# Mechatronic System Case Study: Thermal Closed-Loop Control System

1. Introduction

This case study is a dynamic system investigation of an important and common mechatronic system: a thermal closed-loop control system with a resistive heater, temperature sensor, and microcontroller. The study emphasizes the two key elements in the study of mechatronics:

- Integration through design of mechanical engineering, electronics, controls, and computers
- Balance between modeling / analysis / simulation and hardware implementation

The case study follows the procedure outlined in Figure 1.
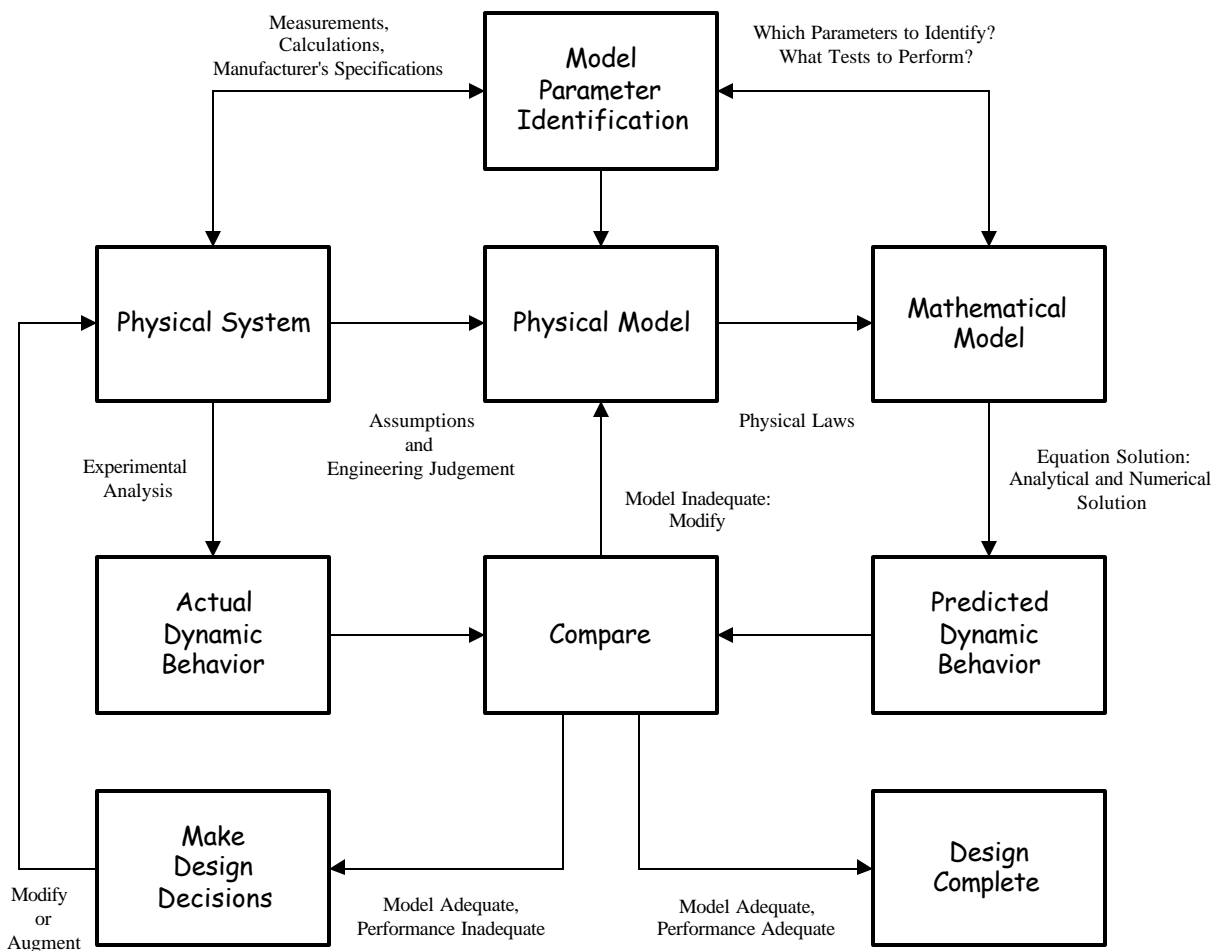
**Figure 1. Dynamic System Investigation**

Thermal regulation is a common control problem. Temperature control systems are found in a host of commercial products and in many environments. In our homes, we find temperature regulation devices that maintain the temperature of our rooms and regulate the temperature of our ovens and refrigerators. In our cars, we find temperature control mechanisms that regulate the temperature of our engine, which help to preserve the integrity of the lubrication and combustion processes. Automobile interiors have mechanisms which allow us to adjust the temperature of the passenger compartment, and, as we physically sense the temperature and adjust the available mechanisms, we become part of the control process. Office equipment, such as xerographic and facsimile machines, have sophisticated control mechanisms that regulate the temperatures of the fuser and thermal transfer rolls in these devices. This case study explores methods of controlling the production of heat energy for the purpose of producing a desired temperature.

The objective of this case study is to control the temperature of a thin aluminum plate, as measured by a temperature sensor positioned in the middle of the top of the plate, by regulating the voltage supplied to a resistive heater positioned under the plate. The temperature is to be regulated to a point 20° C above the temperature of the ambient air. This will be accomplished by:
- applying the general procedure for a dynamic system investigation to the thermal system
- understanding the physical system, developing a physical model on which to base analysis and design, and experimentally determining and/or validating model parameter values
- developing a mathematical model of the system, analyzing the system in MatLab, and comparing the results of the analysis to experimental measurements
- designing a feedback control system in MatLab to meet performance specifications
- implementing the control system and experimentally validating its predicted performance

2. Physical System

The physical system that we are investigating is shown in Figure 2. It consists of an aluminum plate, two inches square and 1/32 inches in thickness, which we desire to control the temperature of. This thin plate is heated on its underside by a thin-film resistive heater, which converts electrical energy to thermal energy. The heat supplied by the heater to the plate depends on the power dissipation across the heater, which is a function of the voltage applied to the heater and the heater's resistance. The resistive heater is insulated on its underside by insulative, ceramic tape 1/8 inches thick to inhibit conductive transfer of heat from the bottom of the resistive heater. The thermal conductivity k of the ceramic insulation is 0.055 W/m-K compared to 177 W/m-K for the aluminum plate. The top of the thin, heated, aluminum plate is exposed to ambient air. Attached to the center of the heated plate is a temperature sensor whose electrical properties vary with the temperature of the surface to which it is bonded.
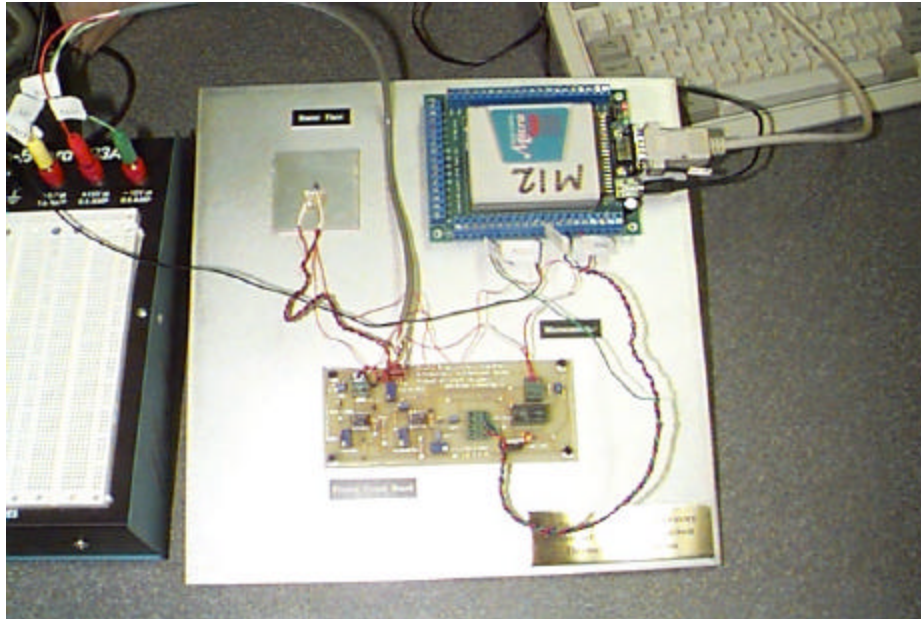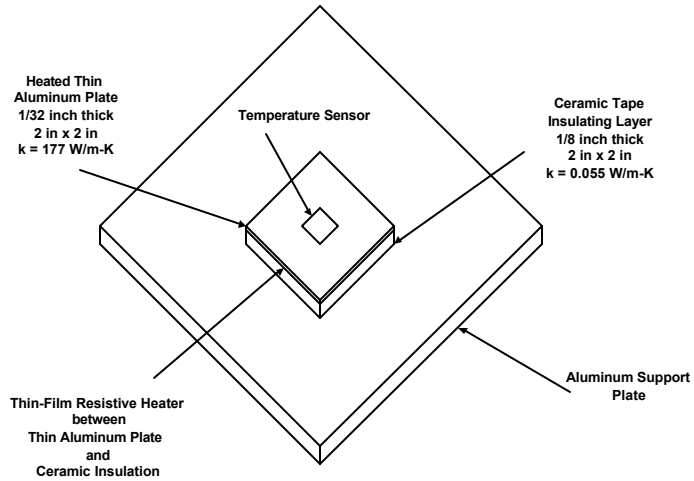
Figure 2. Physical System

The resistive heater is supplied by Minco Products. Its specifications are listed in Table 1.

