Experiments on Robust Vibration Suppression in Mechatronic Systems Using IIR Digital Filters

Abstract—In this paper, a method for suppressing residual vibrations in flexible mechatronic systems is proposed and experimentally demonstrated. The proposed method is based on the preconditioning of the arbitrary inputs to the system using Infinite Impulse Response (IIR) digital filters. To ensure robust behavior, the filter stop band is selected to cover all possible variations of the system dynamic parameters. It is shown that some of the IIR filters are capable of suppressing vibrations when they are properly designed.

Index Terms—Vibration Suppression, Input Preconditioning, IIR Digital Filters.

I. INTRODUCTION

Residual vibration suppression is important in a broad range of mechatronic engineering applications such as the deployment of space structures and cranes or the operation of machine tools and flexible robots. The traditional approaches to minimize the effect of residual vibrations are focused on either increasing the structural stiffness, which increases the system's size and weight or using closed loop control methods, which require accurate on-board electronics which increase the system’s cost and complexity.

An alternative approach for suppressing residual vibrations is the proper conditioning of the pre-specified excitation pattern, the so-called Guidance, so that the system moves exactly to the desired end position without any residual vibrations [1]. This concept is very attractive, since it can simplify the system requirements and complexity. Since methods in this category are traditionally considered to be quite sensitive to variations of the system dynamic parameters, significant research effort has been devoted to increase their robustness features. A brief survey of the methods proposed for the proper conditioning of the guidance function can be found in [2].

Of special interest in the present work is the input shaping approach, based on the convolution of an arbitrary guidance function with a series of impulses [3-5]. A lot of research has been done to propose design techniques for the input shapers so that they suppress multiple modes of vibration and have increased robustness [6-9]. Input shaping has been used successfully in many applications for vibration suppression such as long reach manipulators, cranes and coordinate measuring machines [10-12]. While input shaping methods present good performance in a variety of systems and applications, their robustness is limited in local areas around the system natural frequencies and can be increased only by increasing the total duration of the pulse sequence. This results in unnecessary delays of the total duration of the system motion.

In a first attempt to extend the robustness of the vibration suppression method based on the convolution of the guidance function with a series of impulses, a general approach has been proposed, leading to a set of three different methods for the proper design of the impulse sequence [2]. The impulse sequence approach is significantly extended in [13], by establishing a theoretical framework, according to which the design of an appropriate guidance function can be transformed to the proper design of a conventional low-pass Finite Impulse Response (FIR) digital filter. Furthermore, in order to ensure the proper design of the filters, the “Delay-Error-Order (DEO) Curves” concept has been proposed, as a filter design tool [14]. The effectiveness of FIR filters for residual vibration suppression has been studied thoroughly both theoretically and experimentally [14, 15].

In this work the vibration suppression capabilities of all the conventional IIR filters are examined. First, the requirements for vibration suppression using IIR filters are presented. Then, the DEO curves for the IIR filters are developed as a filter design tool. Last, the vibration suppression capabilities of the IIR filters are experimentally verified using a flexible aluminum beam. The goal of the experiments is to move the flexible beam, using an industrial robot manipulator, from an initial location to a final one, without residual vibrations. Robustness tests are conducted attaching a different mass M at the beam's free endpoint. The experiments, including the robustness tests, showed a dramatic decrease of the amplitude of the residual vibrations when the system input was filtered with a properly designed low-pass IIR digital filter. In some cases these results are even better than those obtained using FIR filters.
II. THEORETICAL DEVELOPMENT

A. Requirements for Residual Vibration Suppression In Flexible Systems Using Input Preconditioning

Using modal analysis, the equations of motion of a Multi Degree Of Freedom (MDOF) system can be written as a set of independent (uncoupled) equations, each one referring to a principal (natural or normal) coordinate of the system:

\[ \ddot{p}_i(t) = A_i \dot{p}_i(t) + b g_i(t) \quad i = 1, \ldots, L \]  

