A Spare Part Inventory Management Model for Better Maintenance of Intelligent Transportation Systems

Kaan Ozbay, Ph.D.
Professor,
Department of Civil and Environmental Engineering,
Rutgers, The State University of New Jersey,
623 Bowser Road, Piscataway, NJ 08854 USA,
Tel: (732) 445-2792
e-mail: kaan@rci.rutgers.edu

Eren Erman Ozguven, M.Sc.
Graduate Research Assistant,
Department of Civil and Environmental Engineering,
Rutgers, The State University of New Jersey,
623 Bowser Road, Piscataway, NJ 08854 USA,
Tel: (732) 445-4012
e-mail: ozguvene@rci.rutgers.edu

Sami Demirobek, M.Sc.
Graduate Research Assistant,
Department of Civil and Environmental Engineering,
Rutgers, The State University of New Jersey,
623 Bowser Road, Piscataway, NJ 08854 USA,
Tel: (732) 445-4012
e-mail: dsami@rci.rutgers.edu

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ABSTRACT

New Jersey’s highway transportation system is highly dependent on the performance of its roadways in order to maximize operational capability and to minimize travel time and congestion. Long-term sustainability of the highways performance is directly related to the efficiency of the Intelligent Transportation Systems (ITS) that need to be maintained and operated properly. One of the most important concerns within the inspection and maintenance procedures of ITS equipment is timely availability of the spare parts of essential components of ITS. Long-term down time of ITS equipment due to the unavailability of spare parts will not only increase personnel and repair time requirements, costs of replacement parts but also might lead to increased delays, poor air quality and fuel consumption. In this paper, we propose an "efficient spare parts inventory control model" that can determine the optimum levels of the safety stocks under probabilistic failure and availability assumptions for components of various ITS equipment. When this inventory control model is fully integrated into Rutgers Intelligent Transportation Systems Inspection and Maintenance Software (RITSIMS) (1), it will allow its users to efficiently manage DOT’s ITS spare parts inventory using historic maintenance and inspection data that is being collected by respective databases of RITSIMS. This inventory control model will also be able to account for the worst case scenarios that DOT can experience in terms of unexpectedly high level of equipment failures due to natural or other disasters such as hurricanes.
INTRODUCTION

Highly dense and complex roadway infrastructure of New Jersey carries heavy traffic of cars and trucks. It is thus important to ensure that deployed ITS equipment works properly and efficiently. Unnecessary ITS equipment failures and malfunctions due to inefficiencies of inspection or maintenance can create many problems (1):

- Increase in maintenance expenditures (time and money), shortening of the useful life of the ITS equipment,
- Increase in requirements for personnel time and spare part inventory,
- Waste of time and fuel and a possible increase in pollutant emissions due to additional congestion that can caused by the longer than expected down-time of ITS equipment,

One of the significant components of any maintenance system is being well-prepared for failures by ensuring availability of spare parts. Currently, many DOTs do not use a system to keep an active inventory of spare parts for their ITS devices. However, it is clearly stated in (2) that “a lack of spare parts inventories and difficulties in procuring critical spare parts in a timely manner” are two key concerns. Patel (2) indicates that if spare parts are not available, repairs may take longer time to complete and may even create safety hazards and can result in liability and/or damages.

First of all, it should be noted that some of these spare parts can be only replaced by the contractor, original manufacturer or a qualified distributor. Anecdotal evidence has suggested that “the ability to get spare parts quickly is one of the most common reasons to rely on contract maintenance, reflecting on the governmental procedures that might tend to delay the reestablishment of ITS operations” (2). However, many traffic operations units have the responsibility of maintaining ITS equipment such as closed circuit television systems (CCTV), variable message signs (VMS), sensors, cabinets, hubs, roadway information and communications systems, etc. Actually, regardless of the contracting method used, (by in-house staff or a contractor) sufficient quantities of key parts should be available for maintenance. "Some agencies routinely keep on-hand a percentage of the original quantity as spare parts without using any mathematical model, but based on experience. Such an approach is considered to result in time-saving and reduces the downtime of expensive ITS devices" (2). Moreover, over the past decade, there have been several manufacturers that have gone out-of-business all around USA, leaving agencies with no immediate sources for spares (3). To mitigate these problems, a maintenance plan of a DOT should incorporate scrapping items and ensure that spares and replacements are anticipated. As the spare parts can be expensive and timely supply of spare parts in terms of replacements is critical, it is also often a significant economical problem to hold sufficient stocks. A trade-off between minimizing the cost for spare part inventories and maximizing the efficiency of the spare parts should be achieved by carefully modeling an inventory management system specifically designed for spare parts. This model should solve an optimization problem concerning the optimal investment allocation to both spare-part inventories and effective usage and maintenance of those spare parts.