Table 1. Properties of the Resistive Heater

| Specifications	Value
Manufacturer	Minco Products
Model Number	HK-5169-R185-L12-B
Heater Resistance	185 ohms $\pm$10%
Heater Area	4 in^2

Heater Thickness	0.010 inches

The temperature sensor is manufactured by Analog Devices, component number AD590. The specifications for this device are listed in Table 2.

Table 2. Properties of the AD590 Temperature Sensor

Specification	Value
Rated Temperature Range	-55° C to 150° C
Power Supply (min)	4 volts
Power Supply (max)	30 volts
Nominal Output Current @ 298.2 K	298.2 µA
Temperature Coefficient	1 µA/K
Calibration Error @ 25° C	± 2.5° C
Maximum Forward Voltage	44 volts
Maximum Reverse Voltage	-20 volts
Case Breakdown Voltage	± 200 volts

The sensor is implemented as shown in Figure 3.
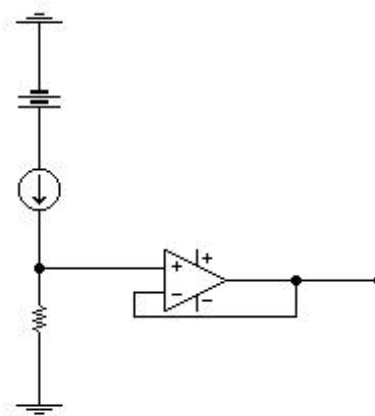
Figure 3. Sensor Voltage Circuit

As can be seen from the circuit in Figure 3, the temperature sensor (with a 15 volt power source) acts like a current source of 1 µA/K and is connected in series with a 1 kΩ resistor. The potential of the resistor with respect to ground is buffered by a follower amplifier. The gain of this sensor in volts/K is given by:

$$K_{sensor}(volts/K) = (1\mu A)(R_{sensor})$$

where the nominal resistance value of R_{sensor} is 1 kΩ. This leads to a nominal sensor gain of 1 mV/K.

The plate is made of aluminum. The relevant physical properties of aluminum are listed in Table 3.

Table 3. Material Properties of 6061 Aluminum

Property	Value
Melting Point	775 K
Density, ρ	2770 kg/m^3
Specific Heat, c_p	875 J/kg-K
Thermal Conductivity, k	177 W/m-K

3. Physical Model

The challenges in physical modeling are formidable:

- Dynamic behavior of many physical processes is complex.
- Cause and effect relationships are not easily discernible.
- Many important variables are not readily identified.
- Interactions among the variables are hard to capture.

The first step in physical modeling is to specify the system to be studied, its boundaries, and its inputs and outputs. One then imagines a simple physical model whose behavior will match sufficiently closely the behavior of the actual system. A physical model is an imaginary physical system which resembles the actual system in its salient features but which is simpler (more "ideal") and is thereby more amenable to analytical studies. It is not oversimplified, not overly complicated - it is a slice of reality. The astuteness with which approximations are made at the outset of an investigation is the very crux of engineering analysis. The ability to make shrewd and viable approximations which greatly simplify the system and still lead to a rapid, reasonably accurate prediction of its behavior is the hallmark of every successful engineer. This ability involves a special form of carefully developed intuition known as *engineering judgment*. Table 4 lists some of the approximations used in the physical modeling of dynamic systems and the mathematical simplifications that result. These assumptions lead to a physical model whose mathematical model consists of linear, ordinary differential equations with constant coefficients.

Table 4. Approximations Used in Physical Modeling

Approximation	Mathematical Simplification
Neglect small effects	Reduces the number and complexity of the equations of motion
Assume the environment is independent of system motions	Reduces the number and complexity of the equations of motion
Replace distributed characteristics with appropriate lumped elements	Leads to ordinary (rather than partial) differential equations
Assume linear relationships	Makes equations linear; allows superposition of solutions
Assume constant parameters	Leads to constant coefficients in the differential equations
Neglect uncertainty and noise	Avoids statistical treatment

Let's briefly discuss these assumptions.

Neglect Small Effects
Small effects are neglected on a relative basis. In analyzing the motion of an airplane, we are unlikely to consider the effects of solar pressure, the earth's magnetic field, or gravity gradient. To ignore these effects in a space vehicle problem would lead to grossly incorrect results!

Independent Environment
Here we assume that the environment, of which the system under study is a part, is unaffected by the behavior of the system, i.e., there are no loading effects. In analyzing the vibration of an instrument panel in a vehicle, for example, we assume that the vehicle motion is independent of the motion of the instrument panel. If loading effects are possible, then either steps must be taken to eliminate them (e.g., use of buffer amplifiers), or they must be included in the analysis.

Lumped Characteristics
In a lumped-parameter model, system dependent variables are assumed uniform over finite regions of space rather than over infinitesimal elements, as in a distributed-parameter model. Time is the only independent variable and the mathematical model is an ordinary differential equation. In a distributed-parameter model, time and spatial variables are independent variables and the mathematical model is a partial differential equation. Note that elements in a lumped-parameter model do not necessarily correspond to separate physical parts of the actual system. A long electrical transmission line has resistance, inductance, and capacitance distributed continuously along its length. These distributed properties are approximated by lumped elements at discrete points along the line.

Linear Relationships
Nearly all physical elements or systems are inherently nonlinear if there are no restrictions at all placed on the allowable values of the inputs. If the values of the inputs are confined to a sufficiently small range, the original nonlinear model of the system may often be replaced by a linear model whose response closely approximates that of the nonlinear model. When a linear equation has been solved once, the solution is general, holding for all magnitudes of motion. Linear systems also satisfy the properties of superposition and homogeneity. The superposition property states that for a system initially at rest with zero energy, the response to several inputs applied simultaneously is the sum of the individual responses to each input applied separately. The homogeneity property states that multiplying the inputs to a system by any constant multiplies the outputs by the same constant.

Constant Parameters
Time-varying systems are ones whose characteristics change with time. Physical problems are simplified by the adoption of a model in which all the physical parameters are constant.

<u>Neglect Uncertainty and Noise</u>

In real systems we are uncertain, in varying degrees, about values of parameters, about measurements, and about expected inputs and disturbances. Disturbances contain random inputs, called noise, which can influence system behavior. It is common to neglect such uncertainties and noise and proceed as if all quantities have definite values that are known precisely.

Thermal System Simplifying Assumptions:

1. Temperature of the plate is uniform.
2. There in no heat loss through the sides of the plate, i.e., one-dimensional heat conduction through the plate.
3. Thermal conductivity of the plate is constant, i.e., independent of time, temperature, space, or direction of heat flow.
4. Heat loss due to radiation is negligible compared to the convective heat loss from the plate.
5. Convection coefficient is constant and is evaluated at the operating temperature of the plate.
6. Heat loss through the insulative layer is negligible, i.e., heat loss through the insulative layer, and subsequent convective heat loss from the supporting plate, is negligible compared to the other heat losses in the system.
7. Sensor dynamics are negligible, i.e., the sensor dynamics are very fast relative to the dynamics of the rest of the system.
8. Ambient air temperature is unaffected by the heat flux from the plate.