(1.a)

\[ A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]  

(1.b)

\[ g_i(t) = V_i^T x(t) = \sum_{m=1}^{L} V_{im} x_m(t) \]  

(1.c)

\[ V_i^T = [V_{i1}, V_{i2}, \ldots, V_{iL}] \]  

(1.d)

\[ x^T(t) = [x_1(t), x_2(t), \ldots, x_L(t)] \]  

(1.e)

where \( L \) is the number of degrees of freedom, \( p_i \) are the generalized coordinates, \( g_i(t) \) are the corresponding generalized forces, \( \omega_i \) are the natural frequencies of the system with corresponding damping ratios \( \zeta_i \) and eigenvectors \( V_i \) and \( x(t) \) is the guidance vector for the system, with spatial distribution according to Eq. (1.d).

Using the Duhamel Integral and the Sylvester expansion method to the transition matrix \( e^{A_i t} \), the expression for each principal coordinate is written as:

\[ \ddot{p}_i(t) = e^{A_i t} \left[ p_i(0) + \sum_{j=1}^{2} H_{ij} b \right] G_i(s; s = q_{ij}) \]  

(2)

where \( G_i(s) \) is the Laplace transform of the guidance generalized forces \( g_i(t) \), evaluated at the system poles \( q_{ij} \).

In view of Eq. (2) the requirement for vibration suppression can be stated as:

\[ G_i(s; s = q_{ij}) = 0 \]  

(3)

This requirement can’t be met for every arbitrary form of the guidance function. For this reason, in the above equations, instead of the original arbitrary generalized guidance forces \( g_i(t) \), conditioned generalized guidance forces \( u_i(t) \) can be used, obtained by the original \( g(t) \) by using a sequence of the form of Eq. (4)

\[ u_i(t) = \sum_{n=0}^{N} c_n g_i(t-nT_{sf}) - \sum_{n=1}^{M} a_n u_i(t-nT_{sf}) \quad i = 1, \ldots, L \]  

(4)

where \( T_{sf} \) is a constant period, characterizing the time instants where the past values of \( g(t) \) and \( u(t) \) are evaluated.

In this case, the corresponding Laplace transform \( U_i(s) \) of \( u_i(t) \) appears in Eqs. (5) and (6) instead of \( G_i(s) \). The corresponding Laplace transform \( U_i(s) \) is written as:

\[ U_i(s) = F(s) G_i(s) \quad i = 1, \ldots, L \]  

(5a)

\[ F(s) = \frac{\sum_{n=0}^{N} c_n e^{-snT_{sf}}}{1 + \sum_{n=1}^{M} a_n e^{-snT_{sf}}} \]  

(5b)

The purpose of introducing the sequence in Eq. (4) is that the conditioned generalized forces \( u_i(t) \) are able to move the system in essentially the same way as the original arbitrary guidance functions \( g_i(t) \), without the effect of the residual vibrations. Taking into account Eqs. (5) and (6), this requirement can be written as:

\[ U_i(s; s = q_{ij}) = 0 \Leftrightarrow F(s; s = q_{ij}) = 0 \]  

(6a)

\[ U_i(s; s = 0) = G_i(s; s = 0) \Leftrightarrow F(s; s = 0) = 1 \]  

(6b)

In view of Eqs. (3), (5a) and (6a), Eq. (6a) implies that the residual vibration effect can be completely canceled by the proper selection of \( F(s) \), quite independently from the properties of the original guidance function \( g_i(t) \). On the other hand, Equation (6b) implies that \( u_i(t) \) maintains all the properties that \( g_i(t) \) possesses, in order to move the system as a rigid body.