In a recent study conducted for NJDOT (4, 5), Rutgers Intelligent Transportation Systems (RITS) researchers successfully developed a state-of-the-art Intelligent Transportation Systems inspection and maintenance manual (ITSIMM) and Rutgers ITS...
inspection and maintenance software (RITSIMS) based on ITSIMM. RITSIMS is currently being tested by the DOT. Over time, the research team expects to obtain substantial amount of data about the usage and performance of the ITS equipment so that this information can be used as an input for the spare parts inventory management model presented in this paper. (6).

The inventory management model proposed in the following sections is an idea that was conceived during this implementation phase of RITSIMS. Integration of the type of an inventory management system similar to the one described in this paper with the equipment history features of RITSIMS (4) will help traffic operations and maintenance personnel to improve their inspection and maintenance activities. Thus, the primary objective of this paper is first to design a theoretically robust inventory management system that would minimize stock-outs and at the same time help in maintaining optimum inventory levels according to the criticality of the equipment. The research team expects that the proposed spare part inventory management model will provide a reduction in maintenance expenditures (time and monetary) and inventory costs due to efficient repairs and replacement of faulty parts, better recording of the failure rates, monitoring the performance equipment, and increasing the useful life of the equipment through better and timely maintenance. First, an overview of the research methodology is given including the details about the proposed “inventory management model”. This is followed by the case studies and the details of an application of the inventory management system within RITSIMS. Finally, conclusions and future recommendations are provided.

THE SPARE PARTS INVENTORY MANAGEMENT PROBLEM

In a technical report prepared for NJDOT (2), it is strongly recommended to have a spare parts inventory and management system for ITS components of NJDOT due to the following reasons:

- "An efficient management of a spare parts inventory integrated with software systems and efficient network management can support transportation operations centers (TOCs) and statewide ITS operations of NJDOT.
- ITS, by definition, is made whole by the sum of its parts, and procuring parts, repairs, management of warranties, etc. is a necessity that cannot be mixed with other activities or generalized and allowed to deteriorate.
- ITS technologies are complex, and swapping a new part for a defective one can be a quick and simple, yet a solution that will keep ITS benefits flowing. TOCs across the country are reporting 10-15 percent spare parts on hand."

These issues create the need for defining and carefully studying the spare parts inventory management problem for ITS systems. First of all, it is important to realize that spare parts inventories are not intermediate or final ITS products, therefore the policies that govern spare parts inventories are different from those which govern classical inventories. Unique aspects of spare parts inventory management for transportation can be listed as follows (adapted from (7) where they present a general spare parts inventory problem):

- The initial driver for spare parts demand is the ITS component failure. However, maintenance policies, rather than the usage of component within the ITS...
equipment, may dictate the need for spare parts inventories. For example, ways to restore the functionality of an ITS equipment that has a broken part include to repair or replace the part. The decision of whether to repair or replace has significant implications on spare parts inventory levels. Moreover, failure of a spare part may lead to the end-use and therefore replacement of the entire ITS equipment.

- A reliability based information is generally not available within the DOT to the degree needed for the prediction of failure times, particularly in the case of a new installed ITS equipment.
- Part failures may create a problem, particularly if the dependence relation is not known.
- Demands for parts are sometimes met through cannibalism of other parts or units.
- Costs of being out of a part generally include quality as well as lost production, and these costs are difficult to quantify. Increased risk to DOT personnel may also be a factor, and costs associated with such risks are not easy to calculate.
- Obsolescence may be a problem as the equipments for which the spare parts were designed become obsolete and are replaced. It is difficult to determine how many units of a part for an obsolescent equipment to stock, and it may be difficult to replace a part that no one still keeps in stock.
- Components of ITS equipment are more likely to be stocked than complete units if the major unit of ITS equipment is expensive, and repair may be preferred to replacement if it is possible.

