficient energy transfer to the resonant circular total internal reflection guided wave in the resonator, representing the WGM, through the evanescent field of a guided wave or a total internal reflection spot in the coupler. Efficient coupling occurs on fulfillment of two main conditions: (1) phase synchronization and (2) significant overlap of the WGM and the coupler mode.

To our knowledge, the best coupling was realized with tapered fiber couplers. A fiber taper can be placed alongside a resonator, allowing simple focusing and alignment of the input beam as well as collection of the output beam. In the context of critical coupling, it very efficiently filters all other waveguide modes, save the fundamental, at both the input and the output. The mode filtering performed by the fiber taper before and after the coupling region results in a taper–resonator system that is, for all intents and purposes, a single-mode coupler and, thus, effectively an ideally matched coupler. Efficiency of tapered-fiber couplers reaches 99.99 percent for coupling fused silica resonators [60]. The couplers were used in photonic crystal resonators as well [68,69]. On the other hand, fabrication of fiber tapers is generally more challenging for the higher-index crystalline resonators since they require fibers with a high index of refraction to be suitable for the critical coupling with the resonators (e.g., the refractive index of the fiber should exceed 2.2 to couple a $\mathrm{LiNbO_3}$ resonator at 1.55 μm). Another disadvantage of the practical application of the couplers is their fragility.

Coupling with comparably low order WGMs can be realized with "symmetry breaking techniques." For example, a directional coupling of light output from WGM microdisk lasers was described in [70]. Patterned asymmetries in the shape of the microdisk resonators provide control of both direction and intensity of light output without dramatically increasing laser thresholds. A "microgear" laser was studied in [71]. Directional emission from asymmetric as well as elliptical WGRs was discussed in [72–76]. Directional coupling via linear defect at some distance away from the circumference of a microdisk resonator was studied in [77].

C. Transparent Materials for WGRs

Micron-sized liquid droplets are widely used for cavity-enhanced spectroscopy (see [78] for review). For example, there were studies of fluorescence and lasing of dye-containing liquids [29,79–81]. Stimulated Raman scattering in $\mathrm{CS_2}$ [82], $\mathrm{CCl_4}$ [83], water [84], glycerol [85], and other droplets was studied. Liquids are transparent and highly nonlinear. However, to be practically suitable, WGRs should be produced from materials that can be easily handled. Applications of liquid WGRs in photonics are rather involved because of difficulty in manipulating the resonators. Solid-state WGRs, fabricated from amorphous and crystalline materials, are more promising here.

Amorphous materials, like fused silica, can possess very small optical attenuation. The highest quality factor of WGMs has remained limited by Rayleigh scattering of residual surface roughness [44]. This is generally the case, even though a cavity formed by surface tension forces has nearly a defect-free surface characterized by molecular-scale inhomogeneities. Any residual roughness would result in serious limitation on the achievable cavity Q-factor. Another limiting factor for the Q of fused silica near the bottom of its transparency window at 1.55 μm is chemosorption of $\mathrm{OH^-}$ ions and water [42,43].

Polymer resonators comprise another example of amorphous WGRs [86–90]. Although fabrication of polymer resonators is simple enough, their performance is limited because of strong absorption of light in the polymers. The Q-values of polymer microresonators is limited by Rayleigh scattering, band-edge absorption, and vibrational absorption in the visible spectral range.

Optical crystals in their turn possess many important properties that make them interesting as host materials for WGRs. It is important to note that the earliest studies of WGRs were initialized by crystalline resonators [16,17]. Very recently crystalline resonators have been realized with photo-refractive crystalline materials [91–94].

It is expected that the crystals would have less loss than fused silica because crystals theoretically have a perfect lattice without the imperfections, inclusions, and inhomogeneities that are always present in amorphous materials. The window of transparency for many crystalline materials is much wider than that of fused silica. Therefore, with sufficiently high purity material, much smaller attenuation in the middle of the transparency window can be expected—as both the Rayleigh scattering edge and the multi-phonon absorption edge are pushed further apart towards the ultraviolet and infrared regions, respectively. Moreover, crystals may suffer fewer, or no, extrinsic absorption effects caused by chemosorption of water.

For example, absorption of sapphire determined by light scattering due to imperfection of the crystalline structure is less than $\alpha = 1.3 \times 10^{-5}$ cm^{-1} at $\lambda = 1$ μm [95], which corresponds to $Q \simeq 8 \times 10^9$. Light absorption for the crystalline quartz should be better than absorption in fused silica, which is extensively studied for fiber-optic applications, with $\alpha \leq 5 \times 10^{-6}$ cm^{-1} at $\lambda = 1.55$ μm [96]; this corresponds to $Q \geq 1.2 \times 10^{10}$ for quartz.

Melting is obviously not suitable for materials with crystalline structure because it destroys the initial crystal purity and stoichiometry. Moreover, during solidification, the original spherical droplet of the melt turns into a rough body with multiple facets and crystal growth steps. It is possible to obtain crystalline WGRs with very high Q-factors ($Q > 10^{10}$) [92], similar to that of surface-tension-formed resonators [42], by adopting simple polishing techniques. With this approach, the original crystal structure and composition is preserved, and the unique linear and nonlinear crystal properties are enhanced with the small volume of the high-Q cavity. Total internal reflection at the walls of the WGRs provides the effect of an ultra-broadband mirror, allowing very high Q-factors across the whole material transparency range. This property makes crystalline WGRs a unique tool for optical materials studies.

There is very little consistent experimental data on small optical attenuation within transparency windows of optical crystals. For example, the high sensitivity measurement of the minimum absorption of specially prepared fused silica becomes possible only because of kilometers of optical fibers fabricated from the material. Unfortunately, this method is not applicable to crystalline materials. Strictly speaking, fibers also have been grown out of crystals such as sapphire [97], but attenuation in those (a few decibels per meter) was strictly determined by scattering of their surface. Therefore, other methods of analysis of optical absorption in crystals should be used. The most sensitive calorimetry methods for measurement of light absorption in transparent dielectrics give an error on the order of $\Delta\alpha \geq 10^{-7}$ cm^{-1} [98]. Several transparent materials have been tested for their residual absorption with calorimetric methods, while others have been characterized by direct scattering experiments [95,99,100], both yielding values at the level of a few parts per million per centimeter (ppm/cm) of linear attenuation, which corresponds to a Q limitation at a level of 10^{10}. The question is whether this is a fundamental limit or the measurement results were limited by the imperfection of crystals used.

WGRs help in studies of the crystal properties. For example, WGRs made of $\mathrm{LiNbO_3}$ recently were used to produce resonant interaction of light and microwaves [93,94,101–105]. The maximum quality factor of WGMs in the resonators reported in [105] was less than 5×10^6 at $\lambda = 1.55$ μm, which approximately corresponded to values expected from the intrinsic absorption of congruent lithium niobate cited by crystal producers ($\alpha \leq 5 \times 10^{-3}$ cm^{-1}). On the other hand, $Q \approx 2 \times 10^8$ at $\lambda = 2.014$ μm

($\alpha \leq 5 \times 10^{-4}$ cm^{-1}) was reported earlier for a multiple total-internal-reflection resonator [106]. The same value of Q at $\lambda = 1.3$ μm was obtained in [92]. This suggests that the Q-factors achieved were either limited by the fabrication process or that the above values for the absorption of congruent lithium niobate are inaccurate.

Selection of material for the highest-Q WGM resonator must be based on fundamental factors such as the widest transparency window, high purity grade, and environmental stability. For this reason, alkali halides that have the widest transparency, as known by spectroscopy, have to be rejected based on their hygroscopic property and sensitivity to atmospheric humidity. Bulk losses in continuous solid transparent materials can be approximated with the phenomenological dependence [99]

$$\alpha \simeq \alpha_{UV}\, e^{\lambda_{UV}/\lambda} + \alpha_R\, \lambda^{-4} + \alpha_{IR}\, e^{-\lambda_{IR}/\lambda} \tag{1}$$

where α_{UV}, α_R, and α_{IR} describe the blue wing (primary electronic), Rayleigh, and red wing (multiphonon) losses of the light, respectively, and λ_{UV} and λ_{IR} stand for the edges of the material transparency window. This expression does not take into account resonant absorption due to possible crystal impurities. Unfortunately, even phenomenological coefficients in Eq. (1) are not always known.

One of the most attractive candidates for fabrication of high-Q WGM resonators is calcium fluoride. It has attracted a lot of attention because of its use in ultraviolet lithography applications at 193 and 157 nm. Very large ultra-pure uniform crystals of this material, suitable for wide-aperture optics, have been grown and are commercially available. According to recently reported measurements on scattering in CaF$_2$, yielding the value $\alpha = 3 \times 10^{-5}$ cm^{-1} at 193 nm [100], extremely small scattering can be projected in the near-infrared band corresponding to the limitation of Q at the level of 10^{13} [92].

D. Linear Properties of WGMs

1. Spectral Properties of WGMs in a Sphere. Spectra of WGRs are determined by their shape and/or by spatial distribution of the refractive index inside the resonator bodies. A general analytic solution to describe those spectra is difficult to obtain, although the structure of WGMs was studied numerically [107,108]. The simplest eigenvalue and eigenfrequency problem for electromagnetic field propagation in a dielectric sphere and cylinder has been solved in, e.g., [1]. Theory of WGM resonances in spherical WGR is developed in [109–111] analogously to quantum-mechanical shape resonances. Orthogonality of WGM was proven in [112]. An analytical expression for the spectrum of the high-order WGMs is derived in [113,114].