B. Input Preconditioning Using IIR Digital Filters

Infinite Impulse Response (IIR) filters are series of constants \( \{c_n\} \) and \( \{a_n\} \). The mathematical foundation of filtering is a convolution procedure, resulting to a discrete filtered signal \( u_i(kT_s) \), which is obtained from an original discrete input signal \( g_i(kT_s) \) according to Eq. (7).

\[ u_i(kT_s) = \sum_{n=0}^{N} c_n g_i(kT_s - nT_{sf}) - \sum_{n=1}^{M} a_n u_i(kT_s - nT_{sf}) \]  

(7)

where: \( k \) is an integer, \( T_s \) is the sampling period of the discrete signals \( u_i(t) \) and \( g_i(t) \), \( T_{sf} \) is the filter sampling period, \( \{c_n\} \) and \( \{a_n\} \) are series of constants of length \( N+1 \) and \( M \) respectively. The maximum of \( N \) and \( M \) is the filter order. The z-transform of the filtered function \( U_i(z) \) is related to the z-transform of the original input function \( G_i(z) \) by:

\[ U_i(z) = \frac{\sum_{n=0}^{N} c_n z^{-n}}{1 + \sum_{n=1}^{M} a_n z^{-n}} G_i(z) = F(z) G_i(z) \]  

(8)

where:

\[ F(z) = \frac{\sum_{n=0}^{N} c_n z^{-n}}{1 + \sum_{n=1}^{M} a_n z^{-n}} \]  

(9a)

with a corresponding frequency response function:
Equations (9.a, b) and (9a,b) clearly imply that the design of a proper function $F(j\omega)$ of the filter is zero at frequencies coinciding with the expected natural frequencies of the dynamic system according to Eq. (6.a), this filter is capable of completely eliminating the residual vibrations effect. In addition, according to Eq. (6.b), the response for zero frequency of this filter should be kept equal to one, in order to ensure the proper motion of the mechanical system as a rigid body. In view of Eq. (9.b), this last requirement becomes:

$$F(0) = \sum_{n=0}^{N} c_n = 1 \quad (10)$$

Considering robust residual vibration suppression, the robustness properties for the preconditioning procedure can be directly met, by extending the requirement for zero frequency response of the filter not only for individual frequencies coinciding with the expected natural frequencies of the system, but also for extended areas (stop-band areas) of the filter frequency response function $F(j\omega)$, in order to cover additionally the possible variations of the system natural frequencies.

### III. Filter Design Procedure

#### A. Low-Pass Filter Design Requirements for Robust Residual Vibration Suppression

If at least one of the coefficients $\{a_n\}$ has non-zero value, then the filter is an Infinite Impulse Response (IIR) filter. The primary advantage of IIR filters is that they typically meet a given set of specifications with a much lower filter order than that of a corresponding FIR filter. The disadvantage of IIR filters is that ideally they have infinite response. Thus, contrary to FIR filters, the total time delay introduced by IIR filters is not equal to the filter duration. Although the duration of IIR filters can be shorter than the duration of FIR filters, the IIR filters present larger total delays.

According to Eqs. (6a, b), an appropriate filter must have a pass-band (response equal to 1) for the lowest frequencies and a stop-band (response equal to 0) for frequencies equal to or higher than the natural frequency of the flexible system. This type of filter is called low-pass filter. Contrary to the ideal filter behavior, real filters present ripples in the stop-band. These ripples prevent the requirement expressed by Eq. (6a) to be exactly satisfied. However, if a very small amount of vibration is accepted instead of zero vibrations, then the requirements for vibration suppression using actual IIR filters can be stated as:

$$F(j\omega) = F(j2\pi f) = \frac{\sum_{n=0}^{N} c_n e^{-jn2\pi fT}}{1 + \sum_{n=1}^{M} a_n e^{-jn2\pi fT}} \quad (9b)$$

where $c_{3,\text{per}}$ is the permissible size of ripples (permissible vibration error).

An ultra insensitive IIR filter, capable of suppressing residual vibrations, should satisfy the following requirements:

(R.a) Cut-off frequency (lower limit of the stop-band) smaller than the system’s smallest natural frequency.

