A SPATIAL OVERCONSTRAINED MECHANISM THAT CAN BE USED IN PRACTICAL APPLICATIONS

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ABSTRACT

In this paper a new overconstrained mechanism is presented that can be used in many practical applications. It is a 5 link 4R1P spatial mechanism with two pairs of revolute joints with parallel axes. Its kinematic properties are such that it can transfer rotational motion to linear and vice versa when the revolute joint axis and the linear axis are any two lines in space. It can also be used to transfer revolute motion from one input shaft to another revolute joint whose axis is any line in space arbitrarily located with respect to the input shaft. A prototype set-up has been built to verify the mechanism kinematic properties.

1. INTRODUCTION

Mechanisms are mechanical devices composed of links connected by joints, forming open or closed mobile chains. Mechanisms are used in almost every machine to transfer motion or force. Conveyors, part handling systems, printers and vehicle suspension systems are some of the mechanisms applications (Chironis, 1991).

The main problem that is addressed when a mechanical device or linkage is assembled from a set of links and joints, is to determine its mobility. Is the linkage mobile and hence a mechanism or is it a structure? From classical mobility analysis of mechanisms, it is known that the mobility m of a linkage composed of n links that are connected with b joints can be determined by the following equation called Grübler or Kutzbach mobility criterion (Hunt, 1978, Kutzbach, 1929):

$$m=6(n-b-1)+\Sigma f \tag{1}$$

where Σf is the sum of kinematic variables in the mechanism.

However, there are linkages, that have full range mobility and therefore they are mechanisms even though they should be structures according to the mobility criterion. These linkages are called *overconstrained mechanisms*. Their mobility is due to the existence of special geometric conditions between the mechanism joint axes that are called overconstraint conditions. The problem of unveiling in a general and systematic way all the special geometric conditions that turn a structure into a mechanism has been an unsolved problem for a long time.

One of the first overconstrained mechanisms was proposed by Sarrus (1853). Since then, other overconstrained mechanisms have been proposed by various researchers. Of special interest are those proposed by Bennett (1914), Delassus (1922), Bricard (1927), Myard (1931a,b), Goldberg (1943), Waldron (1967, 1968, 1969), Wohlhart (1987, 1991, 1995) and Dietmaier (1995). In Baker (1984), Waldron (1973 a, b) and Phillips (1990) references can be found to almost all known overconstrained mechanisms. The most detailed studies of the subject of overconstraint in mechanisms are due to Baker (for example: Baker, 1980 and 1984.)

Single-loop overconstrained mechanisms with lower pairs can have two, three, four, five or six links (linkages with more than six links are mobile.) The question of overconstraint for two, three and four-bar linkages seems to be closed. Waldron (1979) asserted that all four-link overconstrained mechanisms with lower pairs are known. Hence the two-link and three-link overconstrained mechanisms are also known, since they are special cases of the four-link ones (see also Savage, 1972). Some five-link overconstrained mechanisms are known (Pamidi, Soni and Dukkipati, 1973; Hunt, 1967), of these the only ones that have all revolute joints are Goldberg's mechanisms (including as a special case the Myard mechanism.) There are only a few known overconstrained six-link mechanisms, these are mainly with revolute joints.

Two basic problems with the study of overconstrained mechanisms are: a) proof of overconstraint and b) calculation and solution of the input-output equations. Most of the time, methods to solve these problems were specialized to the geometry of the particular mechanism studied. Mavroidis and Roth (1994, 1995a, 1995b) proposed a general and systematic method to prove overconstraint and obtain the input-output equations of any closed loop overconstrained mechanism.

Overconstrained mechanisms have many appealing characteristics. Most of them are spatial mechanisms. Their spatial kinematic characteristics make them good candidates in modern linkage designs where spatial motion is needed. Another advantage of overconstrained mechanisms is that they are mobile using fewer links and joints than it is expected. For example, in normal closed loop revolute joint spatial mechanisms, the linkage should have at least seven links to be mobile. Overconstrained mechanisms can be mobile with four, five or six links. Fewer links and joints in a mechanism means reduction in cost and complexity.