Let us present a simple method for approximate description of high-order WGMs [50,115]. We consider a dielectric sphere with dielectric constant distribution $\epsilon(r)$ that depends on the radius r only. The electric field in the sphere obeys the Maxwell equation

$$\nabla \times (\nabla \times \mathbf{E}) + \frac{\epsilon(r)}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \tag{2}$$

where c is the speed of light in the vacuum. Presenting the electric field as $\mathbf{E} = \int_0^\infty d\omega \mathbf{e}(r)\exp(-i\omega t)$, we rewrite Eq. (2) as

$$\nabla \times (\nabla \times \mathbf{e}) - k^2 \epsilon(r)\mathbf{e} = 0 \tag{3}$$

where $k = \omega/c$ is the wave vector. Equation (3) may be solved in terms of the transverse electric (TE) and transverse magnetic (TM) modes. Keeping in mind that $\nabla \cdot (\epsilon \mathbf{e}) = 0$, we write

$$\mathbf{e}=\sum_{\nu,m}\frac{1}{r}\left[\Psi\mathbf{Y}_{\nu,m}+\frac{1}{\epsilon(r)}\nabla\times(\Phi\mathbf{Y}_{\nu,m})\right] \tag{4}$$

where radial functions Ψ and Φ stand for the TE and TM modes, respectively, and $\mathbf{Y}_{\nu,m}$ are vector spherical functions with angular number ν and magnetic number m. It is worth noting that modes of an infinite dielectric cylinder may be described in a similar way.

Radial field distribution for TE modes, for instance, of a dielectric cavity (sphere or cylinder) can be described by

$$\frac{\partial^2\Psi}{\partial r^2}+\left[k^2\epsilon(r)-\frac{\nu(\nu+1)}{r^2}\right]\Psi=0 \tag{5}$$

where ν is the angular momentum number ($\nu=0,1,2,3,\cdots$ for a sphere; $\nu=1/2,3/2,5/2,\cdots$ for an infinite cylinder). Electric-field distribution has a dependence $\Psi(r)/r$ for a sphere and $\Psi(r)/\sqrt{r}$ for a cylinder.

Equation (5) has an exact solution for homogeneous dielectric cavity $\epsilon(r)=\epsilon_0=\text{const}$. This solution reads $\Psi(r)=J_{\nu+1/2}(kr)$, where $J_{\nu+1/2}(kr)$ is the Bessel function of the first kind. The mode spectrum is determined by the boundary conditions $\Psi(r)\to 0$ for $r\to\infty$ and 0. In the case of high TE mode order $\nu\gg 1$,

$$k_{\nu,q}\simeq\frac{1}{a\sqrt{\epsilon_0}}\left[\nu+\alpha_q\left(\frac{\nu}{2}\right)^{1/3}-\sqrt{\frac{\epsilon_0}{\epsilon_0-1}}+\frac{3\alpha_q^2}{20}\left(\frac{2}{\nu}\right)^{1/3}+O\left(\nu^{-2/3}\right)\right] \tag{6}$$

where a_q is the qth root of the Airy function, $Ai(-z)$ [116], which is equal to 2.338, 4.088, and 5.521 for $q=1,2,3$, respectively [113]. The expression for TM WGM spectrum looks similar:

$$k_{\nu,q}\simeq\frac{1}{a\sqrt{\epsilon_0}}\left[\nu+\alpha_q\left(\frac{\nu}{2}\right)^{1/3}-\frac{1}{\sqrt{\epsilon_0(\epsilon_0-1)}}+\frac{3\alpha_q^2}{20}\left(\frac{2}{\nu}\right)^{1/3}+O\left(\nu^{-2/3}\right)\right] \tag{7}$$

The third terms in the right-hand sides of Eqs. (6) and (7) represent the fact that the dielectric WGRs are open. The optical field tunnels are outside the resonator's surface at the characteristic length $\sim 1/(k_{\nu,q}\sqrt{\epsilon_0-1})$. The larger the susceptibility ϵ_0, the smaller is this length and the closer are properties of the open resonator and ideal closed resonator.

The first-order approximation for the high-order mode eigenfunctions and eigenvalues may be found from the solution of an approximate equation:

$$\frac{\partial^2\Psi}{\partial r'^2}+\left(k^2\epsilon_0-\frac{\nu(\nu+1)}{a^2}-r'\frac{2\nu(\nu+1)}{a^3}\right)\Psi=0 \tag{8}$$

where we assume that $\nu\gg 1$, $r'=a-r$, and $\Psi(0)=\Psi(a)=0$, and a is the radius of the sphere or cylinder. Comparison of the numerical solution of the exact equation, Eq. (5), and of the approximate equation, Eq. (8), shows that the solution of Eq. (8),

$$\Psi_q(r') = \Psi_{q,0} Ai \left[\left(\frac{2\nu(\nu+1)}{a^3} \right)^{1/3} \frac{r'}{\alpha_q} - \alpha_q \right] \tag{9}$$

where $\Psi_{q,0}$ is the field amplitude and k_q is the root of the equation

$$k_{\nu,q}^2 \epsilon_0 - \frac{\nu(\nu+1)}{a^2} = \alpha_q \left(\frac{2\nu(\nu+1)}{a^3} \right)^{2/3} \tag{10}$$

gives satisfactory results for the eigenvalues as well as eigenfunctions of the exact problem. For instance, it is easy to see that Eq. (10) gives a close approximation of the first two terms of Decomposition (6).

There are several experimental studies of WGM spectra in spherical WGRs. Spectra of liquid droplets were studied in [24,25,117–120]. High-resolution spectroscopy of WGMs in large fused silica spheres (3.8-cm diameter) was reported in [50]. Visualization of WGMs in WGRs was also realized [121–127].

2. Spectral Properties of WGMs in a Spheroid. Calculations for spheroid WGRs are more complicated. Deviation of the resonator shape from an ideal sphere results in removal of degeneracy by m and other effects. For a spheroid (ellipsoid) characterized with small deviation from an ideal sphere, the frequency shift of each WGM is given by [128]:

$$\frac{\Delta k_{\nu,m,q}}{k_{\nu,q}(a)} = \frac{\varepsilon}{6} \left[\frac{3m^2}{\nu(\nu+1)} - 1 \right] \tag{11}$$

where $\varepsilon = (r_{pol} - r_{eq})/a$, and r_{eq} and r_{pol} are the equatorial and polar radii, respectively. Hence, $\varepsilon > 0$ corresponds to prolate ellipsoids, and $\varepsilon < 0$ corresponds to oblate ellipsoids. Equation (11) does not depend on number q and average radius $a \approx (r_{pol} + 2r_{eq})/3$ (we derive it from the condition of volume conservation if the sphere is transformed into spheroid $r_{eq}^2 r_{pol} = a^3$).

Approximations for oblate and prolate spheroids with large eccentricity were reported in [105,129,130] and [131], respectively. To estimate the eigenvalues of the WGMs in an oblate spheroid of large equatorial radius r_{eq}, small polar radius r_{pol}, $r_{eq} \gg r_{pol}$, and eccentricity $\widetilde{\varepsilon} = \sqrt{1 - r_{pol}^2/r_{eq}^2}$, we recall that eigenfrequencies of high-order WGMs ($\nu \gg 1$; $m \simeq \nu$) (in an ideal sphere as well as in the spheroid) can be approximated via solutions of the scalar wave equation with zero boundary conditions. This is because most of the energy is concentrated in one component of the electromagnetic field (E_θ for TE modes and E_r for TM modes), and the tangential component of the electric field $\mathbf{E}$ (TE modes), or normal component of induction $\mathbf{D}$ (TM modes), is continuous at the boundary. For the spheroid expression, similarly to Eq. (6), we have

$$\sqrt{\epsilon_0} \widetilde{k}_{m,q} r_{eq} = T_{m,q} - \sqrt{\frac{\epsilon_0}{\epsilon_0 - 1}} \tag{12}$$

where $\widetilde{k}_{m,q} = \sqrt{k_{\nu,m,q}^2 - k_\perp^2}$, $k_\perp$ is the wave number for the angular spheroidal function, and $T_{m,q}$ is the qth zero for cylindrical Bessel function $J_m(T_{m,q}) = 0$. Since whispering-gallery modes are confined in the cavity “equatorial” region, we use cylindrical, not spherical, functions in our calculations.

For our purposes, a rough approximation of $k_\perp$ is enough:

$$k_\perp^2 \approx \frac{2(\nu - m) + 1}{r_{eq}^2 \sqrt{1 - \widetilde{\varepsilon}^2}} m \tag{13}$$

Because $k_{\nu,m,q} \approx \nu \epsilon_0^{-1/2} r_{eq}^{-1}$ and $k_\perp \approx (1 - \widetilde{\varepsilon}^2)^{-1/4} \nu^{1/2} r_{eq}^{-1}$, $k_{\nu,m,q} \gg k_\perp$, we write

$$k_{\nu,m,q} \simeq \widetilde{k}_{m,q} + \frac{k_\perp^2}{2\widetilde{k}_{m,q}} \tag{14}$$

Substituting Eqs. (12) and (13) into Eq. (14), and taking into account that $T_{m,q} - [\epsilon_0/(\epsilon_0 - 1)]^{1/2} \approx k_{\nu,q}/(r_{eq}\epsilon_0^{1/2}) - (\nu - m + 1/2)$ $[k_{\nu,q}(r_{eq})$ is given by Eq. (6)], we finally derive

$$k_{\nu,m,q} - k_{\nu,q}(r_{eq}) = \frac{2(\nu - m) + 1}{2 r_{eq} \sqrt{\epsilon_0}} \left(\frac{1 - \sqrt{1 - \widetilde{\varepsilon}^2}}{\sqrt{1 - \widetilde{\varepsilon}^2}} \right) \tag{15}$$

To compare this expression with Eq. (11), we note that both Eqs. (11) and (15) give the same frequency splitting between two successive modes in the case of small eccentricity:

$$k_{\nu+1,m,q} - k_{\nu,m,q} \simeq k_{\nu,m,q} - k_{nu-1,m,q} \approx \frac{1}{\epsilon_0^{1/2} a} \left(1 + 0.62\nu^{-2/3} + O(\nu^{-5/3}) \right) \tag{16}$$

$$k_{\nu,m+1,q} - k_{\nu,m,q} \simeq k_{\nu,m,q} - k_{\nu,m-1,q} = -\frac{1}{\epsilon_0^{1/2} r_{eq}} \left(\frac{1 - \sqrt{1 - \widetilde{\varepsilon}^2}}{\sqrt{1 - \widetilde{\varepsilon}^2}} \right) \approx \frac{\varepsilon}{\epsilon_0^{1/2} a} \tag{17}$$

Spectra of highly oblate spheroid WGRs were studied in [130,132].