(R.b) Stop-band quite wide in frequency in order to suppress the vibrations while the system frequencies are varying and also to cover higher modes.

(R.c) Response for zero frequency equal to one, in order to achieve appropriate rigid body motion (Eq. (11b)).

(R.d) Ripples on the stop-band smaller than a pre-specified acceptable residual vibration error (Eq. (11a)).

(R.e) Minimum possible filter delay.

#### B. Selection of the Filter Design Parameters

Several methods are used to design an IIR filter (i.e. calculate its coefficients $\{c_n\}$ and $\{a_n\}$) that can approximate the filter desired frequency response. Most of these methods make use of analog prototype filters, such as the Butterworth, Chebyshev Type I and II or the Elliptic prototype. In this work all conventional IIR filters are examined. The Chebyshev Type I and the Elliptic filters fail to satisfy requirement (R.c) and they can’t be used for vibration suppression. The Chebyshev Type II, the Butterworth and Thiran’s filters [16], are further studied in this work.

The input parameters needed for the calculation of the filter coefficients, using the corresponding MATLAB commands [17], differ from method to method. However, they can all be related to four important parameters, which specify the behavior of the filter. We call them "the IIR Filter Design Parameters". These are: the filter order $N$, the cutoff frequency $f_c$, the permissible error (size of ripples) in the stop-band and the sampling frequency $f_{sf}$.

The selection of the IIR Filter Design Parameters is not a trivial problem nor can it be done arbitrarily. The problem is that due to approximations used in all filter design methods and the non-linear phase of the IIR filters, the performance of the calculated filter may be different than the performance expected from the input design parameters. Therefore, the selection of the IIR Filter Design Parameters should be done is such a way so that the filter performance after calculation of the filter coefficients has the desired vibration suppression capabilities. In this section a method is proposed to select the IIR Filter Design Parameters taking into account this important filter design constraint. According to this method, the stop-band width ratio, a non-dimensional index is used:

$$r_e = \frac{f_c}{f_{sf}/2} \quad (12)$$

The purpose of the introduction of the above ratio is the
fact that, for an already designed filter, this ratio has constant value regardless of the value of the sampling frequency $f_{SF}$. Using the ratio $r_C$, the filter can be designed in a general non-dimensional form, independently from the actual values of the frequencies $f_C$ and $f_{SF}$ and therefore independently from the actual dynamic system. Additionally, a filter performance index, the relative delay $d$ introduced by the filter, is defined as:

$$d = \frac{T_D}{T_0} = \frac{LT_{SF}}{T_0} = \frac{L f_0}{f_{SF}}$$

(13)

where $T_D$ is the total delay introduced by the filtering process. Given the infinite nature of IIR filters, total delay is the time needed for a filtered signal to settle within a small range around its steady state value. $L$ is the total delay expressed as number of samples of the filter.

Taking into account that $f_C \leq f_0$, according to requirement (R.a), and using Eq. (12), Eq. (13) can be written as:

$$d \geq \frac{L r_C}{2} \Rightarrow (d)_{\text{min}} = \frac{L r_C}{2}$$

(14)

where $(d)_{\text{min}}$ is the minimum possible value for $d$ and corresponds to $f_C = f_0$. From Eq. (15) it is clearly seen that the relative delay is independent from the dynamic system. Minimization of the total delay $T_D$ introduced by the filtering process, according to requirement (R.e), is equivalent to minimization of the relative delay $d$, which can be performed independently from the dynamic system.

The minimization of the relative delay $d$ can’t be performed just by the minimization of the ratio $r_C$ because the number of samples $L$ is dependent on the value of $r_C$. The way to calculate the IIR Filter Design Parameters to ensure minimum relative delay $d$ is by generating the Delay-Error-Order (DEO) curves [14,15]. Those are generated via an iterative optimization algorithm using the Matlab Signal Processing Toolbox [17]. This algorithm is used to vary the filter design parameters over a large range of possible values and calculate the minimum relative delay $d$ introduced by the filter. Then the minimum $d$ is plotted as a function of the filter order and the permissible vibration error.