Among all these, cost is apparently a determining factor for managing spare parts inventories for transportation systems optimally. An estimated cost analysis for the preventive maintenance of an ITS device for the Maryland State Highway Authority was provided in a report by Institute of Transportation Engineers (ITE) (8). Based on this analysis, the conclusion reached was that in-house maintenance was more cost effective than outsourcing the maintenance tasks. Another related study provided guidelines for funding operations and maintenance of ITS systems where spare parts inventory costs were considered as critical (9). For instance, among the operations and maintenance costs (O&C costs) for ITS systems, given by Research and Innovative Technology Administration of U.S. Department of Transportation, dynamic message signs O&C costs range was between $2300-6000 in terms of 2005 dollars per year, whereas CCTV O&C costs range was between $1000-2300 in terms of 2004 dollars per year, both having a life time of 10 years (7). Considering that NJDOT Traffic Operations North has more than 500 CCTV's all around North Jersey (1) and observing the cost of maintaining and operating one CCTV device gives a rough idea about the importance of being prepared for failures in terms of having spare part stocks. To sum up, these cost-related studies bring out the issue of effectively managing the spare parts inventories, a key issue for the maintenance of ITS equipment.

Another significant issue to consider while modeling the spare parts inventories is that the lifetime of ITS components is shown to vary between 4 and 20 years. Although technological changes happen quickly, the procurement cycles for most DOTs do not. In fact, there are hardware components in the field -e.g., electromechanical controllers - that have been operational for many decades (3). The actual rate or time for replacement of
ITS field components depend on the mean time between failures for each component, which normally should be obtained from the DOT database accumulated over years. It is recommended in (5) that the maximum number of spare parts that may be needed for ITS equipment should be procured, whether or not they are provided by the agency or via an outsource contractor. In this paper, we focus on the optimal amount of spare parts needed for efficient ITS operations and maintenance procedures rather than having maximum number mentioned in (8), which is generally not known, or decided based on experience.

Mathematical Formulation

There are a number of objectives that are sought by the DOT personnel in order to develop an efficient spare parts inventory management methodology. These objectives mainly differ from the objectives of a commercial inventory management since spare parts inventory management in the context of a transportation system is unique and significantly different from classical inventory and manufacturing framework. In our spare parts inventory management problem, we include the following objectives:

- Minimization of failures and maximization of the performance of ITS equipment,
- Maintaining a buffer or safety stock to account for and respond fast for ITS component failures,
- Tracking the usage of spare parts and/or store a list of parts required for each piece of ITS equipment.
- Optimally determining the amount of spare parts and their corresponding costs within the inventory (shortage, surplus, storage, etc.) that can be used effectively in a DOT maintenance plan and within the allocations of funding,

The constraints for the spare parts inventory management problem, on the other hand, include:

- the storage space constraints,
- the minimum tolerable disruption level (possibly due to transportation, extreme failure rates, and supplier related disruptions) constraints.

Figure 1 illustrates the overall problem with objective function and its constraints in the transportation and ITS operations concept.
To solve this spare parts inventory management problem, we propose a stochastic spare parts inventory control (SSIC) model based on Hungarian Inventory Control Model introduced in (10). In our model, demand is defined as the need for replacements of spare parts due to failures, whereas delivery represents delivering the spare parts that are not readily on hand in the spare parts inventory, if it exists. Note that maintenance activities usually follow a scheduled scheme, and therefore delivery for the spare part may not be immediately conducted if the failure does not have high priority.

We will first mathematically describe a multi-spare part model in the context of spare parts inventory logistics. The “stochastic spare parts inventory control” (SSIC) problem, as defined in this study, is to find the amount of safety stock in the spare parts inventories with a probability $1 - \varepsilon$, so that independent delivery and demand processes go on without disruption at minimum cost. For instance, if the value of $\varepsilon$, the probability of disruption, is 0.1, the aim is that disruption will not occur 90% of the time.

In our model, we assume that deliveries, fixed and designated by $n$, take place according to some random process at discrete times within a finite time interval $[0, T]$. These random times have joint probability distributions the same as that of $n$ random points chosen independently from the interval $[\tau, \tau + T]$ according to a uniform distribution. A minimal number of spare parts, $\delta$, is delivered with each delivery $n$. If the total amount of delivery is $D_u$ for some failure rate $u, u \in U$, $U$ being a finite set of different failure rate values, there is also a random amount of delivery obtained by choosing a random sample of size $n - 1$ from a population uniformly distributed in the interval $[0, 1 - n\delta]$. The demand process for spare parts is defined similarly, with parameters $C_u$ for some $u, u \in U$, as the total amount of demand, $\gamma$ as the minimal amount of failures, and $s$ as the number of failures creating the need for spare parts.
assume that delivery and demand processes are independent, and there will be a superscript \( l \) for each spare part.