3. Management of WGM Spectra. Applications of WGMs call for efficient methods of engineering WGM spectra. Originally proposed spherical WGRs are over-moded, with a complex quasi-periodic spectrum and unequal mode spacings translating from both material and cavity dispersion. There are several recipes for control of WGM spectra. Manipulations of the WGR shape result in controllable rarefication of the resonators' spectra [93]. As is shown above, significant reduction in the mode spectral density is achieved in highly oblate spheroidal WGRs [130,132]. Microring resonators, as quasi-single-dimension objects, naturally have very clean spectra [88,133,134].

It is not always practically useful or convenient to tune the spectrum of a big WGR by changing its shape. The dense optical spectrum associated with a big WGR represents a limitation for its application in systems that require large side-mode rejection. The spectral density of the majority of geometries for WGRs could be significantly decreased with application of specially designed mode dampers [135]. A prism, or other polished piece of a material, with an index of refraction higher than the index of refraction of the resonator material, can be used to decrease Q-factors of the majority of the unwanted modes of the resonator. Ideally, only the modes of the main sequence survive. The Q-factor of those modes slightly changes in the mode rarefication process, but many applications tolerate that change. The selective suppression of the modes' Q's is possible because various WGMs are localized in various geometrical places. To avoid deterioration of the Q-factors of the main mode sequence, the dumper should not be in the same plane as the coupling prism, i.e., angle a should not be equal to zero.

Another problem of management of WGM spectra is related to fabrication of a WGR with an equidistant spectrum. Performance and range of applications based on WGRs will be significantly expanded if a method is found to make resonator modes equally spaced with precision corresponding to a fraction of the resonance bandwidth. Such a dielectric resonator with equidistant mode spectrum is similar to the Fabry–Perot resonator. A WGR with an equidistant spectrum may be used, for example, in frequency comb generators, optical pulse generators, broadband energy storage circuits of electro-optical devices, and in other applications where conventional optical cavities are utilized.

Within current technology based on uniform resonator material, the smaller the resonator size, the more the resonator geometrical dispersion is manifested in unequal spectral separation between adjacent modes. The problem is rooted in the fact that the radial distribution of WGMs is frequency dependent. Higher-frequency modes propagate on paths that are slightly closer to the surface than those of lower-frequency modes. Thus, higher-frequency modes travel in trajectories of slightly larger radii and slightly longer optical path lengths.

To describe dispersion of the modes, we compare a value equal to the ratio of frequency separations between two pairs of neighboring modes and the mode width. The number of equidistant modes in the case of large ν can be estimated as

$$N = \max_q \left| \left(\frac{\partial^2 k_{\nu,q}}{\partial \nu^2} \right)^{-1} \frac{k_{\nu,q}}{2Q} \right| \tag{18}$$

From Eq. (6), we derive

$$N_1 \simeq 1.2 \, \frac{\nu^{8/3}}{Q} \tag{19}$$

The mode dispersion can be very high for realistic conditions. For example, for $\nu = 10^3$, resonator modes can already be treated as un-equidistant for $Q \geq 10^8$. Keeping in mind that the maximum achieved quality factor for a WGM in fused silica resonators is 9×10^9 [42], one can see that the geometrical dispersion problem is really important in small WGRs. On the other hand, the material, not geometrical dispersion, is the most significant factor in large resonators [136].

Optical path length is a function of both the physical distance and the index of refraction. One way of creating a WGR with an equidistant spectrum is with mutual cancellation of the material and geometrical dispersion [137] if an optimum size of the resonator is chosen. Another method is based on the fabrication of a resonator out of a cylindrically symmetric material whose index decreases in the radial direction [138]. With the proper choice of gradient of the refractive index, circular trajectories corresponding to a WGM at different frequencies will have identical optical path lengths. This results in an equidistant mode spectrum of the resonator [139].

4. Mode Volume. Basic properties of a WGR include its geometrical characteristics of the field localization (mode volume). The volume of WGMs is especially important for nonlinear applications of the resonators [140]. WGRs can have a mode volume orders of magnitude less than in Gaussian-mode resonators. The mode volume for a spherical WGR can be estimated from [140]

$$V = 3.4\pi^{3/2} \left(\frac{\lambda}{2\pi n} \right)^3 \nu^{11/6} \sqrt{\nu - m + 1} \tag{20}$$

where λ is the wavelength of the pumping light.

Numerical calculations of the mode volume of an arbitrary dielectric spheroid were presented in [141]. The additional confinement provided by the toroid geometry results in a mode volume decrease as compared with the spherical geometry. In the case of an oblate spheroid (toroid) when WGMs are localized near the largest spheroid circumference having radius a, the reduction of modal volume scales as $\sim(d/2a)^{1/4}$ with respect to that of a spherical cavity, where d is the minor diameter of the toroid. This is true if d exceeds the wavelength of the WGMs. For a smaller-diameter d, the spatial confinement becomes strong enough that the WGMs are additionally compressed in the radial direction. This results in a faster reduction of modal volume. The WGMs approach modes of a loop made of an optical fiber. The mode volume reduces until the point at which the optical mode becomes delocalized due to the weak geometrical confinement, causing a finite minimum value.

The comparative characteristics of volumes of WGMs in various kinds of WGRs can be found in [8,141,142]. WGMs with the smallest mode volumes, close to cubic wavelength, were realized in photonic crystal resonators [69,143], disordered media resonators [69], microdisks [145,146], and microrings [147]. However, those tiny resonators have much smaller Q-factors than do bigger WGM resonators.

5. Quality Factor. Amongst many parameters that characterize the resonator, the quality factor (Q) is a basic one. The Q-factor is related to the lifetime of light energy in the resonator mode (τ) as $Q = 2\pi\nu\tau$, where ν is the linear frequency of the mode. The ring downtime corresponding to a mode with $Q = 10^{10}$ and wavelength $\lambda = 1.3$ μm is 7 μs, thus making ultra-high-Q resonators potentially attractive as light storage devices.

To date, the highest Q-factor, $Q = 2 \times 10^{10}$, was achieved in crystalline WGRs [92]. The resonators have spheroidal shape and are hand polished. The size of the WGRs varies from 12 mm to 0.5 mm. Restrictions of the Q-factors result from the absorption of the material. Ideally, Q-factor can reach 10^{13} at 1.55 μm in fluorite resonators.

The highest measured Q-factor in amorphous WGM resonators is $Q = 8 \times 10^9$ at 633 nm [42]. Approximately the same WGM Q-factors were observed in the near-infrared [43] (1.55 μm) and at 780 nm [148]. The WGR radius varies from 60 μm to 200 μm in [148], and from 600 μm to 800 μm in [42,43]. The measured Q-factors are close to the maximum achievable value of Q for fused silica [44].

Q-factors measured in liquid WGRs (e.g., free-flying or trapped droplets of liquid aerosols) are less than 10^5. The problem is in the difficulty of excitation and detection of WGMs with larger Q using a free-beam technique [149]. Theoretical implications from the experimental data for the Q-factors in liquid WGRs are more optimistic: $Q \geq 10^6$ [38,150–153] for $\sim$20-μm droplets. On the other hand, it was shown that a 400-μm liquid-hydrogen droplet can achieve high Q-values that exceed 10^9 for WGMs in the ultraviolet [154].

Quality factor $Q \approx 2 \times 10^8$ at $\lambda = 2.014$ μm ($\alpha \leq 5 \times 10^{-4}$ cm^{-1}) was reported for a multiple total-internal-reflection resonator, analogous to a WGR, used in optical parametric oscillators pumped at 1064 nm [106]. The same order of Q was measured in fused silica resonators of the same shape at 1064 nm [155].

Quality factors of microring and microdisk WGRs typically do not exceed 10^5. For instance, all-epitaxial semiconductor 10-μm-diameter microring resonators vertically coupled to buried heterostructure bus waveguides have $Q = 2.5 \times 10^3$ [156]. An unloaded Q-factor of the order of 10^5 was demonstrated in 80-μm microdisks [31]. Micron-size microdisk semiconductor resonators have Q's of the order of 10^4 [145,146].

E. Nonlinear Properties of WGMs

Whispering-gallery modes play a significant role in modern nonlinear optics. High quality factors and large field densities associated with whispering-gallery modes in dielectric resonators result in resonant enhancement of nonlinear interactions of various kinds [42,43,140,148,157,158]. These modes provide the opportunity to achieve a high nonlinear response with weak electromagnetic fields, even if the cavity is fabricated from a material with low nonlinearity, as is usually the case for optically transparent materials.

Crystalline WGRs with kilohertz-range resonance bandwidths at room temperature and high resonance contrast (50 percent and more) are extremely promising for integration into high-performance optical networks. Because of small modal volumes and extremely narrow single-photon resonances, a variety of low-threshold nonlinear effects can be observed in WGRs based on small broadband nonlinear susceptibilities. As an example, in the following we consider the thermo-optical instability in WGRs.

Thermal nonlinearity is important in high-Q WGRs [159,160,162]. The thermal dependence of the index of refraction for calcium fluoride, for instance, is $\beta = n_0^{-1}\partial n/\partial T \simeq -10^{-5}/K^o$. It means that the frequency of a WGM increases by $10^{-5}f$ if the temperature T increases by one kelvin (this follows from $2\pi f \approx c\nu/an_0(1+\beta)$, where c is the speed of light in the vacuum, $\nu \gg 1$ is the mode number, a is the radius of the resonator, and n_0 is the index of refraction). This shift is five orders of magnitude larger than the width of the resonance if $Q = 10^{10}$.

Because of the thermal nonlinearity, the trace of the resonance on the screen of an oscilloscope changes depending on the laser power and the speed and direction of the laser scan. This effect is produced because of heating of the mode volume by the power absorbed in the material, resulting from the nonzero optical losses. The process can be described with two time constants, one of which is responsible for the flow of the heat from the mode volume to the rest of the resonator, and the other for heat exchange between the resonator and the external environment. To reduce the influence of nonlinearity on the measurement results, the laser scan should be fast compared with the relaxation constants, and the light power must be small as well.