What mathematical simplifications do these assumptions lead to? Figure 4 shows a diagram of the resulting physical model.
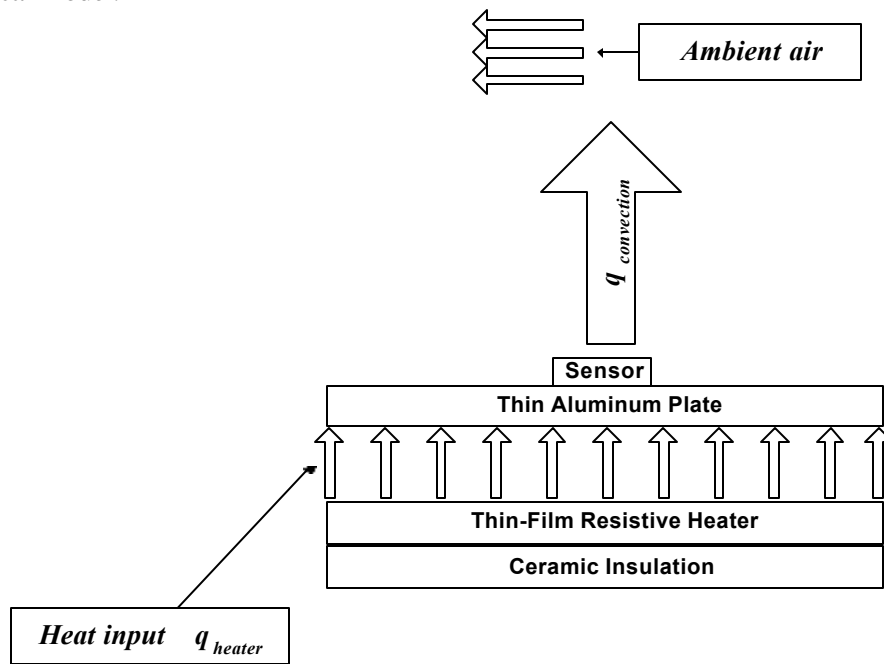
Figure 4. Diagram of Physical Model

4. Mathematical Model

The steps in mathematical modeling are as follows:

- Define System, System Boundary, System Inputs and Outputs
- Define Through and Across Variables
- Write Physical Relations for Each Element
- Write System Relations of Equilibrium and/or Compatibility
- Combine System Relations and Physical Relations to Generate the Mathematical Model for the System

Let's look at each step more closely and then apply these steps to our physical model.

- *Define System, System Boundary, System Inputs and Outputs*: A system must be defined before equilibrium and/or compatibility relations can be written. Unless physical boundaries of a system are clearly specified, any equilibrium and/or compatibility relations we may write are meaningless. System outputs to the environment and system inputs from the environment must be also clearly defined.

- *Define Through and Across Variables*: Precise physical variables (velocity, voltage, pressure, flow rate, etc.) with which to describe the instantaneous state of a system, and in terms of which to study its behavior must be selected. Physical Variables may be classified as:

  1. *Through Variables* (one-point variables) measure the transmission of something through an element, e.g., electric current through a resistor, fluid flow through a duct, force through a spring.

  2. *Across Variables* (two-point variables) measure a difference in state between the ends of an element, e.g., voltage drop across a resistor, pressure drop between the ends of a duct, difference in velocity between the ends of a damper.

- *Write Physical Relations for Each Element*: Write the natural physical laws which the individual elements of the system obey, e.g., mechanical relations between force and motion, electrical relations between current and voltage, electromechanical relations between force and magnetic field, thermodynamic relations between temperature, pressure, etc. These relations are called constitutive physical relations as they concern only individual elements or constituents of the system. They are relations between the through and across variables of each individual physical element and may be algebraic, differential, integral, linear or nonlinear, constant or time-varying. They are purely empirical relations observed by experiment and not deduced from any basic principles. Also write any energy conversion relations, e.g., electrical-electrical (transformer), electrical-mechanical (motor or generator), mechanical-mechanical (gear train). These relations are between across variables and between through variables of the coupled systems.

- *Write System Relations of Equilibrium and/or Compatibility*: Dynamic equilibrium relations describe the balance of forces, of flow rates, of energy which must exist for the system and its subsystems. Equilibrium relations are always relations among through variables, e.g., Kirchhoff's Current Law (at an electrical node), continuity of fluid flow, equilibrium of forces meeting at a point. Compatibility relations describe how motions of the system elements are interrelated because of the way they are interconnected. These are inter-element or system relations. Compatibility relations are always relations among across variables, e.g., Kirchhoff's Voltage law around a circuit, pressure drop across all the connected stages of a fluid system, geometric compatibility in a mechanical system.

- *Combine System Relations and Physical Relations to Generate the Mathematical Model for the System*: The mathematical model can be an input-output differential equation or state-variable equations. A state-determined system is a special class of lumped-parameter dynamic system such that: (i) specification of a finite set of n independent parameters, *state variables*, at time $t = t_0$ and (ii) specification of the system inputs for all time $t \geq t_0$ are necessary and sufficient to uniquely determine the response of the system for all time $t \geq t_0$. The *state* is the minimum amount of information needed about the system at time t_0 such that its future behavior can be determined without reference to any input before t_0. The *state variables* are independent variables capable of defining the state from which one can completely describe the system behavior. These variables completely describe the effect of the past history of the system on its response in the future. Choice of state variables is not unique and they are often, but not necessarily, physical variables of the system. They are usually related to the energy stored in each of the system's energy-storing elements, since any energy initially stored in these elements can affect the response of the system at a later time. The *state-space* is a conceptual n-dimensional space formed by the n components of the state vector. At any time t the state of the system may be described as a point in the state space and the time response as a trajectory in the state space. The number of elements in the state vector is unique, and is known as the *order* of the system. The *state-variable* equations are a coupled set of first-order ordinary differential equations where the derivative of each state variable is expressed as an algebraic function of state variables, inputs, and possibly time.

Let's apply these steps to the thermal system.

- *Define System, System Boundary, System Inputs and Outputs*:
  Figure 4 shows the physical model of the physical system, identifying the system and the system boundary. The input to the system is the voltage supplied to the resistive heater and the output of the system is the plate temperature as measured by the temperature sensor centered on the top of the plate.

- *Define Through and Across Variables:*
  The through variable is the heat flow rate q (J/s or W) and the across variable is the temperature $\boldsymbol{q}$ (K). We assume that all points in the body have the same temperature (average temperature) and temperature deviations from the average at various points do not affect the validity of the single-

temperature model. If this is not the case, the body may be partitioned into segments, each of which has an average temperature associated with it. Since temperature is a measure of the energy stored in a body, we normally select the temperatures as the state variables of a thermal system.

- *Write Physical Relations for Each Element:*
  There are only two types of passive thermal elements: thermal capacitance and thermal resistance. We also need to consider thermal sources.

  1. Thermal Capacitance
     When heat flows into a body of solid, liquid, or gas, this thermal energy may show up in various forms such as mechanical work or changes in kinetic energy of a flowing fluid. If we restrict ourselves to bodies of material for which the addition of thermal energy does not cause significant mechanical work or kinetic energy changes, the added energy shows up as stored internal energy and manifests itself as a rise in temperature of the body. A physical body at a uniform temperature will have an algebraic relationship between its temperature and the heat stored in it. Provided that there is no change of phase and that the range of temperatures is not excessive, this relationship can be considered linear.

     $$q_{in}(t) - q_{out}(t) = \text{ net heat flow rate into body}$$

     $$\int_{t_0}^{t} \left[q_{in}(\lambda) - q_{out}(\lambda) \right] d\lambda = \text{ net heat supplied between times } t_0 \text{ and } t$$

     Assume that heat supplied during this time interval equals a constant C times *Dq*.

     $$C\left[\theta(t) - \theta(t_0)\right] = \int_{t_0}^{t} \left[q_{in}(\lambda) - q_{out}(\lambda) \right] d\lambda$$

     where C is the thermal capacitance (J/K) and is equal to *Ms*, where M is the mass of the body (kg) and *s* is the specific heat of the body (J/kg-K). Differentiating the above equation results in:

     $$\dot{\theta} = \frac{1}{C}\left[q_{in}(t) - q_{out}(t) \right]$$

     We can use this equation only when the temperature of the body is assumed uniform. If thermal gradients within the body are so great that we cannot make this assumption, then the body should be divided into two or more parts with separate thermal capacitances.