In each time interval, a minimal number of spare parts equal to \( \delta \geq 0 \) is delivered. Then, a sample size of \( L (L \geq n) \) is taken from the uniformly distributed population in the interval \([0, D_a - n\delta]\) and given by \( x_1^* \leq x_2^* \leq \ldots \leq x_n^* \). Then, \( n - 1 \) positive integers are taken as \( j_i < j_2 < \ldots < j_{n-1} \leq L \). The delivered quantities of the spare part in the \( n \) time intervals are assumed to be

\[
\delta + x_j^*, \delta + x_j^* - x_{j_i}^*, \ldots, \delta + x_{j_{n-1}}^* - x_{j_n}^*, \delta + (D_a - n\delta) - x_n^*.
\]

The model for the demand process is similar. A sample size of \( N \) is taken from the population uniformly distributed in the interval \([0, C_u - n\gamma]\), and designated by \( y_1^* \leq y_2^* \leq \ldots \leq y_n^* \). Then, \( n - 1 \) positive integers are taken as \( k_1 < k_2 < \ldots < k_{n-1} \leq N \). The failed spare parts of a certain type are assumed to be

\[
\gamma + y_k^*, \gamma + y_k^* - y_{k_1}^*, \ldots, \gamma + y_{k_{n-1}}^* - y_{k_n}^*, \gamma + (C_u - n\gamma) - y_{k_{n-1}}^*.
\]

Assuming that the delivery and demand processes are independent, we let

\[
X_1 = x_j^*, \ X_2 = x_j^* - x_{j_i}^*, \ldots, X_{n-1} = x_{j_{n-1}}^* - x_{j_n}^*, \ X_n = (D_a - n\delta) - x_n^*.
\]

\[
Y_1 = y_k^*, \ Y_2 = y_k^* - y_{k_1}^*, \ldots, Y_{n-1} = y_{k_{n-1}}^* - y_{k_n}^*, \ Y_n = (C_u - n\gamma) - y_{k_{n-1}}^*.
\]

Let \( S_u \) denote the initial safety stock of spare parts for some \( u, u \in U \). Another assumption is that we always want to have a safety stock for the parts, shown as \( S_u + D_a \geq C_u \). Then, the condition of no disruption for each \( u, u \in U \) is formulated as follows:

\[
S_u + \delta + X_1 \geq \gamma + Y_1
\]

\[
S_u + 2\delta + X_1 + X_2 \geq 2\gamma + Y_1 + Y_2
\]

\[
S_u + (n-1)\delta + X_1 + X_2 + \ldots + X_{n-1} \geq (n-1)\gamma + Y_1 + Y_2 + \ldots + Y_{n-1}
\]

\[
S_u + n\delta + X_1 + X_2 + \ldots + X_n \geq n\gamma + Y_1 + Y_2 + \ldots + Y_n.
\]

The last inequality \( S_u + D_a \geq C_u \) is simply removed since it is aimed to have a model without disruption.