While many overconstrained mechanisms have been discovered, only a few of them have been used in practical applications. There are many reasons for this. Most of the engineers are unaware of the existence of spatial overconstrained mechanisms and their properties. For example, very few engineers in industry know the four-bar Bennett or the six-bar Bricard spatial mechanisms and their properties and hence they can not consider these mechanisms in their designs. The other reason for not using overconstrained mechanisms in industrial applications is that most of the known overconstrained mechanisms have complex kinematic properties. This is because these mechanisms have been found using mobility criteria only and not other criteria as well, such as to satisfy a desired input-output relationship. For example, a six-bar Bricard mechanism is shown in Figure 1a. This mechanism satisfies the special geometric conditions shown in the Figure (see Section 2 for the notation on the Denavit and Hartenberg parameters.) If joint angle θ_3 is the input and joint angle θ_1 is the output then their relationship during a mechanism full cycle is shown in Figure 1b. This curve shows a complex input-output relationship that can not have an obvious use. A linear input-output relationship would have been very useful since in this case the mechanism can be used to transfer revolute motion from one shaft to another skew output shaft.

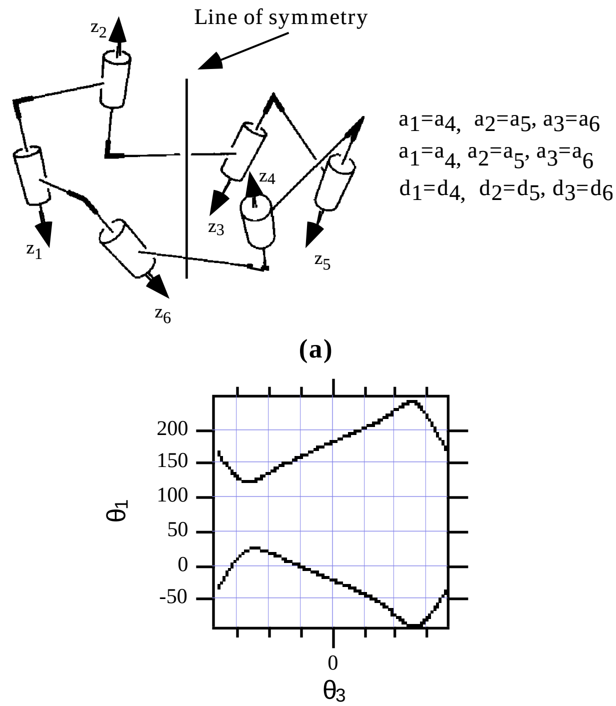


Figure 1: The Bricard Mechanism and Its Input-Output Curve

In this paper an overconstrained mechanism is studied that can be used in many important applications. This mechanism, that was first introduced in Mavroidis and Roth (1995b), can be used to transfer rotational motion to linear and vice versa when the revolute joint axis and the prismatic joint axis are any skew lines in three dimensional space. It can also be used to transfer revolute motion to another revolute joint when the two joint axes are again any skew lines in space. Such a mechanism can eliminate the need to use various types of gears such as conic, spur, bevel or worm gears which are expensive and heavy when change of direction of the axis of revolute motion is needed. Computer Aided Design models and an experimental prototype system were built to validate the kinematic properties of the proposed mechanism.

2. NOTATION

In this work, revolute and prismatic joints are denoted with the letters R and P respectively. It is easy to show (Mavroidis and Roth, 1995a) that studying the six-link mechanisms with only revolute and prismatic joints includes all other mechanisms with fewer links and multiple degree of freedom joints.

The relative position of links and joints is described using the variant of Denavit and Hartenberg notation (Denavit and Hartenberg, 1955,) in which the parameters a_i, α_i, d_i and θ_i are defined so that: a_i is the length of link i, α_i

is the twist angle between the axes of joints i and i+1, d_i is the offset along joint i and θ_i is the rotation angle about joint axis i (see Figure 2.)

For a six-link chain there are twenty-four Denavit and Hartenberg parameters that define its assembly configuration. In general a six-link chain is immobile. However, since we are interested in overconstrained six-link mechanisms, our chains will be mobile and six of these parameters will be motion variables (for a revolute (prismatic) joint, θ_i (d_i), i=1,...,6). The other eighteen parameters are constants, called the structural parameters, and they define the special geometry that makes the chain mobile. The special values of the structural Denavit and Hartenberg parameters, can be considered to be due to special geometric conditions between the joint axes called “overconstrained conditions.”