  2. Thermal Resistance
     Whenever two objects have different temperatures, there is a tendency for heat to be transferred from the hot region to the cold region in an attempt to equalize temperatures. For a given temperature difference, the rate of heat transfer varies depending on the thermal resistance of the path between the hot and cold region. The nature and magnitude of the thermal resistance depend on the modes of heat transfer involved: conduction, convection, or radiation.
     a) Conduction: In conduction, heat flows from one body to another through the medium connecting them at a rate proportional to the temperature difference between the points:

     $$q(t) = \frac{1}{R}\left[\theta_1(t) - \theta_2(t) \right]$$

where R is the thermal resistance (K-s/J or K/W) which equals L/Ak, where A is the cross-sectional area of the path, L is the length of the path, and k is the thermal conductivity of the material (W/m-K). We can use this equation only when the body being treated as a thermal resistance does not store any heat.

b) Convection: Many practical situations involve heat flow through fluid / solid interfaces by convection. Here heat flows by conduction through a thin layer of fluid (called the boundary layer) which adheres to the solid wall. At the interface between the boundary layer and the main body of fluid, the heat is carried away by constantly moving fluid particles into the main stream. This overall process is called convection and is described by the equation:

$$q = hA\left[\theta_1(t) - \theta_2(t)\right]$$

where A is the surface area and h is the film coefficient of heat transfer (J/s-m^2-K), which is experimentally determined and usually varies with temperature.

c) Radiation: Two bodies can exchange thermal energy with no physical contact whatever by the process of radiation. The rate of heat transfer depends on a surface property of each body called the emissivity, geometrical factors involving the portion of the emitted radiation from one body that actually strikes the other body, the surface areas involved, and the temperatures of the two bodies. For a given configuration and materials, the defining equation takes the form:

$$q = C\left[\theta_1^4(t) - \theta_2^4(t)\right]$$

where C includes all the effects other than the temperatures and is usually quite small. The temperatures are absolute.

3. Thermal Sources: A thermal source can be one that adds or removes heat at a specified rate, or can take the form of a temperature of a body as a known function of time regardless of the rate at which heat flows between the body and the rest of the system. For this thermal system, the resistive heater electrical network is shown in Figure 5, where R_h is the heater resistance, i_h is the heater current, and V_h is the heater voltage.
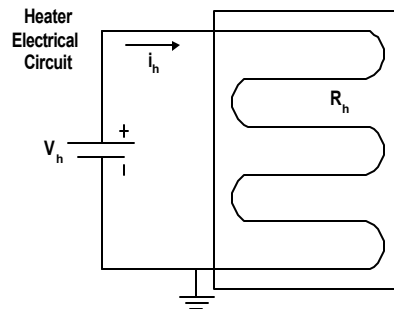
Figure 5. Electrical Network of the Resistive Heater

If we assume 100% conversion efficiency, the heat flux produced by the resistive heater is equivalent to the power dissipated across the resistive element. The power dissipated across the resistive heater is given by:

$$P = V_h i_h = V_h \frac{V_h}{R_h} = \frac{V_h^2}{R_h}$$

- *Write System Relations of Equilibrium and/or Compatibility:*
  The general procedure for this step is:
  a) Select the temperature of each thermal capacitance as a state variable and use

  $$\dot{\theta} = \frac{1}{C}\left[q_{in}(t) - q_{out}(t) \right]$$

  to obtain the corresponding state-variable equation.
  b) The net heat flow rate into a thermal capacitance depends on the heat sources and heat flow rates through thermal resistances. Use

  $$q(t) = \frac{1}{R}\left[\theta_1(t) - \theta_2(t) \right]$$

  to express the heat flow rates through the resistances in terms of the system's state variables.

- *Combine System Relations and Physical Relations to Generate the Mathematical Model for the System*:

  $$q_{in}(t) = \text{ heater input} = \frac{V_h^2}{R_h}$$

  $$q_{out}(t) = \frac{1}{R}\left[\theta - \theta_{ambient} \right] = \frac{1}{hA}\left[\theta - \theta_{ambient} \right]$$

  $$\dot{\theta} = \frac{1}{C}\left[q_{in}(t) - \frac{1}{R}(\theta - \theta_{ambient}) \right]$$

  $$\dot{\theta} + \frac{1}{RC}\theta = \frac{1}{C}q_{in}(t) + \frac{1}{RC}\theta_{ambient}$$

This is a linear, first-order, ordinary differential equation with constant coefficients. The time constant is $\tau = RC$ and the inputs are $q_{in}(t)$ and $\theta_{ambient}$.

5. Predicted Dynamic Response

In order to predict the dynamic response of the physical system (as represented by the physical model), we must solve the mathematical equations either analytically or numerically. To gain the most complete insight into the dynamic behavior of the physical system, both methods of solution should be used, if possible.

At the operating point of the thermal system

$$\theta = \overline{\theta} \qquad q_{in}(t) = \overline{q} \qquad \dot{\theta} = 0$$

The system differential equation

$$\dot{\theta} + \frac{1}{RC}\theta = \frac{1}{C}q_{in}(t) + \frac{1}{RC}\theta_{ambient}$$

reduces to

$$\frac{1}{RC}\overline{\theta} = \frac{1}{C}\overline{q}_{in} + \frac{1}{RC}\theta_{ambient}$$

$$\overline{\theta} = \theta_{ambient} + R\overline{q}_{in}$$

When the system is in equilibrium, the temperature of the thermal capacitance is constant, and the heat flow rate $\overline{q}_{in}$ supplied by the heater must equal the rate of heat flow through the thermal resistance. Then the temperature difference across the resistance is $R\overline{q}_{in}$.

Define incremental variables:

$$\hat{\theta}(t) = \theta(t) - \overline{\theta} \qquad\qquad \hat{q}_{in}(t) = q_{in}(t) - \overline{q}_{in}$$

Therefore

$$\dot{\hat{\theta}} + \frac{1}{RC}\left(\hat{\theta} + \overline{\theta}\right) = \frac{1}{C}\left[\hat{q}_{in}(t) + \overline{q}_{in}\right] + \frac{1}{RC}\theta_{ambient}$$

$$\dot{\hat{\theta}} + \frac{1}{RC}\hat{\theta} = \frac{1}{C}\hat{q}_{in}(t)$$

Observations:
- If $\overline{q}_{in} > 0$ then $\overline{\theta} > \theta_{ambient}$ and the capacitance is being heated.
- If $\overline{q}_{in} < 0$ then $\overline{\theta} < \theta_{ambient}$ and the capacitance is being cooled.
- If $\overline{q}_{in} = 0$, then the nominal value of the temperature is $\overline{\theta} = \theta_{ambient}$ and $\hat{q}_{in}(t) = q_{in}(t)$.
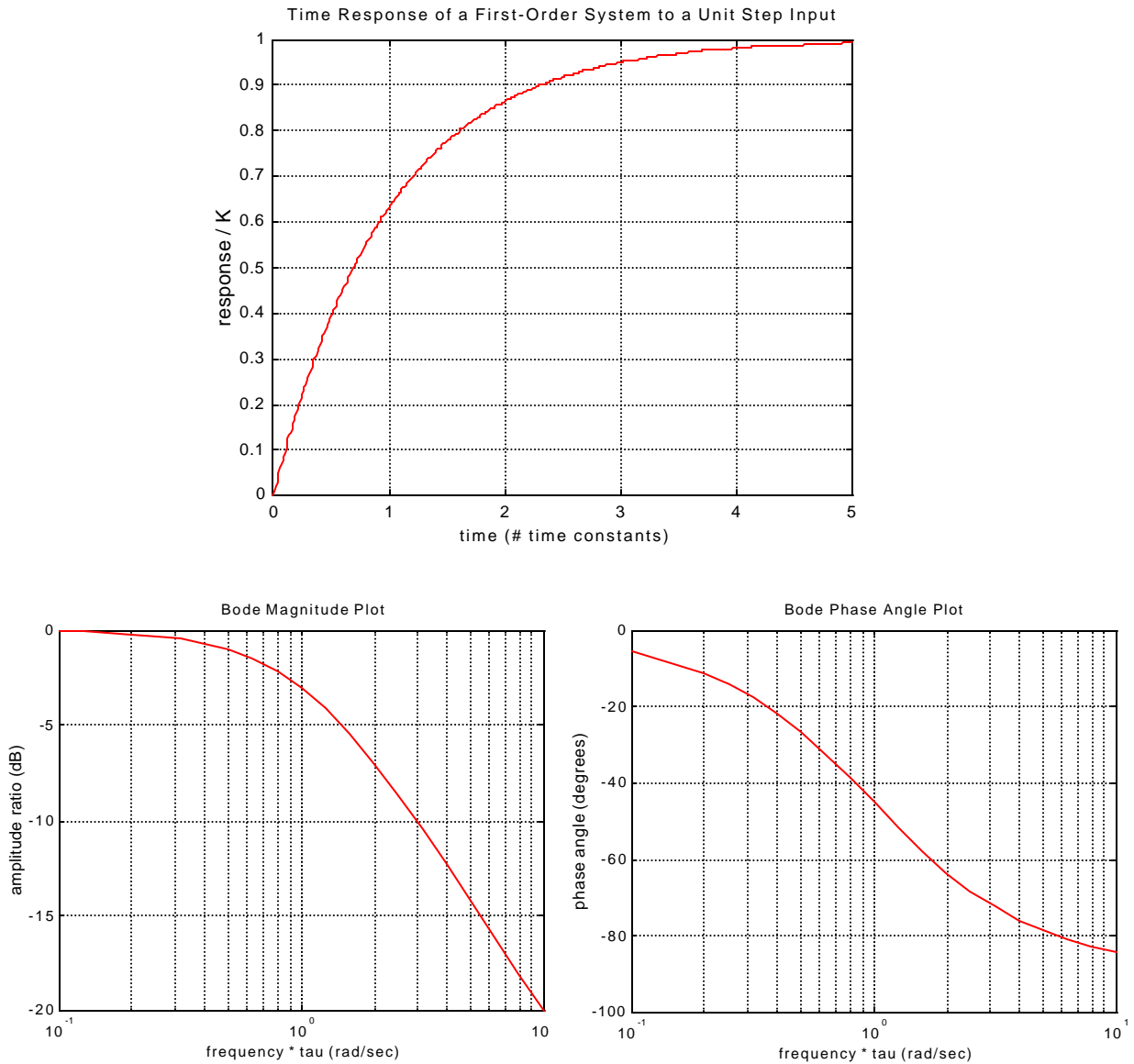
Take the Laplace transform of the differential equation:

$$s\hat{\Theta}(s) + \frac{1}{RC}\hat{\Theta}(s) = \frac{1}{C}Q_{in}(s)$$

$$\frac{\hat{\Theta}(s)}{Q_{in}(s)} = \frac{\frac{1}{C}}{s + \frac{1}{RC}} = \frac{R}{RCs + 1} = \frac{R}{\tau s + 1} \qquad \text{where} \quad R = \frac{1}{hA} \quad \text{and} \quad \tau = RC$$

This differential equation is a standard first-order, linear, ordinary differential equation with constant coefficients. Step response and frequency response plots for this system are shown in Figure 6.

$$\tau \frac{dq_o}{dt} + q_o = Kq_i \qquad\qquad q_o = Kq_{is}\left(1 - e^{\left(\frac{-t}{\tau}\right)}\right)$$

$$\frac{Q_o}{Q_i}(s) = \frac{K}{\tau s + 1} \qquad\qquad \frac{Q_o}{Q_i}(i\omega) = \frac{K}{\sqrt{(\omega\tau)^2 + 1}} \angle -\tan^{-1}\omega\tau$$

Figure 6. Step Response and Frequency Response Plots for a First-Order Dynamic System

6. Experimental Testing: Model Parameter Identification

In the mathematical model derived from the physical model of the physical system, two parameters need to be determined before the response of the dynamic system can be predicted. The thermal capacitance C can be computed as the product of the mass of the aluminum plate $M = 0.0057$ kg (M = ρV where density $\rho = 2770$ kg/m^3 and volume V = 2.048E-6 m^3) and the specific heat of aluminum c_p = 875 J/kg-K. The thermal capacitance C of the aluminum plate then is 4.96 J/K.

The thermal resistance R = 1/hA must be obtained by experimental testing. If we can measure the time constant of the system τ = RC, we can compute the value of R from the calculated value of C and the measured value of τ. An open-loop step response test is proposed to measure τ. The thermal resistance can also be determined from the steady-state temperature of the plate, since in the steady state

$$q_{in} = q_{out} \quad \text{or} \quad \frac{V_h^2}{R_h} = \frac{(\theta - \theta_{ambient})}{R}$$

6.1 Experimental Set-Up
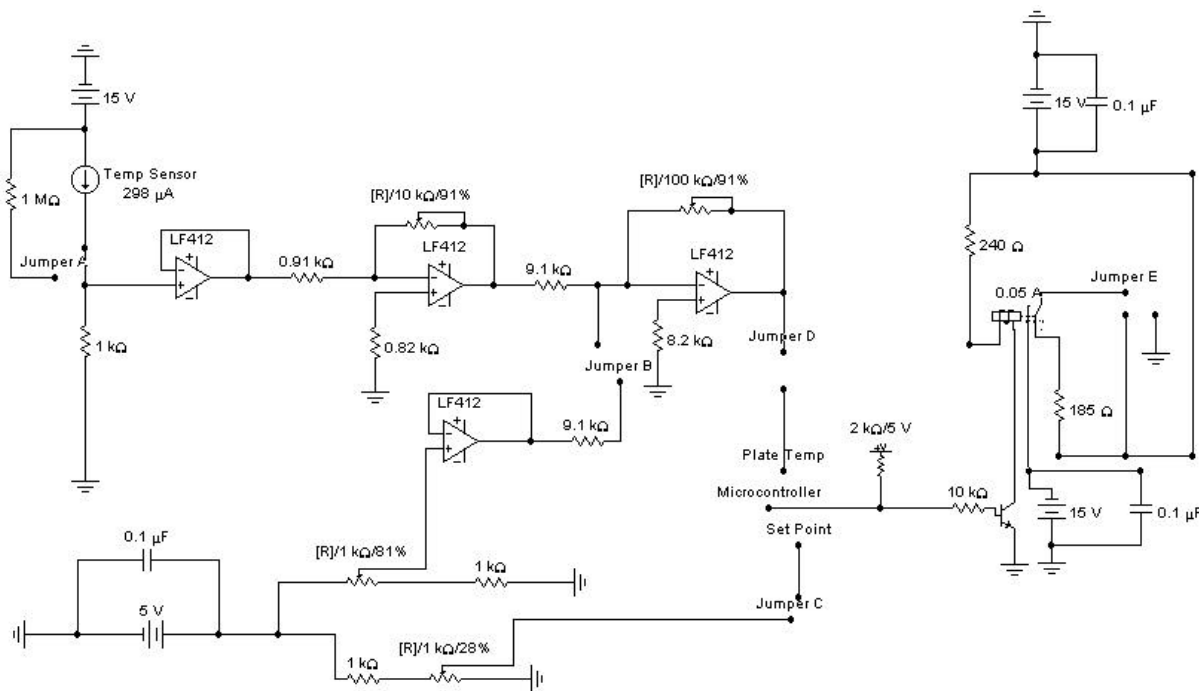The circuit diagram of the physical system is shown in Figure 7.

Figure 7. Circuit Diagram

The sensor circuit appears on the left side of Figure 7. The AD590 temperature sensor is represented by the current source in the upper left hand corner, simulated as 298 μA. With the sensor gain of 1 μA /K, this corresponds to 298 K, or 25 °C. The collection of four points in the upper left hand corner of the system, marked J-A, represent Jumper A, which allows the user to switch between a constant voltage input to the circuit formed by the resistive divider of the 1 MΩ resistor and the 1 kΩ resistor, and the input from the AD590 sensor source. The use of this feature will be explained later in the case study. For the experimental parameter identification, it is only necessary to note that the output of the first amplifier on the left most LF 412 op-amp represents a buffered version of the sensor signal.

For the experimental parameter identification, it is necessary to measure the resistance marked R_{sens} on the schematic diagram. This resistor determines the sensor gain of the AD590 temperature sensor, i.e., $K_{sensor}(volts/K) = (1μA)(R_{sensor})$. In addition, it is necessary to measure the resistance of the heater, the nominal resistance of which is 185Ω. The heater resistance is located in the lower right hand corner of the schematic drawing. It is denoted as R_h.

The sensor-gain resistor and the heater resistance are experimentally quantified in the following way. The sensor resistance, R_{sens}, determines the sensor gain. The resistance can be measured by removing Jumper A, located in the upper left hand corner of the circuit schematic, and measuring the resistance across the leads of R_{sens} on the printed circuit board. This measurement assumes the input impedance of the operational amplifier that R_{sens} is connected to is very high and that loading effects are therefore negligible. The next step in the open-loop experimental analysis of the system is to disconnect the heater from the heater control board. This connection is made on the green terminal block located on the upper right hand corner of the printed circuit board. Disconnection is accomplished by slackening the two screws in the green header block, and removing the heater wires. The resistance of the heater, R_h, can be determined by measuring the resistance across the two red wires that lead to the heater.

To conduct the model parameter identification, it is necessary to place a voltage across the heater terminals. The system must be disconnected from the controller for the open-loop experiment. The most direct way to do this is to disconnect the control signal line.

The experimental data is acquired by applying power to the printed circuit board, placing a voltage across the heater terminals, and recording the voltage at Pin 2 on the printed circuit board. Pin 2 represents the buffered voltage signal from the AD590 temperature sensor. The signal should be measured every thirty seconds, and the experimental step response data should be plotted.

A more thorough experimental determination of the thermal resistance R should be conducted. Such an experimental determination would involve applying a series of voltage potentials to the heater, and waiting for the response of the heater to achieve a steady-state temperature. The thermal resistance could be determined for each of these steady-state plate temperatures. A plot of R as a function of temperature would allow a more sophisticated model to be implemented. Such a model would use this experimental plot of the thermal resistance as a look-up table, and determine a thermal resistance for each time step that best matches the temperature of the plate at that time.