In the simplest approximation, the evolution of the system can be described with a set of two equations [160,161]:

$$\dot{e} + e\left[\gamma + i\left(\omega + \delta\right)\right] = F(t)$$

$$\dot{\delta} + \Gamma\delta = \Gamma\xi|e|^2$$

where e is the complex amplitude of the field in the resonator mode, γ is the mode linewidth, $\omega = 2\pi f$ is the mode frequency, δ is the thermal frequency shift, $F(t)$ stands for the external pump, Γ characterizes thermal relaxation rate, and ξ is the thermal nonlinearity coefficient.

II. Passive WGM Devices

Unique spectral properties of WGMs, including narrow linewidth, tunability, and high stability to environment conditions, make WGRs useful for numerous practical applications. In this section, we review applications of WGRs for filtering and sensing.

Photonics filters based on optical WGRs are among the most developed devices that involve WGMs. The filters have been devised primarily to address the shortcomings of microwave filters. Microwave filters with narrow bandwidth and wide tunability are crucial to the realization of advanced communications and radar schemes. The narrow bandwidth naturally allows increased channel capacity in a communications band, while tunability provides spectral diversity and increased efficiency. Unfortunately, microwave filters that also have a bandpass with a flat top and high side-mode rejection cannot simultaneously provide narrow bandwidths (high quality factor, Q) and wide tunability. Typical high-Q filters in the 10-GHz regime have bandwidths in the range of a few megahertz, with cavity filters providing the highest Q's.

These filters, nevertheless, have either a fixed center frequency or are tunable through bandwidths that represent a minor fraction of the center frequency. The high-Q filters also introduce several decibels of insertion loss in microwave circuits; the typical insertion loss for a multi-pole filter with a few megahertz linewidth at a 10-GHz center frequency is about 10 dB. It also should be mentioned that the quality of microwave filters generally degrades as the center frequency increases above several tens of gigahertz. As is shown in the following, WGM-based photonics filters give a solution to the problems.

Research and development studies and follow-up commercialization of devices that utilize WGMs in laser stabilization, spectroscopy, and metrology are on the way.

A. Optical and Photonic Single-Resonator Filters

Transmission of a monochromatic electromagnetic wave of frequency f by an optical lossless WGR in a single prism configuration may be characterized by coefficient

$$T = \frac{\gamma_c - \gamma - i(f - f_0)}{\gamma_c + \gamma + i(f - f_0)} \tag{21}$$

where T describes the amplitude transmission, and γ, γ_c, and f_0 are the absorption, coupling linewidth, and resonance frequency of a mode of the resonator, respectively (we assume that $|f - f_0|$ is much less than the cavity-free spectral range). The power transmission $|T|^2$ through the resonator is Lorentzian. Condition $\gamma = \gamma_c$ corresponds to critical coupling of the resonator [163,164].

It is important to note that the filter is characterized by absorption resonance in a single coupling prism (single coupler) configuration (stopband filter), while in the two-prism configuration it is characterized by transmission resonance (passband filter). In the latter case, the transmission and reflection coefficients through the resonator are

$$\left.\begin{aligned} T &= \frac{\gamma_c}{\gamma_c + i(f - f_0)} \\[2ex] R &= \frac{i(f - f_0)}{\gamma_c + i(f - f_0)} \end{aligned}\right\} \tag{22}$$

where T and R describe the amplitude transmission (light goes into one prism and exits the second prism) and reflection (light goes into and exits the same prism), respectively, and $\gamma_c \gg \gamma$ is assumed for simplicity.

Single ring-shaped WGR-based filters were studied in [88,156,166,167]; see also [8,165] for a review. The filter response of single-ring resonators with integrated semiconductor optical amplifiers based on GaInAsP-InP is presented in [168].

Two dielectric waveguides that are evanescently coupled to a square or rectangular region of increased refractive index can serve as a very compact integrated optical microresonator, similar to a ring WGR. Applications of the device for filtering were discussed in [169].

Unfortunately, the Lorentzian line shape of the filter function associated with a single microresonator represents a limitation for its application in many systems that require large side-mode rejection in addition to a narrow bandpass and a large tuning range.

B. Tunable Filters

While delivering high optical Q's and desirable passband spectra, optical WGM filters based on silica ring resonators are limited in their microwave Q and in availability of convenient tuning technique. Mechanical trimming of WGMs with applied strain [170–172] and temperature [173] have been previously used. Although the mechanical as well as temperature tuning ranges are large, e.g., on the order of a few to several tens of nanometers, these methods are not very convenient for many applications because of small speeds and low accuracy. The tuning accuracy is especially important for high-Q resonators with a narrow filter bandwidth. An all-optical tunable filter design based on discontinuity-assisted ring resonators that does not have the above-mentioned disadvantages also has been proposed theoretically [174], but, to our knowledge, no experimental implementation of the configuration has been reported.

A technique for WGM resonance tuning was demonstrated using microring resonators with a photosensitive coating. In that study, glass microrings were dipped in a polymer coating material and were exposed to ultraviolet (UV) light. This method produced resonators with relatively small Q (about 800) because of the polymer-induced absorption, but it still allowed large tunability of the optical resonance of the microring, enough for wavelength selective applications [175].

A method for trimming the wavelength of polymer optical microresonator-based optical notch filters was proposed in [176]. The method is based on photobleaching chromophores. A maximum wavelength shift of -8.73 nm was observed. The resonators had an intrinsic Q-value of $\sim 2\times10^4$ and a finesse of $\sim$10.

Another approach for trimming the frequency of microresonators exploits the photosensitivity of the germanate silica glass. When exposed to UV light, this material undergoes a small permanent change in structure that alters its index of refraction. In the case of a WGR, the spatially uniform change in the index of refraction results in a uniform translation of the resonant frequencies. Such a tunable resonator, as well as a second-order optical filter based on two coupled resonators, one of which was tunable, was experimentally realized for optical high-Q (10^8) WGMs [177–179].

Recently, fabrication of optical WGM resonators with lithium niobate [180] has led to the demonstration of a high-Q microwave filter with a linewidth of about 10 MHz and a tuning range in excess of 10 GHz [181]. The best tunability for a LiNbO_3 single resonator filter was ±20 GHz by applying DC voltage of ±50 V to an electrode placed over the resonator.

Theoretically, the maximum frequency shift of the TE and TM modes in a crystalline (LiNbO_3, LiTaO_3, or any other host material with the same symmetry) WGR with crystalline symmetry axis-z (3) corresponding to the axis of the resonator can be found from [182]

$$\left.\begin{aligned}\Delta f_{TE} &= f_0\frac{n_e^2}{2}r_{33}E_Z\\[2ex] \Delta f_{TM} &= f_0\frac{n_o^2}{2}r_{13}E_Z\end{aligned}\right\}\tag{23}$$

where f_0 is the carrier frequency of the laser, r_{33} and r_{13} are the electro-optic constants of the crystalline host material ($r_{33} = 31$ pm/V and $r_{13} = 10$ pm/V for LiNbO_3), n_e and n_o are the extraordinary and ordinary refractive indices ($n_e = 2.28$ and $n_o = 2.2$ are the refractive indices of LiNbO_3 at $f_0 = 2\times10^{14}$ Hz), and E_Z is the amplitude of the electric field applied along the cavity axis.

In experiments [181], TM modes were used because they have larger quality factors than do TE modes. If the quality factor is not very important, it is better to use the TE modes because their electro-optic shifts are three times as large as those of TM modes for the same values of the applied voltage. Numerical estimations obtained with Eq. (23) are in good agreement with the experimental measurements. Theoretically, Δf_{TE} and Δf_{TM} do not depend on the resonator properties and are related to the fundamental limitations of optical resonator-based high-speed electro-optic modulators [183].

Optical filters with bandwidths of about 10 kHz using CaF_2 WGM resonators were demonstrated in [92]. The CaF_2 resonators have very stable ultra-high Q-factors compared with fused silica resonators, where Q degrades with time. The CaF_2 filters can be tuned by their temperature change. The insertion loss of the filter was at the 5-dB level.

Absolute tunability of optical resonator-based photonic filters is characterized by the ratio of their free spectral range (FSR) and linear tunability range. Tuning the filters does not change the FSR but only shifts the comb of the optical modes, making it overlap with itself for each frequency shift proportional to the FSR. Hence, the filter can be tuned at any prescribed single frequency if the linear tunability exceeds the FSR.

Some photonic applications call for narrowband filters passing simultaneously both the carrier and side bands. For example, this is important for generation of spectrally pure microwave signals in opto-electronic oscillators [184], where beating of the optical side bands and the carrier on a fast photodiode generates microwaves. Tunability of the microwave frequency of the oscillator requires that the frequency difference between the filter passbands change controllably. This property is lacking in existing tunable filters, in which the entire filter spectrum shifts as a whole as the tuning voltage is applied. A critical component of a novel miniature filter with an electro-optically reconfigurable spectrum was recently reported. The filter is based on a WGR fabricated from a commercially available lithium niobate wafer having a specially engineered domain structure [185].

C. High-Order Filters

Coupled optical fiber resonators are widely used as optical and photonic filters [186,187]. Newly developed WGRs also are promising for achieving that goal because of their small size, low losses, and integrability into optical networks.

Multi-pole, high-Q filters based on cascaded WGRs fabricated with silica have been demonstrated, allowing compact packages and robust performance at 10- to 100-GHz bandwidths and corresponding optical Q's on the order of 10^5 to 10^4 [133,188–192], and they are in fact commercially available. These filters provide passbands with a flat top and sharp skirts, suitable for high-performance applications. Since the microwave signals in photonic systems are side bands of an optical carrier, these filters, in principle, can be used at any microwave frequency, providing the same characteristics throughout the band, from 10 to 100 GHz, and higher.

High-Q WGR resonators can be used to create microwave photonic filters with a several orders of magnitude narrower bandwidth. The first narrowband second-order optical filter based on two coupled high-Q WGRs, one of which was tunable, was realized in [178]. The tunable resonator was made of germanate glass.

A three-resonator narrowband photonic filter made of $LiNbO_3$ WGRs was demonstrated in [193]. The filter had the following distinctive features/advantages over other WGM filters: (1) agile tunability accompanied by a high-order filter function, (2) narrow linewidth, and (3) low fiber-to-fiber loss. The combination of these three features made the filter a unique device for a wide range of applications in optics and microwave photonics.