The generated DEO curves for the Chebyshev Type II, the Butterworth filter and the Thiran’s filter [16], are shown in Fig. 1. The permissible residual vibration error (size of ripples of the stop-band) for the calculation of the DEO curves is varied from 1% of the size of the pass-band up to 10%. The filter order was varied from 2 up to the maximum possible value for each design method. The ratio $r_C$ could start from very small values, but small values for $r_C$ lead to very large values for the sampling frequency $f_{SF}$, as can be derived from Eq. (12) and these values for $f_{SF}$ may be undesired. The value of $r_C$ is varied from 0.025 up to 0.25. Because only two of the IIR Filter Design Parameters can be determined from the DEO curves, look-up tables are also created during the optimization process to store the values for all the design parameters that lead to a minimum $d$.

Conclusively, the design of an IIR digital filter for any specific physical system must follow the following procedure. First, the system’s lowest expected damped natural frequency $f_0$ is calculated. This is the only information necessary from the flexible system and can be calculated either from experimental data or using a system model. According to requirement (R.a) in Section III.A, the filter cutoff frequency $f_C$ is set equal to the system’s lowest damped natural frequency $f_0$. Then, according to the specific task specifications and constraints for the filter total delay (and hence the relative delay $d$) and/or for the maximum permissible error, a specific DEO curve is selected. Then the value of the filter order $N$ is selected, either taking into account the overall minimum relative delay on the selected DEO curve or by limitations for the desired number of filter coefficients. This leads to a specific point on the DEO curve. Finally, using the parameters from the look-up table corresponding to this specific point on the DEO curve, the optimal filter is designed and the actual $r_C$ is derived from the frequency response of the designed filter. The sampling frequency of the filter $f_{SF}$ is obtained from Eq. (12).

Fig. 1: Delay-Error-Order (DEO) curves for a) Butterworth Filter, b) Chebyshev Type II Filter, c) Thiran’s Filter.
IV. EXPERIMENTAL DEMONSTRATION

A. Experimental System and Procedures

The proposed method is experimentally verified using a thin, long, rectangular, flexible, aluminum beam that has the physical characteristics shown in Table 1. The beam is attached at the end-effector of a five-degree-of-freedom Mitsubishi RV-M2 robot manipulator. Figure 2 shows a schematic and a picture of all mechanical and electrical components of this system. The input function to the flexible beam is the angular displacement \( \theta(t) \) introduced by one of the manipulator wrist joints, as it is shown in Fig. 3.a. The beam is rotated by an angle of 30 degrees of the vertical position and the plane of the rotation is perpendicular to the plane formed by the beam's body. The joint is under closed-loop PID control to ensure accurate implementation of the desired input function.

Table 1: Physical Properties of the Aluminum Beam

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>2.7x10^3 Kg/m^3</td>
</tr>
<tr>
<td>Weight</td>
<td>0.0697 Kg</td>
</tr>
<tr>
<td>Length</td>
<td>0.386 m</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.62x10^-3 m</td>
</tr>
<tr>
<td>Width</td>
<td>0.108 m</td>
</tr>
</tbody>
</table>

Prior to the execution of the experiment, the input function is off-line preconditioned using a low-pass IIR filter as it is described in Section II.B. The original (non-filtered) input function used in these experiments is the fastest rotation that the manipulator's wrist is able to follow. The duration of the original input function is 0.3 sec. The duration of the inputs, which are preconditioned using the two filters described in Section IV.B, is 0.9 sec and 1.1 sec, introducing a delay of 0.6 sec or 0.8 sec respectively. Fig. 3.b shows, the original and the preconditioned inputs.