As the intended control scheme for this system is on-off control, such a sophisticated model is not deemed necessary for this investigation. For the remaining analysis, the current model with a constant thermal resistance will be employed.

7. Control System Design

With an understanding of the nature of the system dynamic behavior, as well as a model that predicts reasonably well the performance of the system, it is now necessary to design a control system that will regulate the temperature of the plate. The control system proposed for this experiment is an on-off controller. The architecture of this control method is discussed here.

The goal of an on-off control scheme is to switch the actuator between two states, on or off, based upon a measure of an error signal. Typically, the on-off controller is implemented with a deadband, or a region in which no control action is taken. An illustrative diagram of on-off control is shown in Figure 8.
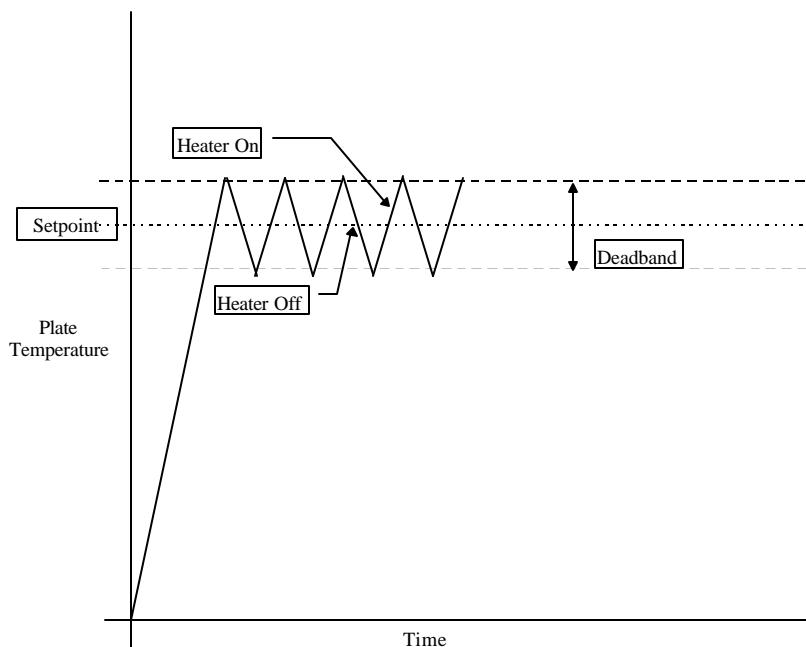
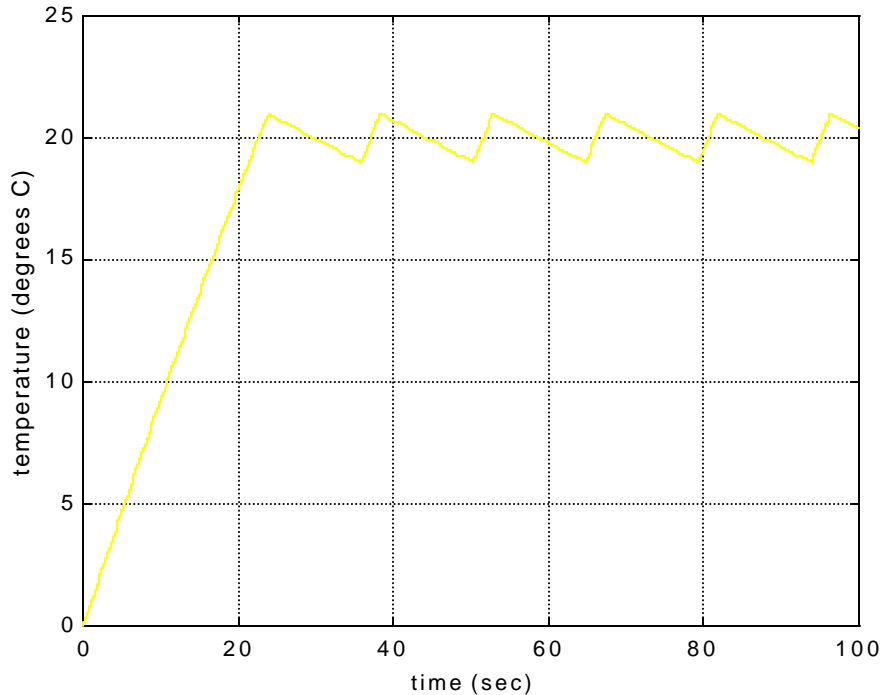**Figure 8. Typical On-Off Control Response**

Examine the response shown in Figure 8. As we can see from the figure, the error, or difference between the setpoint and system response, is initially quite large. The heater is on, and the system acts to decrease the difference between the desired response and the measured response. As the plant temperature crosses the lower part of the deadband we continue to apply corrective action. When the plant response reaches the setpoint, the heater remains on, and does so until the response reaches the upper limit of the deadband, at which point the heater is turned off, and the temperature of the plate begins to fall. When the plate temperature falls below the lower limit of the deadband, the heater is again turned on, and the plate is heated until it has again reached the upper limit of the deadband. This repetitive behavior is known as limit cycling, and it is a characteristic of on-off control. We can see from the response that the average plate temperature is the setpoint temperature.

Simulation of an on-off controller applied to the thermal system could be accomplished using a non-linear MatLab / Simulink block diagram, as shown in Figure 9.
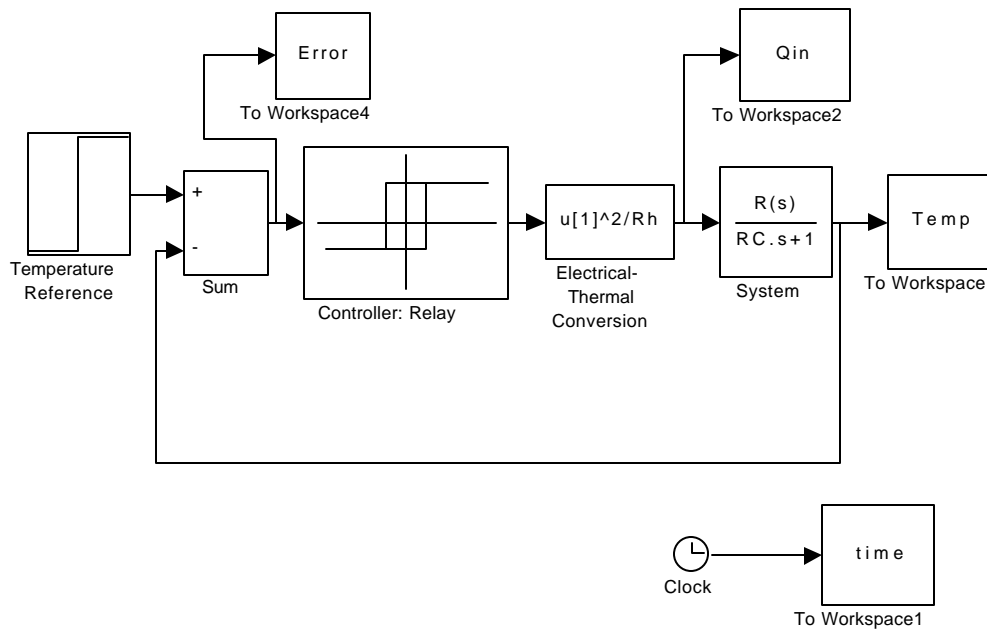
Figure 9. MatLab / Simulink Block Diagram of Thermal Closed-Loop Control System

The simulated response of the system is shown in Figure 10 for a setpoint of 20° C and a deadband of 2° C.

Figure 10. Simulation of Control about 20° C Sepoint with a 2° C Deadband

As can be seen in Figure 10, the system behavior closely resembles classic on-off control response as depicted in Figure 8. Initially, the plate temperature is below the setpoint temperature of 20° C. The heater is on, and the temperature of the plate begins to climb until it reaches 21° C, at which point the heater is turned off. The plate cools due to natural convection until the plate reaches the lower limit of the deadband, 19° C, and then the heater is turned back on. We can see from the figure that the average response centers around a plate temperature of 20° C.

It is important to note that the rise to 20° C under closed-loop control is much faster than what is possible under open-loop control. This is one primary benefit of feedback control, as we can use a much higher voltage command signal, which produces much more power and thus faster response than the voltage command possible under open loop control to achieve the same steady-state temperature.