At each link a reference system R_i is attached. The position and orientation of frame R_{i+1} into the previous frame R_i is described by the 4x4 homogeneous matrix A_i:

$$A_i = \begin{pmatrix} c_i & -s_i\lambda_i & s_i\mu_i & a_i c_i \\ s_i & c_i\lambda_i & -c_i\mu_i & a_i s_i \\ 0 & \mu_i & \lambda_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where: $s_i=\sin(\theta_i)$, $c_i=\cos(\theta_i)$, $\mu_i=\sin(\alpha_i)$ and $\lambda_i=\cos(\alpha_i)$.

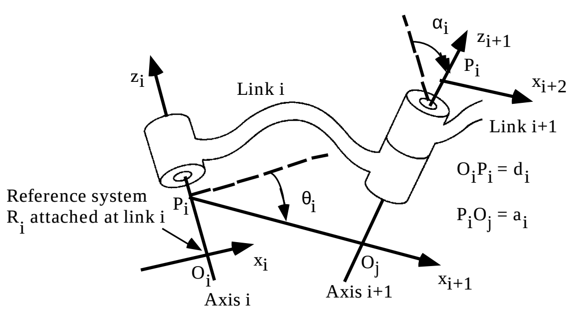


Figure 2: Denavit and Hartenberg Parameters

3. THEORY OF OVERCONSTRAINT

In this section, a brief description of a systematic methodology to prove mobility and obtain the input-output equations of any overconstrained mechanism is presented (Mavroidis and Roth, 1994, 1995a, 1995b). The mechanism that is presented in Section 4 was found using this method.

The method to prove overconstraint was based on the solution of inverse kinematics of general six-joint manipulators. Raghavan and Roth (1993) proposed a method that solves the inverse kinematics of any general non-degenerate, six-joint, open loop serial manipulator at any configuration of its end-effector. The inverse kinematics problem is the problem of finding the values of the manipulator’s joint variables that correspond to a given end-effector position and orientation. Characteristic polynomial is called a polynomial in one of the joint variables that gives all the solutions to the inverse kinematics problem for this joint variable for a specific configuration.

In order to study overconstrained mechanisms using the solution of inverse kinematics problem, six-joint manipulators must be considered in configurations where the end-effector is reaching its base (i.e. in configurations where the end-effector reference system R_7 coincides with the base reference frame R_1). In these configurations the manipulator becomes a six-link closed loop structure and the loop closure equation is written as:

$$A_1\, A_2\, A_3\, A_4\, A_5\, A_6 = I \Leftrightarrow A_3\, A_4\, A_5 = A_1^{-1}\, A_2^{-1}\, A_6^{-1} \quad (2)$$

where I is the 4x4 unitary matrix. The elements of the third column of the matrix equation (2) forms a vector called **l** and those of the fourth column form a vector called **p**. Equating the left and right expressions of **l** and **p** six scalar equations are obtained that are the main equations in solving the inverse kinematics problem.

The overconstraint criterion can be stated as: calculate the roots of the characteristic polynomial of the corresponding open loop manipulator when the end-effector frame is coincident to the base frame. If the characteristic polynomial has no real roots then the linkage can not be assembled. If the characteristic polynomial has a finite number of roots, then the linkage can be assembled in a finite number of configurations but it is not mobile. It is a structure. If the characteristic polynomial coefficients are all zero then the linkage is mobile.

The solution of the inverse kinematics problem, starting from the equations of **l** and **p**, after an elimination procedure, reaches a point where a homogeneous linear system of equations is obtained. This linear system, for a 6R manipulator, can have the following form:

$$\Sigma(\theta_3)X=0 \quad (3)$$

where Σ is a 12x12 matrix, whose elements depend on the manipulator structural parameters, the end-effector coordinates and only one of the kinematic joint variables which in the case of Equation 2 is joint angle θ_3. The vector X is a 12x1 vector whose elements are power products of the other joint variables.