Multi-resonator filters have significantly more sparse spectra as compared with a stand-alone WGM resonator. This feature is due to the so-called Vernier effect [194] and is similar to the coupled fiber-ring resonators [186,187], which are noted for a rare spectrum. The efficient finesse of such multi-resonator systems is very large, e.g., the FSR of the filter reported in [193] exceeds a terahertz.

Based on the Vernier effect, polymer double microring filters with thermo-optic as well as electro-optic tuning are demonstrated [195]. It was noted that one of several advantages of using coupled microrings for filtering is in the possibility of high tuning enhancement factor M given by

$$M = \frac{1}{1 - a_2/a_1} \tag{24}$$

where a_1 and a_2 are the radii of the two rings. The tuning range of the double microring filter is M times the tuning range of a single ring [196]. Since M can be very large in double-WGR structures, the faster but much smaller electro-optic effect can be used for tuning.

Each ring resonator has a set of transmission spectra and a wavelength period determined by its free spectral range, f_{FSR}. The two rings have slightly different radii (or effective indices). Therefore, the two sets of transmission peak combs have small different peak spacings. Wavelength tuning is achieved by aligning the peaks in the two sets of combs with the adjustment of index in one or both ring resonators. The filter transmission discreetly jumps from transmission at wavelength f_0 to $f_0 + f_{FSR}$, with M times less voltage applied to one of the rings as compared with the voltage necessary to continuously shift the spectrum of the ring by f_{FSR}.

A tuning enhancement factor of $M = 40$ in a double-ring filter is achieved at wavelengths near 1.55 μm [195]. The tuning rate for the thermo-optic device is 120 GHz/mW, and for the electro-optic device it is 120 GHz/12 V. A tunable laser with a side-mode suppression ratio greater than 30 dB was demonstrated using this filter and erbium-doped fiber amplifier gain. Thermal tuning over 35 nm was achieved [195].

Large group delays in chains of multiple coupled WGRs have been studied in [197,198]. It was shown that the Q-factor of the coupling-split modes for a system of N identical coupled resonators is greater than that of a single resonator in the chain by a factor of N, and even more in the case of optimum coupling [199]. Manipulation by the group velocity of light, and even reaching zero group velocity ("stopping light"), using a chain of interacting tunable optical resonators was discussed in [200].

Generally, WGR tunable filters allow shifting of the spectrum of the resonators; however, they do not provide linewidth-tuning capabilities. Cascaded resonators were proposed to be used for real-time shaping of their modal structures [201,202]. A key feature of the approach is that it points to a simple tuning of the frequency and the width of the filter transmission window, resulting in the tuning of the group delay of optical signals, a highly desirable feature for signal processing applications.

The transmission spectral window of the filter could be curiously narrow. Theoretically, in the case of resonators without absorption, the width of the window can be arbitrarily narrow [201]; however, in reality, the minimum width of the resonance is determined by the material absorption. The physical principle of the filter operation that results in the narrow spectral window has been recognized in [133,186,187]. The existence of the window also has been demonstrated experimentally [175].

D. WGM Filters in Opto-Electronic Oscillators and Lasers for Stabilization

1. WGM Filters for OEO. Generation of spectrally pure signals at 1 to 100 GHz supports new developments in communications, radar, and navigation. The advent of high-throughput optical communications links points to the prospect of networks operating at data rates as high as 160 Gb/s and consisting of multiples of channels separated by a few gigahertz. Schemes for realizing this type of capability rely on sources capable of providing high-frequency, low-phase-noise signals, without which an efficient high-data-rate system would not be possible. Similarly, high-performance radar systems require low-phase-noise oscillators to allow for detection of feeble signals from a dense background clutter.

The opto-electronic oscillator (OEO) is a device that produces spectrally pure signals at many tens of gigahertz based on photonic techniques and, thus, overcomes some of the inherent limitations of the conventional electronic devices [204–208]. The OEO is a generic architecture consisting of a laser as the source of light energy. The laser radiation propagates through a modulator and an optical energy storage element before it is converted to electrical energy with a photodetector. The electrical signal at the output of the modulator is amplified and filtered before it is fed back to the modulator, thereby completing a feedback loop with gain, which generates sustained oscillation at a frequency determined by the filter.

Since the noise performance of an oscillator is determined by the energy storage time, or quality factor Q, the use of optical storage elements allows for the realization of extremely high Q's and thus spectrally pure signals. In particular, a long fiber delay results in micro-second storage times, corresponding to Q's of about a million at a 10-GHz oscillation frequency. This is a high value compared to conventional dielectric microwave cavities used in oscillators [209,210]. The fiber delay line also provides for wideband frequency operation unhindered by the usual degradation of the oscillator Q with increasing frequency. Thus, spectrally pure signals at frequencies as high as 43 GHz, limited only by the modulator and detector bandwidth, have been demonstrated.

In a generic OEO [184], the long fiber delay line supports many microwave modes imposed on an optical wave. A narrowband electrical filter should be inserted into the electronic segment of the OEO feedback loop to achieve stable single-mode operation. The center frequency of this filter determines the operational frequency of the OEO. While this approach yields the desired spectrally pure high-frequency signals, it nevertheless calls for an OEO configuration limited in size by the kilometers of fiber delay needed. Moreover, the long fiber delay is very sensitive to the surrounding environment, so the OEO does not produce an output with high long-term frequency accuracy and stability. The OEO typically is phase locked to a stable reference for long-term stability.

The properties of the OEO with a high-Q WGR in place of the electronic filter were studied in [211]. It was shown that the method allows one to choose virtually an arbitrary frequency of oscillation by tuning the resonator.

2. WGM Filters for Laser Stabilization. Except for stabilization of the OEO, WGRs can be used for laser stabilization. Optical feedback from a high-Q microsphere resonator was used to narrow the spectrum of a miniature high-coherent diode laser, and a nearly half-pitch gradient-index lens served as a coupling element [64]. As was estimated from the variation in frequency-tuning range (the chirp-reduction factor), the fast linewidth of the laser was reduced by more than three orders.

A modification of external optical feedback that includes a WGR was used to narrow the line of a diode laser [63]. A WGM of a high-Q microsphere was excited by means of frustrated total internal reflection while the feedback for optical locking of the laser was provided by the intracavity Rayleigh backscattering. The close to 600-MHz beat note of the two laser diodes optically locked to a pair of orthogonally polarized modes of the same microresonator had the indicated spectral width of 20 kHz and a stability of 2×10^{-6} over an averaging time of 10 s. A theoretical model for the laser stabilization with a WGR was presented in [212].

Finally, instead of locking a laser to a WGM, the opposite was realized in [213]. A WGM of a fused-silica microsphere was locked to a frequency-scanning laser. The resonance frequency was modulated by axial compression of the microsphere, and phase-sensitive detection of the fiber-coupled optical throughput was used for locking.

E. Spectroscopy, Analysis of Chemical and Biological Agents

Starting from liquid WGRs used for resonator-enhanced spectroscopy (see [78] for review), solid-state WGRs were utilize to enhance interaction between light and atoms/molecules. One of the first experiments on the subject was realized in the frame of cavity quantum electrodynamics (QED) [21].

The radiative coupling of free atoms to the external evanescent field of a WGM was detected. It was proven that the evanescent field of the high-Q (5×10^7) and small mode volume (10^{-8} cm^3) fused-silica microsphere enable velocity-selective interactions between a single photon in the WGM and a single atom in the surrounding atomic vapor. An ultrasensitive spectrometer based on a stretched silica microsphere was proposed [172,215].

The next twist in the sensor development was related to WGR-based biosensors [62,129,216]. Optical biosensors typically are transducers that detect the presence of molecules at a surface. They have several desirable features, particularly for the detection of biological molecules, that include: (1) high sensitivity (less than nanomoles); (2) non-destructivity to the sample; (3) high selectivity; and (4) applicability to various substances. The transduction processes in optical biosensors generally take place on a surface and can be tailored to sense almost any kind of molecule, chemical and pre-biotic as well as biological.

Among the most sensitive classes of biosensors are the evanescent wave sensors [217,218], in which an evanescent wave produced by the total reflection of light within the waveguide interacts with analytes on the waveguide surface. The evanescent wave protrudes above the waveguide surface by ∼100 nm (the actual distance depends on the relative index of refraction of the waveguide and the sample medium) and samples only analyte on the surface. Surface treatments such as antibodies or oligonucleotide strands can provide specificity for the analyte; the sensor then detects only that bound to the surface. Transduction mechanisms for bound analyte include fluorescence, mass change in the evanescent region [219], and change in index of refraction [220]. The typical sensitivity of evanescent wave biosensors based on fiber-optic sensors or planar waveguide sensors is in the range of nanomole (nM) to picomole (pM).

The basic detection scheme that utilizes WGRs is that binding of molecules to the micro-resonator surface induces an optical change proportional to the amount of bound molecules. The cavity Q changes as the surface bound molecules affect the photon storage time in WGMs either through increased scattering or increased absorption. The analyte spoils the Q, and that change can be measured. However, any protein will adhere to glass, and hence fire-polished spheres are entirely nonspecific. Two conditions must be met for chemical modification of the microsphere surface: first, the glass must be coated with a compound that minimizes nonspecific binding. Second, an antibody or other protein with sensitivity to a particular ligand must be linked to the sphere in such a way that both the protein's functionality and the sphere's Q are preserved. A thin film of a material with thickness smaller than the WGM's evanescent field will not significantly alter the Q of the micro-resonator; thus, a thickness of ∼10–100 nm can be applied to the microsphere while retaining its high Q.

A possibility of enhancement of the detection sensitivity of evanescent-wave optical biosensors was discussed in [221–225]. It was shown that the resonant coupling of power into the WGR allows for efficient use of the long photon lifetimes of the high-Q WGMs to increase the interaction of the light and the particles under study. This enhancement results in stronger fluorescence and in change to the resonator's parameters.

A spectroscopic technique for high-sensitivity, label-free deoxyribonucleic acid (DNA) quantification was developed in [226]. It was demonstrated that a micron-sized silica sphere can be used to detect and measure nucleic acids. The surface of the silica sphere is to be chemically modified with oligonucleotides.