Sets of experiments are performed with different mass \( M \) attached at the beam's free endpoint (see Fig. 2). To test the robustness of the proposed method, the mass \( M \) is allowed to vary from zero up to 287% of the total mass of the beam causing a variation of the natural frequency of the beam. Five cases of additional mass at the end of the beam are considered and the corresponding natural frequencies of the first mode for the flexible beam / mass system are shown in Table 2.

An Entran Accelerometer (Model EGE-732B-2000D/RS) is attached at the free-end of the flexible beam to record the beam's oscillations. WinRec v.1, a software developed at the Robotics and Mechatronics Laboratory at Rutgers University, is used in both real-time control and data acquisition [18].

Table 2: Measured System First Mode Frequencies in Hz for 5 Different Added Masses at the Beam's Tip.

<table>
<thead>
<tr>
<th>Case</th>
<th>( M )</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>200 g</td>
<td>2.5</td>
</tr>
<tr>
<td>B</td>
<td>100 g</td>
<td>3.2</td>
</tr>
<tr>
<td>C</td>
<td>40 g</td>
<td>4.13</td>
</tr>
<tr>
<td>D</td>
<td>20 g</td>
<td>4.8</td>
</tr>
<tr>
<td>E</td>
<td>0 g</td>
<td>6.4</td>
</tr>
</tbody>
</table>

B. Filter Design

For these experiments, the permissible vibration error is set at 5%. From the DEO curves shown in Fig 1, it is found that the minimum values of the relative delay is about 1.5 for the Thiran's filter, 1.85 for the Chebyshev Type II filter and 2.55 for the Butterworth filter. The last one is excluded from the experiments due to the large delay that it introduces.

Based on the filter design parameters stored in the look-up tables, two filters were designed: a low-pass filter of order 7 using the design method proposed by Thiran and a low-pass Chebyshev Type II filter of order 3. The frequency response in the stop-band is equal or less than 5% of the amplitude of the pass-band. The duration of the first filter is 0.6 sec and also the total delay \( T_D \) is 0.6 sec, which is 1.5 times the system's largest eigenperiod and the corresponding sampling frequency of the filter is equal to 13.3 Hz. The duration of the second filter is 0.06 sec, the total delay \( T_D \) is 0.8 sec, which is 2 times the system's largest eigenperiod and the corresponding sampling frequency is equal to 66.6 Hz. The coefficients of the filters are shown in Table 3 and the frequency responses are shown in Fig. 4.
Table 3: Coefficients of the Filters Used.

<table>
<thead>
<tr>
<th>N</th>
<th>Filter by Thiran</th>
<th>Chebyshev type II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c_n$</td>
<td>$a_n$</td>
</tr>
<tr>
<td>0</td>
<td>0.0206</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>-3.4855</td>
</tr>
<tr>
<td>2</td>
<td>5.5166</td>
<td>-0.0139</td>
</tr>
<tr>
<td>3</td>
<td>-5.0929</td>
<td>0.0149</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>2.9410</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-1.0564</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>0.2175</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>-0.0197</td>
</tr>
</tbody>
</table>

Fig 4: Filter Frequency Response: a) Thiran's Filter, b) Chebyshev type II Filter.

C. Experimental Results

Table 4 presents a summary of the experimental results. By preconditioning the inputs with the selected filters the level of vibrations of the flexible beam is dramatically reduced to only 5.5% of the level of vibrations obtained with no preconditioning, using Thiran's Filter and 7.5% using the Chebyshev Type II filter. Although the filters were designed for the natural frequency of Case A, they performed equally well in all five cases.

Table 4: Residual Vibrations Introduced from the Original and the Filtered Inputs; Cases A to E.