8. Implementation

The next step in the design process is to develop circuitry to perform the functions of the control design proposed in Section 7. In the case of the on-off controller, the logical branching of the control algorithm will be implemented in software on a microcontroller. The use of a microcontroller allows for a great deal of flexibility in implementing logical decision branching. With this increased flexibility comes the price of additional signal conditioning.

Microcontrollers are digital devices, and the signals generated by the temperature sensor and received by the heater system are analog voltages. We are required, therefore, to design circuitry that will condition the sensor and command signals such that they can be read by the analog-to-digital converters of the microcontroller, as well as develop power-drive circuitry that can switch the necessary current on and off through the heater, based on the relatively low-current output signals of the microcontroller.

8.1 Analog Control Implementation

The printed circuit board on the plate temperature control system has been designed to implement on-off control of the resistive heater. The schematic diagram of the board circuit is shown in Figure 11 (identical to Figure 7).
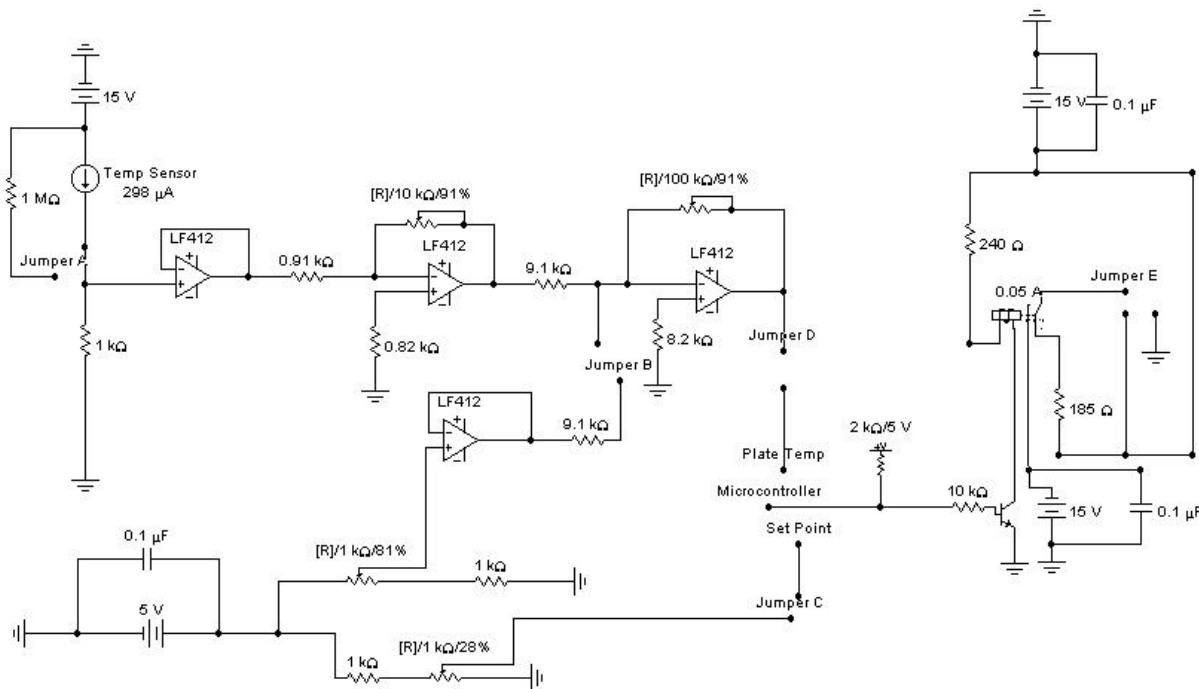

Figure 11. Circuit Diagram of On-Off Controller for Plate Temperature Control

Working from left to right on the above circuit diagram, the function of each of the components will be described. Two signals are selectable via jumper A on the upper left hand corner of the board. The sensor signal, selected when jumper A is in the lower position, places the temperature sensor, modeled here as a current source, across the 1 kΩ resistor shown, resulting in a 1 mV/K sensor gain. The first stage of a LF 412 operational amplifier acts to buffer the sensor signal to prevent loading. From the output of the LF 412, the sensor signal is fed into an inverting amplifier, whose gain is set by the potentiometer shown above the amplifier. The gain is selected such that the output of the amplifier is ten times the input. The sensor gain is now -10 mV/K. From this amplifier, the output is fed into a summing amplifier, with a gain of -10. The summing amplifier takes the signal from the sensor and adds it to a signal generated by the resistive divider powered by the five-volt supply at the bottom of the page. Jumper B selects whether this divider signal is part of the summation. The resistive divider in combination with the summing amplifier allows the subtraction of room temperature from the signal, and the output measured at the point marked "Plate Temp" on the schematic can be tuned by the potentiometer of this temperature offset to be zero volts when the plate is at room temperature. The problem at this point is setting the gain of the second stage amplifier. We need a way to set this gain so that it is precisely a minus 10 times increase of the output of the first stage amplification. We must do this when no offset is applied (Jumper B), so that we ensure that the output of the amplifier is a pure minus 10 times increase on the previously amplified sensor signal. Its effective amplification of the room temperature offset is of no concern, as we will use this offset only to zero the sensor signal at room temperature, and the amount of offset needed to perform this function is of no concern. The problem, however, is that room temperature is 298 K, and based on a sensor gain of 1 mV/K, the output of the first stage amplification will be 2.98 volts. This signal cannot be amplified by a factor of ten, as the resulting voltage (29.8 volts) is well beyond the saturation point of the amplifier.

At this point the function of Jumper A is realized. Recall that the 1 µA/K signal that the sensor generates is converted to a 1 mV/K signal by the 1 kΩ resistor. If this signal is replaced with a signal of lower potential, then the inverting operational amplifier and the summing amplifier can each be tuned so that they yield their intended gain of minus 10 on the signal. Jumper A, therefore, is used to replace the signal provided by the sensor with the potential created by a resistive divider consisting of a 1 MΩ resistor in series with the 1 kΩ resistor. The potential between the top of the 1 kΩ resistor and ground, the substitute sensor signal, will be in the neighborhood of 0.015 volts when the divider is powered by a 15-volt supply. The effect of amplifying this signal by a factor of 100 will be to create a potential of 1.5 volts at the output of the second stage of amplification, which is well within the saturation limits of the amplifier.

With each of the gains tuned to minus 10, it is clear that a sensor signal has been created with a net gain of 0.1 V/K, or a signal in the neighborhood of 2 volts for the desired plate temperature of 20° C above ambient.

It is now necessary to create a signal that will be the setpoint of the controller. Looking to the bottom of the schematic, a resistive divider can be seen which is driven by a five-volt source, and consists of a 1 kΩ potentiometer and a 1 kΩ resistor. This network can produce a setpoint signal that ranges from 0 to 2.5 volts.

Recall that both the setpoint signal and the plate temperature signal are connected to the microcontroller via analog-to-digital converters, which have an input range of 0-5 volts. Each input signal, the setpoint signal and the temperature sensor signal, should therefore be within the capacity range of the converters for the intended control temperature of 20° C above ambient.

The function of the microcontroller is to compare the levels of the set point and plate temperature signals, and take appropriate control action. The control action is to turn the heater on or off, and in this case the microcontroller command is a digital signal, either a high (+5 volts) or low (GND). For now we will just assume that the microcontroller is capable of performing this task, and we will look at the circuitry downstream of the microcontroller to see how the control signal is delivered to the heater as a command.

Let us look at the two cases of high and low separately. First, assume the signal from the microcontroller is low. A transistor, connected to the microcontroller through the 10 kΩ resistor, switches a relay on and off . The transistor is utilized because the output current capability of the microcontroller is on the order 1 mA @ 5V, and the relay coil requires 15 volts at 50 mA. The resistive heater foil is connected through the contacts of the relay. When the output is low, the current signal to the base of the transistor is zero, and the coil of the relay is not energized. The relay is shown in the schematic in the off position, and it is seen that the path of current is from the 15-volt supply, through a 185 Ω resistor that represents the heater, through the contacts of the relay to Jumper E. Jumper E is a selection switch that allows two forms of control to be implemented. When the jumper is in the left-hand position, connecting vertically the terminal at the top to the terminal at the bottom, the output of the relay is connected back to the 15-volt supply. With no potential drop across the heater element, there is no current flow, and the heater is off. This is the configuration for **on-off** control. For the case where the jumper is in the right-hand position, the output of the relay is connected to ground. This applies a 15-volt potential across the heater. The use of this **on-bias** control maintains a nominal current flow through the heater, even in its off state. This nominal dissipation prevents the plate from cooling as quickly it would in a zero-current state. The requirement to utilize this biased control scheme is that the desired plate temperature be higher than the temperature created by a steady 15-volt potential applied across the heater.