The characteristic polynomial of the manipulator at any configuration is equal to the determinant of Σ. In cases where the manipulator forms a mobile six-joint closed loop chain, the characteristic polynomial coefficients are zero because the lines of Σ become linearly dependent. Since the closed loop chain is mobile, one of the joint variables is the mechanism input and therefore it is known. For example, in Equation 2, θ_3 can be the mechanism input, and one of the other joint variables can be the mechanism output, let's say θ_4. Then Equation (2) can be written as:

$$\Sigma'(\theta_4)X'=0 \tag{4}$$

where Σ' is a 3x3 matrix. The determinant of Σ' will result in a polynomial in θ_4 with roots that give the values of the output that correspond to a certain value of the input.

A detailed description of the method to prove overconstraint and obtain the input-output equations can be found in Mavroidis and Roth (1994b, 1995a and b.)

4. A NEW MECHANISM

In this section an overconstrained mechanism is presented that can be used in important practical applications. This mechanism was first presented briefly in Mavroidis and Roth (1995b).

This is a 5 link mechanism with four revolute and one prismatic joints. Its Denavit and Hartenberg parameters and the special conditions they satisfy are shown in Table 1. This mechanism is shown in Figure 3 in an open configuration and in Figure 4 in a closed assembly configuration. Both Figures are captions from the mechanism drawings generated by an automated program using the computer aided design software I-DEAS (Integrated Design and Engineering Analysis Software, Structural Design and Research Corp., 1994).

	α	a	d,θ
1	α_1	a_1	θ_1
2	0	a_2	d_2
3	$\alpha_3 = \alpha_1 + \alpha_5$	$a_3 = a_1 + a_5$	d_3
4	0	$a_4 = a_2$	d_4
5	$\alpha_5 = \alpha_1$	a_5	$d_5=d_2+d_3-d_4$

Table 1: Denavit and Hartenberg Coordinates of the New Mechanism

The mechanism has two pairs of revolute joints with parallel joint axes. The link that connects the two parallel joints in a pair is called "parallel pair link". Both pairs have equal length links. The two parallel pairs are connected with each other with a link called in this paper the "coupler link." Each pair of parallel joints is connected with the prismatic joint with a link called the "crank". Both cranks are of equal length and form equal angles with the prismatic joint axis. The coupler link has twice the length of each crank and its twist angle is two times the twist angle of each of the cranks. The sum of the offsets in each pair of revolute joints is equal. Using the theory of Section 3 it can be shown that the mobility of this mechanism is one.