The problem has two stages, hence there are first and second stage decision variables: a subscript \( u \) is given to each second stage variable. The first stage decision variables in the model are \( M^{(i)} \), the storage capacity (which is more important if we store a large component such as power supplies) of each spare part \( l, l = 1, \ldots, r \). The second stage variables are \( m_u^{(i)} \geq 0 \), the additional safety stock for each spare part \( l, l = 1, \ldots, r \), and for each \( u, u \in U \). We have an initial safety stock in the interval \([0, T] \) and to satisfy the needs for spare parts needed for ITS equipment, we are trying to find the optimum additional safety stocks, \( m_u^{(i)*} \) values and the optimum storage capacities \( M^{(i)*} \). The convex cost functions of storage capacities \( M^{(i)}, l = 1, \ldots, r \) are \( g^{(i)}(x), l = 1, \ldots, r \). The second stage problem comes up after the delivery values \( D_a = (D_a^{(i)}, \ldots, D_a^{(r)}) \) are observed. The corresponding probabilities for each \( \{D_a, C_u, u \in U\} \) are given by \( p_u \). We
prescribe that no disruption occurs in any of the spare parts demand in the time intervals 
\[(kT + \tau; (k + 1)T + \tau), l = 1, \ldots, r,\] with probability \(1 - \varepsilon\). This parameter, \(\varepsilon\), represents the 
probability of disruption due to the unavailability of the required item or disruptions in 
the transportation system, etc. The optimal values of the second stage variables \(m_u^{(i)} \geq 0\) 
are the adjustment values of the safety stocks. If at time \(kT + \tau\), the safety stock levels 
are \(m_u^{(i)} \geq 0\), the new stock levels calculated are \(m_u^{(i)} + m_u^{(i)} \geq 0, u \in U, l = 1, \ldots, r\). Here, 
the adjustments to stock levels incur some additional costs. Thus, the adjustment cost 
function of emergency spare part \(l\) is denoted by \(f^{(i)}(x), l = 1, \ldots, r\).

We approximate the joint distribution of the random demand and delivery 
variables using an approximate multivariate normal distribution with the random 
variable, \(W_{u}^{(i)}\) for each spare part \(l = 1, \ldots, r,\) for \(i = 1, \ldots, n,\) and for each support \(u, u \in U\). 
Therefore, \(W_{u}^{(i)}\) simply represent the values of the probability distribution of the spare 
parts in terms of demand minus delivery for any time step:

\[
W_{iu}^{(i)} = i\gamma^{(i)} + Y_1^{(i)} + \ldots + Y_u^{(i)} - i\delta^{(i)} - X_1^{(i)} - \ldots - X_u^{(i)}, l = 1, \ldots, r, i = 1, \ldots, n \text{ for each } u, u \in U
\]  
(1)

This allows us to calculate the total cost where the highest and lowest values have 
the lowest probabilities according to a pre-determined discretized normal distribution of 
delivery and demands. The expectations, variances and elements of the covariance matrix 
for the random variable \(W_{iu}^{(i)}, l = 1, \ldots, r, i = 1, \ldots, n \text{ for each } u, u \in U\) are calculated 
following the normal approximation guidelines given in (10).

With this information, an overview of the model that shows the inputs and outputs 
can be seen as follows:

**Inputs**
- \(n\) : Number of deliveries
- \(m_u^{(i)}\) : Initial safety stock for spare parts
- \(D_u\) : Total amount of delivery
- \(C_u^{(i)}\) : Total amount of consumption
- \(W_u^{(i)}\) : Approximate normal 
distribution variable of the random consumption and delivery 
distributions
- \(g^{(i)}, f^{(i)}\) : Associated costs
- \(M\) : Total capacity
- \(\varepsilon\) : Probability of disruption

**Outputs**
- \(m_u^{(i)}\) : Additional amount of spare parts safety stock 
  required to satisfy the needs for the vital supplies
- \(M^{(i)}\) : Storage capacity for 
each spare part
Constraints

It is significant to consider the probabilistic nature of the demand and delivery processes given the high stochasticities of the problem domain dealing with the maintenance and repair of ITS equipment. Thus, our model should take these stochasticities into account in terms of probabilistic constraints. There are two types of constraints in the model, the probabilistic constraints, and the capacity constraints.

As the probabilistic constraints ensure the minimal disruption of the usage of spare parts for the ITS equipment with a given probability, the sum of initial stocks and deliveries has to be greater than or equal to the demand for any time step. By replacing $W_{iu}^{(l)}$'s with their expectations and variances of the approximate normal distribution, our probabilistic constraint is defined as

$$P(W_{iu}^{(l)} \leq m_{iu}^{(l)} + m_{iu}^{(l)} \geq 1 - \varepsilon \Rightarrow \prod_{l=1}^{r} \Phi \left( \frac{m_{iu}^{(l)} + m_{iu}^{(l)} - \mu_{iu}^{(l)}}{\sigma_{iu}^{(l)}} \right) \geq 1 - \varepsilon \right)$$  (2)

Other constraints in the SSIC model are the capacity constraints. At any time step, the initial safety stock plus the optimal additional stock must be smaller than the storage capacity for that spare part, and the sum of storage capacities for each spare part ($a_{iu}^{(l)}$, space occupied by each spare part $l$, $l=1,\ldots,r$, multiplied by the storage capacity for each spare part, $M^{(l)}$) must be smaller than the overall capacity, $M$.