<table>
<thead>
<tr>
<th>Added Mass at Beam End (gr)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (damped) (Hz)</td>
<td>2.50</td>
<td>3.20</td>
<td>4.13</td>
<td>4.80</td>
<td>6.40</td>
</tr>
<tr>
<td>Original input function</td>
<td>85.7</td>
<td>78.2</td>
<td>46.9</td>
<td>57.0</td>
<td>64.9</td>
</tr>
<tr>
<td>Peak-to-peak amplitude (mm)</td>
<td>4.7</td>
<td>3.2</td>
<td>2.6</td>
<td>3.0</td>
<td>1.9</td>
</tr>
<tr>
<td>Filtered input</td>
<td>5.46%</td>
<td>4.06%</td>
<td>5.56%</td>
<td>5.32%</td>
<td>2.90%</td>
</tr>
<tr>
<td>Peak-to-peak amplitude (mm)</td>
<td>6.3</td>
<td>5.9</td>
<td>2.8</td>
<td>2.9</td>
<td>4.2</td>
</tr>
<tr>
<td>Filtered Input</td>
<td>7.31%</td>
<td>7.49%</td>
<td>5.94%</td>
<td>5.05%</td>
<td>6.39%</td>
</tr>
</tbody>
</table>

Fig 5: Beam Vibration With: A) 200 g Attached Mass, B) 100 g Attached Mass, C) 40 g Attached Mass, D) 20 g Attached Mass and E) Without Attached Mass.
Figure 5 presents the vibrations of the tip of the beam for the five cases, when Thiran's filter is used. These vibrations are compared with the residual vibrations induced by the original input function. The transient vibration (vibrations during the rotation of the beam) is presented using a different line type (first portion of each curve). The length of this portion presents the duration of the transient motion. It is clearly seen that the preconditioned input functions result in a significantly reduced amount of vibrations compared with the amount of vibrations obtained from the original input function. It is also seen that, although the rising time (transient portion) has been increased, the settling time is reduced dramatically due to the significant vibration suppression obtained by the preconditioning approach.

The satisfactory cancellation of residual vibration in all five cases demonstrates the robustness to variations of the dynamic characteristics of the system. Even if the system mass changes as much as 287% of the original mass (causing a natural frequency change equal to 156% of the frequency used for the filter design), the amount of vibrations obtained from the filtered input function is less than 5.56% of the residual vibrations obtained from the original input function, when the Thiran's filter is used.

It must be noticed that, although the vibrations of the preconditioned input in some cases is larger than the theoretically expected 5%, the cancellation of residual vibrations is quite satisfactory. The difference between the theoretically expected and the experimentally measured cancellation is small and the main reason for this difference is that the actual filtered input, provided by the manipulator's wrist, is only an approximation of the desired preconditioned input.

Comparing the results obtained when the IIR filter, proposed by Thiran, is used, with the results concerning the FIR filters studied by Economou et al. [14, 15], the IIR filter gave better vibration suppression results with smaller number of coefficients. However, the total delay is 20% larger than the delay introduced by the FIR filters and the robustness (the width of the stop-band) is quite smaller. When the Chebyshev Type II filter is used, the cancellation of vibrations and the robustness are almost the same with these obtained from the FIR filters, but the total delay is 60% larger than the delay introduced by the FIR filters.

V. CONCLUSION

The preconditioning of any guidance function by filtering with a properly designed low-pass IIR digital filter, drastically reduces residual vibrations in mechatronic systems. The cost paid for this reduction is a delay larger or equal to 1.5 times the largest eigenperiod of the flexible system. The residual vibration error can be practically suppressed over an extended frequency range and thus satisfactory robustness is achieved, capable to cover extended variations of the dynamic characteristics of the flexible system. Since the practical implementation of the method requires just the application of a digital filter, the application of the method for residual vibration suppression is quite simple and versatile. The corresponding filtering operation can be performed either online or offline, quite independently from the type of the original guidance. Thus, the method can be easily applied in practice to any mechanical system, with any form of original guidance, either derived through mathematical approaches (e.g. path planning methods), or input directly to the system (e.g. through direct operator commands).

VI. REFERENCES