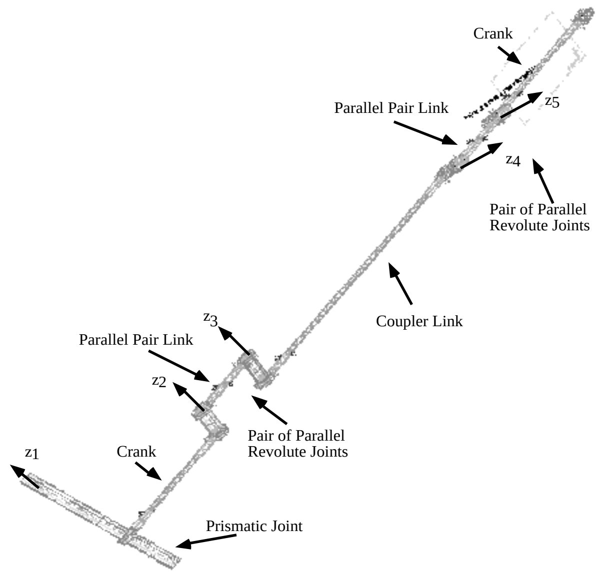


Figure 3: The New Mechanism in an Open Configuration

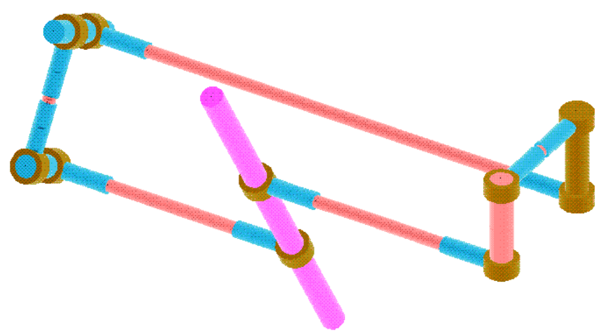
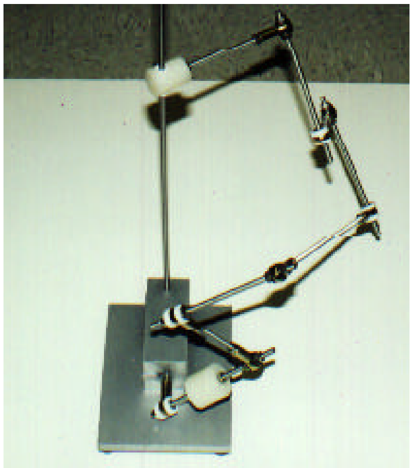
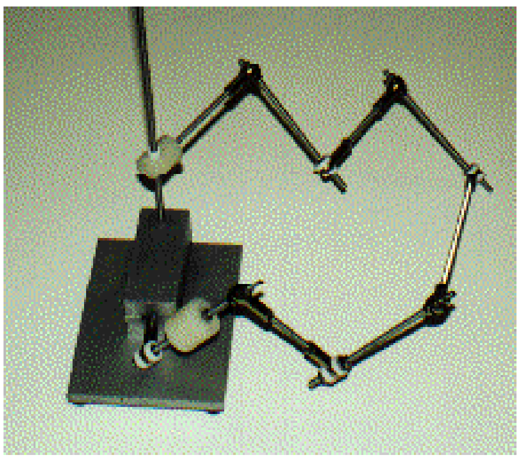


Figure 4: The New Mechanism in a Closed Configuration

A prototype of the new mechanism has been built in our laboratory that demonstrates the mechanism mobility. This prototype is shown in Figures 5a and b, in two different configurations. The dimensions of the experimental set-up are shown in Table 2. The mechanism links are 1/4" steel rods. The mechanism has full cycle mobility with no intersection between its members. The range of motion of the prismatic joint is approximately 14".



(a)



(b)

Figure 5: The Prototype System in Two Configurations

	α (deg)	a (in)	d_i (in), θ_2 (deg)
1	45°	6	θ_2=0
2	0°	5	5
3	90°	12	5
4	0°	5	5
5	45°	6	5

Table 2: Parameters of the Prototype System

It can easily be shown, using vectors **l** and **p**, that the kinematic variables of this mechanism satisfy Equations (5)-(8) where, θ_2 is chosen as the mechanism input and the other variables are considered to be the mechanism outputs:

$$\theta_3 = -\theta_2 \tag{5}$$

$$\theta_4 = \theta_2 \tag{6}$$

$$\theta_5 = -\theta_2 \tag{7}$$

$$d_1 = -2[\, a_2\mu_1 s_2 + \lambda_1(d_4+d_5)] \tag{8}$$

Using Equation (5) to (8) the input-output curves of the mechanism prototype are drawn in Figure 6. The same curves were also obtained using the method described in Section 3 and verified experimentally.