A first-order perturbation theory was developed for WGMs in a dielectric microsphere [227,228]. The theory was applied to three sensor applications of the microsphere to probe the medium in which the sphere is immersed: a refractive-index detector, an adsorption sensor, and a refractive-index profile sensor.

Biosensors based on the shift of WGMs in microspheres accompanying protein adsorption were described by use of a perturbation theory in [227]. For random spatial adsorption, theory predicts that the shift should be inversely proportional to microsphere radius a and proportional to protein surface density and excess polarizability.

Hybrid zinc oxide/silica microdisk lasers were utilized to sense volatile organic compounds, such as toluene and nitrobenzene [229]. Nonspecific adsorption of these organic molecules onto the WGR surface causes an increase in the disk refractive index, ultimately resulting in a red shift of the observed lasing wavelengths.

Improvement of photonic WGM sensors using a Fano-resonant line shape was proposed in [230]. Polystyrene microring resonators were fabricated by the nanoimprinting technique, and the optical spectra were measured in glucose solutions of different concentrations. The shift in resonant wavelength and variation of the normalized transmitted intensity were linearly related to the concentration of the glucose solution.

Application of WGRs in high-field, high-frequency electron magnetic resonance spectroscopy was discussed in [231].

F. Mechanical Sensors

High-Q WGMs resulted in an increase in sensitivity of various mechanical experiments. For instance, WGMs could be used for measurement of an optical-fiber strain [170]. A twin-resonator sensor of small displacements that utilizes high-Q and mechanical tunability of normal modes in coupled optical WGRs was proposed in [232].

An accelerometer utilizing high-Q WGRs was presented in [233]. Induced flexure-arm displacements were monitored through changes in the resonance characteristics of a spherical optical cavity coupled to the flexure. Instantaneous measurement sensitivity of better than 1 mg at a 250-Hz bandwidth and a noise floor of 100 μg were achieved.

The idea of using passive and active optical ring interferometers for detection of rotation was developed and implemented a couple of decades ago [234–236]. A miniature integrated WGM optical sensor for gyroscope systems was proposed recently [237]. It was predicted that the sensor may possess high enough sensitivity even on a millimeter-size scale. A passive WGM gyroscope was discussed in [238]. The basic difference of the gyroscope compared with the existing ring resonator gyroscopes is in the usage of crystalline WGR instead of the usual ring resonator.

G. Fundamental Physics with Passive WGMs

WGRs are interesting from both the classical and quantum fundamental points of view. The high Q-factor, as well as small mode volume, of WGMs results in a multitude of interesting and important phenomena. In this section, we discuss those phenomena related to “passive” WGMs that do not lead to generation of light, leaving the fundamental properties of WGM lasers and other active devices to another section.

One of the fundamental problems is related to WGMs in an asymmetric WGR. It was shown that chaotic behavior is possible there. This has been predicted to give rise to a universal, frequency-independent broadening of the WGRs and to highly anisotropic emission [239–243]. A solution of the problem that confirms these predictions but also reveals frequency-dependent effects characteristic of quantum chaos was presented in [244]. It was shown that for small WGR deformations the lifetime is controlled by evanescent leakage, the optical analogue of quantum tunneling [111]. The WGR lifetime is significantly shortened by a process known as “chaos-assisted tunneling” [245]. In contrast, even for large deformations, some resonances were found to have longer lifetimes than predicted by the ray chaos model due to the phenomenon of “dynamical localization” [246]. The problem of the directional emission from egg-shaped asymmetric resonant cavities was discussed theoretically in [247]. The first experiment on chaos-assisted tunneling in a two-dimensional annular billiard was reported in [248]. Highly directional emission from WGMs was demonstrated in deformed nonaxisymmetric fused-silica microspheres [74].

On the other hand, there is great activity in both theoretical and experimental investigations of cavity quantum electrodynamics [249–253] effects in WGRs. For instance, spontaneous emission processes may be either enhanced or inhibited in a cavity due to a modification of the density of electromagnetic states compared with the density in a free space [254,255]. This effect was studied theoretically [256–258] as well as experimentally [157,259] in WGRs.

Methods for the state control of atoms coupled to single-mode and multi-mode cavities and microspheres were discussed in [260]. Those methods include excitation decay control, location-dependent control of interference of decay channels, and decoherence control by conditionally interfering parallel evolutions.

Ponderomotive interaction of an atom and a WGM was discussed in [158,261,262]; see also [263,264] for review. In particular, it was shown that the external fields of optical WGMs may be used to confine atoms in stable orbits around a dielectric microsphere [158]. The bound-state structure and dynamics for the atom trap were investigated in [261]. The dynamics of the center-of-mass of an ultracold excited atomic oscillator in the vicinity of a dielectric microsphere were studied in [262].

The ponderomotive interaction of an atom and photons confined in a WGM can be used for quantum nondemolition measurements. It was shown [265–267] that the dipole force experienced by an atom in an off-resonant spatially inhomogeneous light field is quantized by the discrete nature of the photon. Similar schemes to perform quantum nondemolition detection of optical photons by observing the deflection of a beam of atoms flying close to an open dielectric resonator were proposed in the studies.

A radiative coupling of a nanoparticle/atom with a WGM was studied in [214,268]; see also [253,272] for a review. A possibility of strong coupling between a photon confined in a WGM and an atom was analyzed in [269–271]. The resonant interaction of a dipolar $J = 0 \leftrightarrow J = 1$ angular-momentum transition with the quantized field in dielectric spheres and spheroids was studied in [269].

A measurement of cavity-QED effects for the radiative coupling of atoms in a dilute vapor to the external evanescent field of a WGM was reported in [214]. Experiments on the coupling of a single nano-emitter and WGMs were discussed in [268]. A composite system consisting of a GaAs quantum-well structure placed in the evanescent field of a fused-silica microsphere and evanescent coupling between excitons in the quantum well and WGMs of the composite system was demonstrated in [273].

III. Active WGM Devices

Small volumes and high Q-factors of WGMs result in enhancement of nonlinear optical processes. Due to this enhancement, WGR-based nonlinear optic devices possess unique characteristics. For example, usage of WGMs results in lasing and wave-mixing having micro-watt thresholds. Narrow linewidths of WGMs warrant narrow spectral characteristics of the lasers. In this section, we review results of recent studies in the field.

A. Continuous-Wave WGM Lasers

Lasers belong to one of the most obvious applications of WGRs. The high quality factor of the resonators allows for significant reduction of the lasing threshold. The first WGM lasers were realized in solid materials [16,17,274]. However, probably because of the lack of an input–output coupling technique for WGMs, the work was discontinued at that point. The next development step of the WGM-based lasers occurred in liquid aerosols and individual liquid droplets [29,275–279]. Finally, lasers based on a sole solid-state WGR were rediscovered, demonstrated experimentally, and intensively studied during the last decade. In this section, we review recent results on WGR continuous-wave (CW) lasers, leaving WGR Raman lasers for the following section.

1. Lasing in Capillaries. A WGR laser can be realized in a cylindrical resonator. The simplest resonator of this kind is a capillary. The gain medium could propagate inside the capillary, where WGMs are localized. For instance, laser emission from WGMs in a highly refractive dye-doped solvent flowing in a normally illuminated silica capillary fiber was demonstrated in [280]. The cylindrical WGM laser differs from the spherical droplet laser [29] in that it has an internal refractive index discontinuity. The light penetrates into the active medium if the refractive index of the medium is higher than that of the capillary material; e.g., no laser peaks are observed when the refractive index of the solvent is less than that of the silica [280].

Lasing based on the evanescent field coupling with the gain medium is also possible. The layered microcavity was realized in [281,282] by flowing dye-doped ethanol through a thin-wall fused-silica capillary tube whose refractive index was larger than that of the liquid. The lasing spectrum showed strong mode selection, and even nearly single constructive interference peaks, due to the interferential coupling of WGMs at the inner boundary. Various mode orders, which are not allowed in the ray optics picture, were made to oscillate due to the evanescent propagation of WGMs at the outer boundary. The estimated cavity quality factors were higher than 10^6. The lasing characteristics of resonance modes in a thin dye-doped dielectric ring cavity made on the inner wall of a cylindrical capillary were studied in [283].

A WGM laser with pulsed optical pumping fabricated by surrounding a small section of a glass capillary with a solution of Rhodamine 6G and by coupling the pump light into the capillary wall was demonstrated in [284]. The lasing threshold pump energy was 100 nJ/pulse at a pump pulse duration of 6 ns.

2. Lasing in Doped Microspheres. Another way to create a WGR laser is by the use of solids doped with active elements, e.g., rare Earth ions.

A WGM laser based on neodymium-doped silica microspheres with a 200-nW threshold was realized [285]. Microspheres of radius $a \sim 25$ to 50 μm, formed by heat-fusing the tip of a length of doped silica wire, were used. Neodymium ions provide a favorable four-level laser system that can be pumped on the $^4I_{9/2}$–$^4F_{5/2}$ transition at ~810 nm with a diode laser. The laser transition $^4F_{3/2}$–$^4I_{11/2}$ in the 1060- to 1090-nm range connects a long-lived upper level to a lower level that is depleted by strong phonon relaxation so that population inversion is easily achieved. A similar experiment with a neodymium-doped silica microsphere laser operating at 2 K and absorbing the same pump power (200 nW) was reported in [286].

A WGM laser utilizing a microsphere made of highly doped erbium:ytterbium phosphate glass was used to generate light at 1.5 μm [287]. Laser threshold pump power of 60 μW and fiber-coupled output power as high as 3 μW with single-mode operation were obtained. A bisphere laser system consisting of two microspheres attached to a single fiber taper also was demonstrated.

A green-room temperature upconversion laser was demonstrated in a 120-μm-diameter microsphere of Er^{3+}-doped ZBLAN [288,289]. Lasing occurred around 540 nm with a 801-nm diode laser pump. The lasing threshold was 30 μW of absorbed pump power.

Experimental results on the realization and spectral characterization of Er:ZBLAN microsphere lasers at 1.56 μm were presented in [290,291]. The lasing was obtained with the external 1.48-μm pumping. Multimode operation and a laser threshold as low as 600 μW were observed.