Objective Function

The objective cost function is the sum of individual costs listed below:

- **Cost of Storage**, $g_{iu}^{(l)}$: It is obvious that there is a cost for storing each spare part $l$, $l=1,\ldots,r$. In case of ITS maintenance and operations, it is important to consider storage costs since the occurrence of an ITS equipment failure is not known a priori.

- **Cost of Surplus**, $q_{iu}^{(l)}$: This is incurred for each spare part if there is more inventory than demand. It can be modeled as a fixed cost or as a step function that allows very low or no cost for a certain surplus level and then a steep increase for higher levels of surplus.

- **Cost of Shortage**, $q_{iu}^{(l)}$: This cost is incurred for each spare part if there is not sufficient spare part inventory to satisfy the demand for the ITS equipment. This is the most important cost component as the shortage of spare part supplies can cause loss of time, fuel and lead to congestion and accidents due to the inefficiently working or broken ITS equipment.

- **Cost of Adjustment**, $f_{iu}^{(l)}$: This cost is incurred by the nature of the two-stage model. Suppose we have an initial amount of safety stock, but to satisfy the probability constraint, we need more. This adjustment can be due to the unexpected factors such as the increased number of demand for spare parts of a specific ITS equipment due to a sudden failure. Of course, this has to be penalized. It can be chosen as a linear function of the additional stock.

The objective function includes the demands for the spare parts multiplied by their corresponding probabilities. This allows us to calculate the total cost where the highest and lowest demands have the lowest probabilities according to a pre-determined
discretized normal distribution. Therefore, the model is minimized according to this total cost that is simply sum of the costs for different demand scenarios. The total cost of an equipment failure due to the unavailability of a spare part may be higher than others due to the additional safety stocks required, however, the probability associated with this high demand will be smaller than lower levels of demand closer to the mean (normal distribution assumption). This self-controlling mechanism gives the DOT personnel the chance to determine the safety stocks more accurately.

With this mathematical information, the formulation of the SSIC model is as follows:

$$\min \left( \sum_{i=1}^{r} g^{(i)}(M^{(i)}) + \frac{1}{T} \sum_{u \in U} p_u \left[ f^{(i)}(m_u^{\text{lower}}) + \sum_{i=1}^{n} q_i^{(i)} + q_i^{(i)} \right] \int_{m_i^{\text{upper}}+m_i^{\text{lower}}}^{\infty} \left( 1 - \Phi \left( \frac{z - \mu_u^{(i)}}{\sigma_u^{(i)}} \right) \right) dz \right)$$

Subject to

$$\prod_{i=1}^{r} \Phi \left( \frac{m_i^{(i)} + m_u^{(i)} - \mu_u^{(i)}}{\sigma_u^{(i)}} \right), \ i = 1, \ldots, n - 1, \Sigma(g) \geq 1 - \epsilon$$

$$m_i^{(i)} + m_u^{(i)} \leq M^{(i)}, \ u \in U, \ i = 1, \ldots, r$$

$$m_u^{(i)} \geq 0, \ u \in U, \ i = 1, \ldots, r$$

$$\sum_{i=1}^{r} a_i^{(i)}(M^{(i)}) \leq M$$

(3)

**Solution Approach: Prékopa-Vizvari-Badics Algorithm (11)**

Solving the nonlinear SSIC problem with an exact solution technique requires extensive programming and optimization knowledge. pLEPs method (12), however, serves as a practical and easily applicable approximate method so that DOT personnel can use the results to decide on the inventory levels for the spare parts needed for ITS equipment. For this purpose, the Prékopa-Vizvari-Badics algorithm will be used to generate the pLEP sets for the relaxed disjunctive programming problem.

The algorithm is based on the discrete random variable $\xi^{(j)}$ where $Z = Z_1 x_1 \ldots x_r$ is the product set containing the support of $\xi$, the vector of the discrete random variable $\xi^{(j)}$. For the sake of illustration, we consider the r-dimensional vector as $\xi = (\xi_1, \ldots, \xi_r)$.

The algorithm is as follows:

Step 0. Initialize $k \leftarrow 0$.