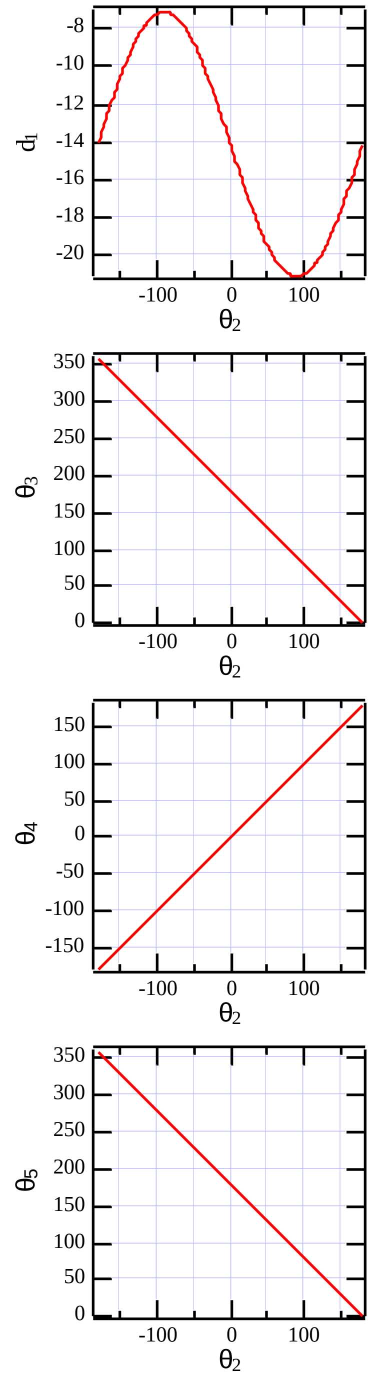


Figure 6: Input-Output Curves of the Mechanism

From equations (5)-(8) it is obvious that a linear, one to one relationship, exists between the joint angles of any two revolute joints. Hence, if the coupler link is grounded, then any one of the two revolute joints connected with this link, can be the input and the other one can be the output. For example, for the mechanism shown in Figure 3, θ_3 can be the input and θ_4 can be the output. As the relative position of the two revolute joint axes connected by the coupler is a design parameter, any two lines in space can be selected. Therefore this mechanism can be used to transfer revolute motion from one input shaft to another output shaft, that has any position with respect to the input shaft. This is an important result since this mechanism can substitute various types of special geared mechanisms that are used to change the direction of the revolute motion and which are very expensive and their fabrication is very complicated.

From Equation (8), it can also be seen that a sinusoidal relationship exists between the kinematic variable of the slider and the joint angle of any one of the revolute joints. Therefore, if anyone of the cranks is grounded then this mechanism can be used to change revolute motion to linear motion and vice versa when the axis of the slider and the axis of the revolute joint are any two lines in space. This mechanism is the exact equivalent of the planar 4-bar slider crank. In the planar slider crank the axes of the revolute and prismatic joint are always perpendicular. In the new mechanism, these axes can have any distance and any orientation with each other. The experimental prototype shown in Figure 5 was built having one of the cranks grounded.

If the mechanism is used to transfer revolute motion to revolute, then the input-output relationships are not affected by the mechanism dimensions and any values will result in the desired function. If the mechanism is used to change revolute motion to linear and vice versa, then a desired specification is the range of motion of the slider. This specification affects the mechanism dimensions. From Equation (8) it can be seen that larger length of the parallel pair link or larger crank twist angle increase the range of motion of the slider. Also the offsets in the parallel pairs affect the mean position of the travel of the slider.

Theoretically the mechanism has full cycle mobility. However, in practice there is a possibility of link intersection that will impede the mechanism to complete its cycle. In both applications mentioned above, link intersection must be avoided. There are three types of possible link intersections. The first type is intersection of two adjacent links connected by a revolute joint. This intersection is due to small offset between the two links and can easily be corrected. The second type of link intersection can occur between the two cranks that are connected by the prismatic joint. Assume for example that one of the cranks is grounded and that the end of the other crank slides on the prismatic joints. There are mechanisms where the range of motion of the moving crank will require that it gets the other side of the fixed crank and this is impossible. This type of link intersection can be predicted and avoided using Equation (8). From this equation, the mechanism dimensions can be found for which the variable d_1 does not change sign during a full cycle of the mechanism. The third type of link intersection is very difficult to predict analytically. In this case the coupler link intersects with the prismatic joint. In this work, a graphical approach has been developed to predict and avoid this type of link intersection. A program has been written using the computer aided design package I-DEAS to detect this type of link intersection.

This mechanism belongs to the large class of 6-link linkages with revolute and prismatic joints that have three pairs of parallel joint axes. The mechanism studied in this paper which is a 5 link, can be seen as a 5R1P 6 link mechanism with one prismatic and one revolute joint axes coincident. We did an extensive search to find which other linkages with three pairs of parallel joint axes can be mobile, and it can be shown that this is the only mobile linkage of this class. All other 6R, 4R2P and 3R3P linkages with three pairs of parallel joint axes are structures.

5. CONCLUSIONS

In this paper a new overconstrained mechanism has been presented that can be used in many practical applications. It is a 5 link 4R1P spatial mechanism with two pairs of revolute joints that have parallel joint axes. Its kinematic properties are such that it can transfer rotational motion to linear and vice versa when the revolute joint axis and the linear axis are any two lines in space. It can also be used to transfer a revolute motion to another revolute joint whose axis has any general location with respect to the input shaft. This mechanism shows that overconstrained linkages can not only be important from a theoretical point of view but they can find some useful applications as well.

6. ACKNOWLEDGMENTS

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