Green lasing having a 4-mW threshold was demonstrated in an erbium-ion-doped fluoro-zirconate glass WGR [292]. Periodic narrow peaks of the emission spectra corresponding to the WGRs were observed.

An erbium-doped microlaser on silicon operating at a wavelength of 1.5 μm that is characterized by a pump threshold as low as 4.5 μW was demonstrated in [293]. The 40-μm-diameter toroidal laser WGR was made using a combination of erbium ion implantation, photolithography, wet and dry etching, and laser annealing, using a thermally grown SiO_2 film on a Si substrate as a starting material. Single-mode lasing was observed.

Instead of using doped materials, a passive WGR can be coated with gain medium. For example, erbium-doped solgel films were applied to the surface of silica microspheres to create low-threshold WGR lasers [294].

3. WGM Lasers with Semiconductor Gain Media. WGM-based lasers can be created with semiconductor quantum dots coupled to the WGMs. One of the most important problems here is fabrication of a single quantum-dot microlaser. Such a microlaser, made by capturing the light emitted from a single InAs/GaAs quantum dot in a WGM of a glass microsphere, was proposed theoretically in [295]. A master equation model of a single quantum-dot microsphere laser was described in [296]. The operation of a single quantum-dot microsphere laser and a semiconductor microsphere bistable element was theoretically studied in [297].

A quantum-dot microcavity system consisting of CdTe nanocrystals attached to a melamine formaldehyde latex microsphere was realized experimentally [298]. The high optical transparency and the thermal and mechanical stability of melamine formaldehyde make it interesting as a potential candidate in optical applications. The refractive index of melamine formaldehyde in the visible region ($n = 1.68$) is greater than that of silica ($n = 1.47$) or other glass materials ($n \approx 1.5$). Photoluminescence spectra of the microspheres covered by a thin shell of CdTe nanocrystals were studied in order to examine the emission intensity as a function of excitation power.

Ultra-low-threshold (the pump was less than 2 μW) continuous-wave lasing was achieved at room temperature in a fused-silica microsphere that was coated with HgTe quantum dots (colloidal nanoparticles) [299].

WGRs can significantly improve operation of semiconductor well lasers. A microlaser design based on the high-reflectivity WGMs around the edge of a thin semiconductor microdisk was described and initial experimental results were presented in [300]. It was shown that optically pumped InGaAs quantum wells provide sufficient gain when cooled with liquid nitrogen to obtain single-mode lasing at 1.3- and 1.5-μm wavelengths with threshold pump powers below 100 μW.

A realization of an InGaAs/InGaAsP room-temperature quantum-well disk laser 1.6 μm in diameter and 0.18 μm in thickness, operating at 1.542 μm and using 0.85 μm optical pumping, was reported in [301]. Methods for directional coupling of light output from and to WGR microdisk lasers were described in [70].

A "microgear" laser composed of a microdisk and a rotationally symmetric Bragg grating was described in [71]. A micron-sized GaInAsP-InP device was fabricated, and the room-temperature continuous-wave operation was obtained by 17-μW pumping.

B. Resonator-Modified Scattering

There are at least three scattering processes playing significant roles in WGRs. These are Brillouin, Raileigh, and Raman scattering.

1. Brillouin Scattering. Stimulated Brillouin scattering (SBS) was demonstrated in liquid droplets [302–311], although no SBS in high-Q solid WGRs was registered because of selection rules [308].

2. Rayleigh Scattering. Rayleigh scattering leads to limitation of the Q-factor of WGMs as well as to inter-mode coupling. The scattering is largely suppressed in high-Q WGRs because of restrictions imposed on scattering angles by cavity confinement, so very high-Q WGMs become feasible [44]. The scattering, on the other hand, couples initially degenerate counterpropagating modes in the WGRs and creates the intracavity feedback mechanism instrumental to the laser frequency-locking application [63]. Rayleigh-scattering-mediated intracavity backscattering reaches 100 percent, as was shown theoretically [44] and demonstrated experimentally [312]. In the frequency domain, intracavity backscattering is observed as the splitting of initially degenerate WGM resonances and the occurrence of characteristic mode doublets [159,313].

3. Raman Scattering. Substantial optical power enhancement within a high-finesse optical cavity has recently yielded continuous wave Raman lasers with low threshold and large tunability (see, e.g., [314,315]). Such properties make cavity-enhanced CW Raman lasers attractive for high-resolution spectroscopy, remote sensing, atomic physics, and telecommunications. Reducing the cavity size may further improve the performance of the lasers. Open dielectric spherical microcavities are promising for those purposes.

An enhancement of stimulated Raman scattering (SRS) is one of the effects demonstrated in spherical microcavities. Low-threshold SRS was observed with pulsed [27,28,85,152,306,316–318] and continuous-wave [82,319] optical pumping in micrometer-sized liquid droplets. A theoretical description of the process was presented in [320–323].

SRS was investigated in a liquid parahydrogen droplet characterized with a WGM having a Q-factor exceeding 10^9 [324]. The SRS was registered not only for a vibrational transition but also for a rotational transition, as in the gas-phase H_2 system, leading to multiorder SRS side bands covering the whole visible spectral range.

SRS in ultra-high-Q surface-tension-induced spherical and chip-based toroid microcavities is considered both theoretically and experimentally in [325]. These fused-silica WGRs exhibit small mode volume (typically 10^3 μm^3) and possess whispering-gallery-type modes with long photon storage times (in the neighborhood of 100 ns), significantly reducing the threshold for stimulated nonlinear optical phenomena.

Studies of Raman gain in isolated high-Q WGRs are important to understanding the cavity QED properties of Raman lasing. Previously, microcavity QED enhancement of Raman gain has been inferred as the result of measurements of a dependence of the SRS threshold on the size and material of the microdroplets, and its comparison with the values of SRS threshold were reported for liquid core fibers having equivalent interaction lengths and core compositions [82,319]. This enhancement has been linked to the cavity modification of the properties of a usual laser. A theory of the Raman gain modification that explains the experimental results was developed [326,327]. Recent experiments with silica microspheres have not shown any significant change in SRS gain which might be attributed to quantum effects [325,328]. This issue was addressed in [329], where it was shown that no cavity-QED-associated Raman gain enhancement exists, unlike the cavity enhancement of the spontaneous emission.

C. Switches and Modulators

1. WGM Switches. WGRs can be used as efficient and compact optical switches and modulators. A possibility of nonlinear optical switching and applications of WGRs to create a quantum-mechanical computer was first recognized in [140].

The majority of studies of optical switches that utilize WGMs are theoretical. It was shown theoretically that WGM-based microdisk lasers are stable and switch reliable [330] and, hence, are suitable as switching elements in all-optical networks.

An integrated all-optical switch based on a high-Q nonlinear cylindrical microcavity resonator was proposed in [65]. The switch consists of two planar waveguides coupled to a WGR. It was argued that, due to the high Q-factor and the small dimensions, fast switching at low power is feasible with the devices based on presently available nonlinear polymers as the active material.

A general electrodynamic theory of a high-Q optical microsphere resonator in an external alternating magnetic field was reported in [331]. It was shown that that such a system can change a polarization state of the WGM photons confined in the sphere due to the Faraday effect. This property was proposed for use in all-optical switches and logical devices.

Numerical evaluation of the optical response of a prism-coupled nonlinear microsphere was presented in [332]. The numerical results have shown that the control and/or the signal lights can induce the optical switching-like variation in the light reflectance. To increase the nonlinearity, it was proposed to coat the resonator with Kerr material.

Coupled WGRs possess different, frequently more advanced, properties as compared with a single WGR. Sequences of optical microresonators can be used to construct integrated structures that display slow group velocity of light, ultra-high or -low dispersion of controllable sign, enhanced self-phase modulation, and nonlinear optical switching [333].

It was pointed out that there should be a reduction in switching threshold for nonlinear optical devices incorporating fiber ring resonators [334,335]. The circulating power in such WGRs is much larger than the incident power, and the phase of the transmitted light varies rapidly with the single-pass phase shift. It was shown that the combined action of these effects leads to a finesse-squared reduction in the switching threshold [334], allowing for photonic switching devices that operate at milliwatt power levels in ordinary optical fibers. A set of coupled differential equations that describe Kerr nonlinear optical pulse propagation and optical switching in systems coupled by a few microresonators was derived in [336]. Gap-soliton switching in a system composed of two channel waveguides coupled by microresonators was studied in [337].

A numerical demonstration of the feasibility of constructing an all-optical "AND" gate by using a microresonator structure with Kerr nonlinearity was presented in [338]. It was shown that the gate can be much smaller than similar "AND" gates based on Bragg gratings and has lower power requirements.

There are a few experimental studies of all-optical switches that utilize WGMs. For instance, laser-induced modification of cavity Q's was achieved in a microdroplet containing a saturable absorber [339]. The elastic-scattering spectra from such droplets for higher incident intensities demonstrated that cavity Q's are increased when the absorption is bleached. The lasing spectra from a droplet containing a saturable absorber and laser dye were modified when an intense bleaching field was injected into the droplet cavity after the pump field has initiated the lasing.

All-optical nonlinear switching in compact GaAs-AlGaAs microring resonators at the 1.55-μm wavelength was demonstrated in [340]. Switching was accomplished in the pump/probe configuration in which the pump/probe signals were tuned to different resonance wavelengths of the microring. Refractive index change in the microring due to free carriers generated by two-photon absorption was used to switch the probe beam in and out of resonance.

An all-optical switching technique utilizing a silica microsphere optical resonator coated by a conjugated polymer was developed in [173]. A 250-μm-diameter silica microsphere was coated by dipping into a toluene solution of the polymer. WGM resonant frequency shifts as large as 3.2 GHz were observed when 405-nm pump light with a power density on the order of 10 W/cm^2 was incident on the microsphere. The time constant of the observed frequency shifts was approximately 0.165 s, i.e., the frequency shift can be attributed to thermo-optic effects. Such a system is capable of switching the WGM resonant frequency having a 2-MHz linewidth at speeds on the order of 100 ms.

Finally, optical memory elements were developed using WGM devices. A memory element constructed by interconnecting WGM microscopic lasers was demonstrated in [341]. The device switches within 20 ps with 5.5-fJ optical switching energy. On the other hand, it was shown theoretically and demonstrated experimentally that a random distribution of spherical microparticles may be used as a spectral hole burning memory [342].