In the case in which the controller commands a logic high, current will flow into the base of the transistor, the relay will switch to its other set of contacts, and the bottom terminal will be switched to minus 15 volts, putting 30 volts across the heater element. This is the **on** mode, and nearly 5 watts of power are delivered to the heater when the microcontroller commands the heater to this state.

With the description of the control circuitry complete, we will now discuss the selection of the microcontroller based on the requirements of this system.

8.2 Microcontroller Selection

The requirements of the microcontroller for this application are fairly straightforward. The microcontroller must be capable of two channels of analog-to-digital conversion, with the range of that conversion at the minimum being 0-3 volts, corresponding to a temperature change of thirty degrees on the plate. In addition, the microcontroller must be capable of at least one digital out. As the dynamics of this plant are very slow, the bandwidth requirements of the microcontroller are not an issue.

If we look at the wide variety of microcontrollers that are commercially available, we see that a good majority of them have these capabilities. The unit we selected for this application, the Micro-485 by Blue Earth Research in Mankato, Minnesota, has these features, plus a host of other capabilities. The specifications of the Micro-485 are shown in Table 5.

Table 5: Summary of Micro 485 Specifications

Feature	Specification
Microprocessor	Intel 8051 running at 12 MHz
Digital I/O	27 Bi-directional TTL compatible pins
Analog Inputs	4 12-bit 0-5 volt A/D converter channels
Serial Communication	RS-422, RS-232
RAM	128K, battery-backed for retention after power down
ROM	32K, contains on-board Basic and Monitor

It is important to note that in addition to meeting the minimum requirements, the Micro 485 has several additional features that make it an attractive microcontroller for real-time control. The first of these is the RS-232 communication capability, which allows the user to communicate with the controller using a personal computer and a serial communication package. In addition, the microcontroller has an on-board BASIC interpreter, which allows the control software to be prototyped in a relatively straightforward manner, with a simple, high-level language. Because of the relatively slow dynamics of the heater system, fast clock speed or compiled code are not required in this control application.

8.3 Software Design

The control software performs the logic branching based on signals measured by the microcontroller. The use of software in implementing the control code allows tremendous flexibility in the implementation of the controller. The control code in this application must implement the logic of Figure 12.
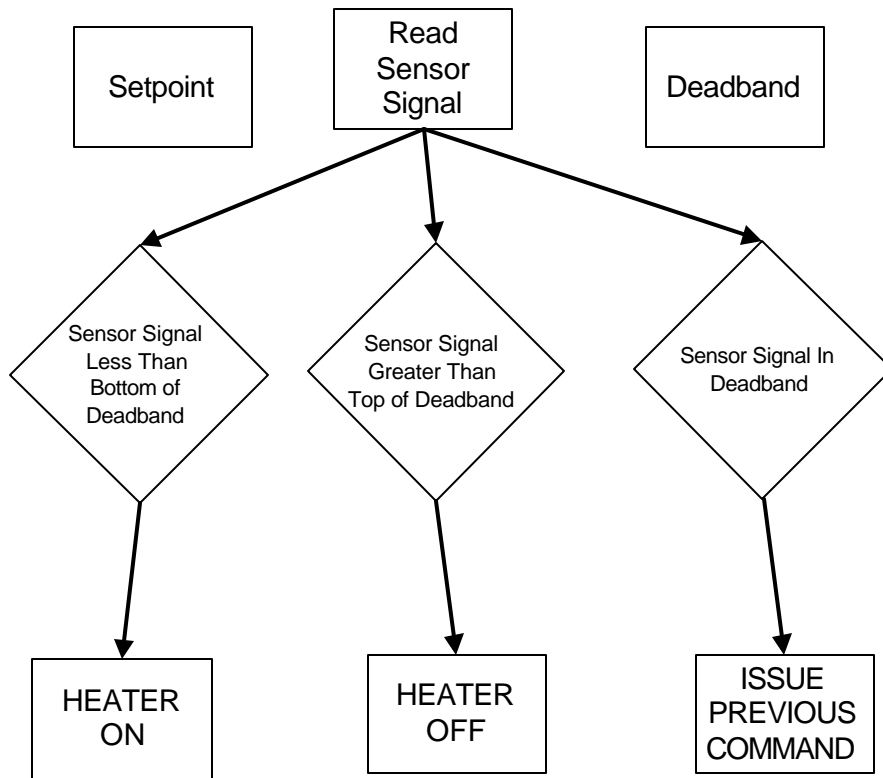
Figure 12. Flow Diagram of Decision Logic

The basic function of the code is described in Figure 13.
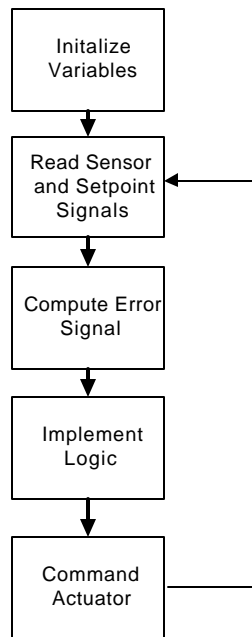
Figure 13. Software Flow Diagram

We will now look at the segments of code that perform the functions described in Figure 13.

```
100 REM set up Port A as an output, reset Port B
110 xby(0fd03h)=128:xby(0fd01h)=6
120 PRINT USING(####)
130 REM calibrate the A to D converters
140 XBY(0ff03h)=2
150 DO: SFR=XBY(0ff03h):WHILE SFR.AND.2
160 IF SFR.AND.40h THEN GOTO 140
```

Line 110 configures the digital outputs pins of the Micro 485, setting up Port A as an output, and reasserting communication with the host computer. Line 120 configures future print statements to print numbers in a four-decimal-place format. Lines 140-160 are standard code in the Micro 485 architecture, which initialize and calibrate the analog-to-digital converters.

```
170 REM read current temperature
180 XBY(0ff00h)=1
190 CURTEMP=256*XBY(0ff01h)
200 CURTEMP=CURTEMP+XBY(0ff00h)
210 REM read setpoint temperature
220 XBY(0ff00h)=0
230 SETEMP=256*XBY(0ff01h)
240 SETEMP=SETEMP+XBY(0ff00h)
```

Lines 180 and 220 read the plate temperature and setpoint temperature through channels 1 and 0 of the analog-to-digital converter, respectively. These are 12-bit conversions, and **CURTEMP** and **SETEMP** are assigned a bit value that corresponds to the voltage level of the signals present on these channels when the conversion was initiated. The conversion is performed in the following manner. Address 0FF00h is assigned a value corresponding to the channel to be converted (0-3). As the result of a conversion is 12-bits, and each memory address in the Micro 485 can store one 8-bit byte, the result must be read in two steps. First, the high byte is read from address 0FF01h. This byte contains zeros in the upper four bit locations, while the lower four bits contain the MSB of the conversion result. This value is summed with the 8 LSB, located at 0FF00h. The upper four bits are weighted by a factor of 256, which represents the significance of their location in the upper byte. The result is a decimal number, 0-4095, that represents the 12-bit conversion of the voltage signals.

```
250 ERROR=SETEMP-CURTEMP
```

Line 250 computes an error, **ERROR**, that is the difference between the setpoint and plate temperature.

260 IF ERROR<-82 THEN COMMAND=0
270 IF ERROR>82 THEN COMMAND=255
280 IF (ERROR>-82).AND.(ERROR<82) THEN COMMAND=COMMAND

Lines 260 through 280 implement the logic branching. Line 260 determines if the error is more than 82 bit values below zero, indicating the plate temperature is greater than the upper limit of the deadband temperature. (Recall that we are using 12-bit converters, with a range of 0-5 volts. One bit is therefore equivalent to 0.00122 volts, and eighty two bits corresponds to a voltage of just about 0.1 volts, or one-half of our deadband.) If the plate is more than one degree above the setpoint, the command is set to zero. Line 270 determines if the plate temperature is more than 82 bit values below the command temperature, in which case the plate temperature is below the bottom of the deadband, and the heater must be turned on. Line 280 determines if the error lies somewhere in the deadband, in which case we issue the previous command.

290 XBY(0fd00h)=COMMAND

Line 290 commands the heater using the value of COMMAND determined by the logic branching. Address 0FD00h is the memory location of Port A, which controls the state of the heater.

300 PRINT "CURRENT TEMP=",CURTEMP,"SET TEMP=",SETEMP
310 GOTO 170

Lines 300 and 310 print the current plate temperature and setpoint temperature, and then send the program flow back to line 170, to allow the control loop to be executed again.

9. Testing and Comparison with Predicted Dynamic Response

The final stage is to experimentally determine the response of the closed-loop system and compare the result to the predicted dynamic behavior based on the physical and mathematical models.