2. WGM Modulators. Microwave cellular phone systems and personal data assistant networks require devices capable of receiving, transforming, and processing signals in the millimeter-wavelength domain [343]. Electro-optic modulators based on electromagnetic wave interaction in nonlinear optical cavities with high-Q WGMs will play an enabling role for these and similar applications.

An approach to creating coupling between light and a microwave field in a WGR was recently proposed [6,129]. In that study, an efficient resonant interaction of several optical WGMs and a microwave mode was achieved by engineering the shape of a microwave resonator coupled to a micro-toroidal optical cavity. Based on this interaction, a new kind of electro-optic modulator as well as a photonic microwave receiver was suggested and realized [93,94,101–105,344].

D. Opto-Electronic Oscillator

WGRs can be used as counterparts in sources of coherent microwave radiation. An opto-electronic oscillator (OEO) is an example of such a source. An OEO produces microwave signals using photonic techniques [184,203–208]. The modulator is one of the main sources of power consumption in the OEO because of the large power required to drive the conventional modulators. Both broadband Mach–Zender modulators and free-space microwave cavity-assisted narrowband modulators typically require from one to a few watts of microwave power to achieve a significant modulation. This means that either the photocurrent in the OEO should be amplified significantly or a powerful laser should be used as the source of the drive power for the OEO.

OEO based on WGM resonant modulator was recently proposed and fabricated [345]. The device is characterized by a low threshold and low power consumption. The disadvantages of the device are low saturation, low output power, and a possibility of transforming the noise of the light field into the microwave signal. In general, resonant and conventional OEOs have nonoverlapping characteristics, and both are useful depending on the application.

E. Pulse Propagation and Generation

It is convenient to distinguish between two regimes of optical pulse propagation in a WGR: (1) the pulse duration that exceeds the inverse of the free spectral range (FSR) of the cavity and (2) the pulse duration that is shorter than the inverse cavity FSR. Studies presented in [346–349] are primarily focused on the first regime. Specifically, the transient behavior of light intensity inside a dielectric sphere excited by a light pulse was discussed in [346,347]. Long optical pulses were used for pumping of polymer microlasers [348]. Linear and nonlinear optical properties of a waveguide-coupled WGR also have been studied theoretically [349].

The second case, propagation of short pulses in WGRs, also was examined [350–352,354], and a general theoretical analysis of the propagation was presented in [350]. Time-resolved measurements of picosecond optical pulses propagating in dielectric spheres [351] and subpicosecond terahertz pulse propagation in a dielectric cylinder [352,353] recently were reported, and microcavity internal fields created by picosecond pulses were discussed theoretically [354]. The behavior of ultra-short light pulses coupled into the resonant modes of spherical microcavities was explored in [355]. A noninvasive pulse-tracking technique was exploited to observe the time-resolved motion of an ultra-short light pulse within an integrated optical microresonator [356].

The minimum pulse width as well as the period of the optical pulse train generated by a system that involves a high-Q cavity are determined by the resonator dispersion. Depending on the dielectric host material and the geometric size, a WGR may possess either a positive, a negative, or a zero group velocity dispersion (GVD) [137]. This dispersion is important when the pulse duration is shorter than the inverse cavity FSR. Resonators possessing a positive group velocity dispersion may be used for GVD compensation in optical fiber links. Negative GVD cavities with Kerr nonlinearity (e.g., fused-silica cavities) sustain nonlinear Schrodinger soliton propagation and may be used for pulse shaping and soliton shortening in conventional mode-locked lasers (see, e.g., [358–360]). Zero GVD cavities may be used as high-finesse etalons to stabilize actively mode-locked lasers (as in [361]). Integrated optical WGM all-pass filters also can be used for tunable dispersion compensation in the optical transmission line if the pulse duration exceeds the inverse of the FSR of the resonator [333,357].

WGRs can substitute Fabry–Perot resonators in various applications. For instance, short resonators are important for the stable generation of optical pulses with high repetition rates. For example, 2-ps pulses at a 16.3-GHz repetition rate were obtained for a 2.5-mm-long actively mode-locked monolithic laser [362]; 420-GHz subharmonic synchronous mode locking was realized in a laser cavity of approximate total length 174 μm [363]. A significant supermode noise suppression was demonstrated by inserting a small high-finesse Fabry–Perot resonator into the cavity of an actively mode-locked laser [361,364].

It was proposed that WGRs be used to generate short optical pulses with a monolithic actively mode-locked laser [137,369]. The idea of this laser is based on two recently realized WGM devices: the electro-optic modulator and the erbium-doped microsphere glass laser [62,287,289,292,294].

It also is known that an electro-optic modulator placed in an optical resonator can generate a frequency comb [365–368], and that the output of such a device is similar to that of a mode-locked laser. However, unlike the mode-locked laser, the pulse duration is not limited by the bandwidth of the laser gain because the system is passive. The pulse width decreases with the modulation index increase and the overall cavity dispersion decrease. The modulation index may be very large in a WGM modulator, which can significantly improve the performance of the system [137].

F. Wave Mixing and Oscillations

WGRs were used in optical parametric as well as hyper-parametric wave-mixing processes.

1. Hyper-Parametric Oscillators. Hyper-parametric optical oscillation [370], also known in fiber optics as modulation instability [96], is based on four-wave mixing (FWM) among two pump, signal, and idler photons, and it results in the growth of the signal and idler optical side bands from vacuum fluctuations at the expense of the pumping wave. The hyper-parametric oscillations are different from the parametric ones. The parametric oscillations (1) are based on $\chi^{(2)}$ nonlinearity coupling three photons and (2) have phase-matching conditions involving far-separated optical frequencies that can only be satisfied in birefringent materials in the forward direction. By contrast, the hyper-parametric oscillations (1) are based on $\chi^{(3)}$ nonlinearity coupling four photons and (2) have phase-matching conditions involving nearly-degenerate optical frequencies that can be satisfied in most materials both in the forward and backward directions.

Recently, the study of hyper-parametric oscillations had a new twist connected with the development of WGM as well as photonic crystal [8,371] microresonator technology. The oscillations occurring in cavities or cavity-like systems filled with transparent solids were analyzed theoretically, e.g., in isotropic photonic crystals [372], and were observed experimentally in crystalline WGM resonators [136,373]. It was suggested, in particular, that the narrowband beat-note signal between the optical pump and the generated side bands emerging from a high-Q WGM resonator could be used as a secondary frequency reference [136,374].

The phase stability of the frequency reference signal increases with an increase of the Q-factor of the resonator modes for the same given value of the pump power. There exists a maximum of the phase stability (minimum of the phase diffusion) of the beat-note signal that does not depend on either the pump power or the Q-factor of the modes. Keeping in mind that a WGM's Q-factor can exceed 10^{10} (a few tens of kilohertz resonance linewidth) [92], the Allen deviation factor of the oscillations was found to be smaller than $10^{-12}\mathrm{s}^{-1/2}$ for sub-milliwatt optical pumping. The pump threshold could reach microwatt levels for reasonable experimental parameters.

2. Parametric Processes. Optical parametric oscillators (OPOs) have been extensively studied since the discovery of lasers [375–377]. Properties of OPOs are well understood by now [378–380]. The CW OPO is considered an ideal device that can generate a broad range of wavelengths.

Efficient frequency doubling at $\lambda = 1.55$ μm and $\lambda = 1.319$ μm was realized [180] using the same WGR made of periodically poled $\mathrm{LiNbO_3}$ (PPLN) [381]. The WGR was doubly resonant, both at fundamental and second harmonic frequencies. The follow-up studies of the parametric processes in PPLN WGRs are important because it has been predicted that an optical parametric oscillator based on the resonator might have a power threshold below a microwatt [382]—orders of magnitude less than that of the state-of-the-art OPOs, typically at the 0.5-mW level [383].

It was shown theoretically [384] that a nondegenerate multi-frequency WGR parametric oscillator has different properties compared with the usual three-wave parametric oscillator. Such an oscillator may have an extremely low threshold and stable operation, and may be used in spectroscopy and metrology. The oscillator mimics devices based on resonant $\chi^{(3)}$ nonlinearity (a hyper-parametric process) and can be utilized for efficient four-wave mixing and optical comb generation.

G. Fundamental Physics with Active WGMs

WGRs can be used for generation of nonclassical states of light. There exists a possibility for the generation of heralded single photons and of sub-Poissonian laser light in the electrically pumped single-quantum-dot microsphere laser [296].

The reduced-density matrix method was used to calculate the quantum-statistical properties of the radiation of a quantum-dot laser operating on the WGM of a dielectric microsphere [385]. It was shown that under the conditions of strong coupling between the quantum dot and an electromagnetic field the radiation of such a laser can be in a nonclassical (sub-Poissonian) state. The laser scheme was characterized by an extremely low lasing threshold and a small number of saturation photons, as a result of which lasing is possible with close to zero population inversion of the working levels.

IV. Summary

In this review, we covered recent developments in applications of whispering-gallery mode resonators in optics and photonics. We tried to mention all the activities in the field, although we admit that some of the recent advances could escape our attention because the area grows very quickly, and literally each month brings new studies related to the subject.

Although whispering-gallery modes are interesting physical objects themselves, we foresee the fastest growth in their practical applications. Filters, modulators, lasers, and other whispering-gallery mode devices have multiple advantages over their “ordinary” analogs. Most of the applications are interesting and important for modern telecommunications.

Finally, let us list the basic problems, from our point of view, that have to be solved to promote high-Q open dielectric resonators from research and development stands to commercially available devices.

First of all, the efficient and robust unified coupling technology with the resonators should be developed. Existing coupling methods, although very efficient in bread-board implementations, are fragile and picky to environmental conditions.

Second, evacuated packaging of the high-Q resonators should be realized. Crystalline dielectric resonators are much more tolerant of the environment, as compared with fused-silica resonators, but the dirt in the air spoils their Q's sooner or later.

Third, mass production technology for the high-Q resonators of the desired shape should be developed and merged with the coupling technology.

Those three problems are under massive attack now; however, no complete solution has been found yet.

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