Step 1. Determine $z_{j_1}, z_{j_2}, \ldots, z_{j_r}$ such that

$$z_{j_1} = \arg \min \left\{ y | F \left( y, z_{j_1+1}, \ldots, z_{j_r+1} \right) \geq 1 - \epsilon \right\}$$

$$z_{j_2} = \arg \min \left\{ y | F \left( z_{j_1}, y, z_{j_3+1}, \ldots, z_{j_r+1} \right) \geq 1 - \epsilon \right\}$$

$$M$$

$$z_{j_r} = \arg \min \left\{ y | F \left( z_{j_1}, \ldots, z_{j_r-1} + 1, y \right) \geq 1 - \epsilon \right\}$$
and let \( E \leftarrow \{ z_{i,j}, \ldots, z_{r,j} \} \).

Step 2. Let \( k \leftarrow k + 1 \). If \( j_i + k > k_i + 1 \), then go to Step 4. Otherwise, go to Step 3.

Step 3. Enumerate all the pLEPs of the function \( F(z_{i,j+k}, y) \), \( y \in R^{r-1} \) and eliminate those which dominate at least one element in \( E \) (\( y \) dominates \( z \) if \( y \geq z \), \( y \notin z \)). If \( H \) is the set of the remaining pLEPs, then let \( E \leftarrow E \cup H \). Go to Step 2.

Step 4. Stop, \( E \) is the set of all pLEPs of the CDF \( F(z) = P(\xi \leq z) \).

This methodology provides discretized set of points, which gives the lower bound of a specific probability distribution (13). These are used to create the deterministic equivalent of the probabilistic constraints, and they assure that the constraints will satisfy the given reliability level \( p = 1 - \varepsilon \). However, first of all, before applying the algorithm, continuous distribution functions in our model have to be converted into approximate discrete distributions (14). For that purpose, for each demand \( u, u \in U \), we approximate the random variable \( W_{iu}^{(l)} \) by a discrete variable \( z_{iu}^{(l)} \) with possible values \( \sigma_{iu}^{(l)} < \sigma_{iu}^{(l+1)} < \ldots < \sigma_{iu}^{(l)} \), where the distribution function is:

\[
P(\left\{ W_{iu}^{(l)} \leq m_{iu}^{(l)} + m_{iu}^{(l)} \right\}) = \Phi \left( W_{iu}^{(l)} \right) = F_{W_{iu}^{(l)}}(\sigma_{iu}^{(l)}) = \begin{cases} F_{\zeta_{iu}^{(l)}}(\sigma_{iu}^{(l)}), & \sigma = 1, \ldots, L - 1 \\ 1, & \sigma = L \end{cases}, \quad l = 1, \ldots, r \tag{4}
\]

where \( \sigma_{iu}^{(l)} < \sigma_{iu}^{(l+1)} \) are chosen to be equidistant on some interval \( [0, B_{iu}^{(l)}] \) where \( F_{\zeta_{iu}^{(l)}}(B_{iu}^{(l)}) = 1 - \zeta \) for a prescribed small tolerance \( \zeta \). Here, \( B_{iu}^{(l)} \) is the selected upper boundary of the interval of \( \sigma_{iu}^{(l)} \) values for each commodity \( l = 1, \ldots, r \), and for each demand \( u, u \in U \). Using this process, we obtain the following probabilistic constraint in the form of multiplication of the cumulative distribution functions of \( z_{iu}^{(l)} \):

\[
\prod_{l=1}^{r} F_{\zeta_{iu}^{(l)}}(\sigma_{iu}^{(l)}) \geq 1 - \varepsilon, \quad \sigma = 1, \ldots, L - 1, \quad l = 1, \ldots, r \text{ for each } u, u \in U
\]

That is, we discretize our continuous cumulative distribution function associated with \( W_{iu}^{(l)} \) on its entire domain. During the analysis, we try to keep a substantial amount of accuracy while choosing the possible values of \( z_{iu}^{(l)} \) in the interval \( [0, B_{iu}^{(l)}] \) selecting \( B_{iu}^{(l)} \) and \( N \) accordingly. With this idea, we try to preserve most of the information related with the original function in the discretized one. The important point is that we focus on the upper regions of the distribution function that contribute to the calculation of pLEPs with respect to the selected disruption probability.

**CASE STUDIES**

First, a base case two-spare parts scenario is created where independent distributions of power supplies and LEDs (light-emitting diode) are considered. The reason to choose these components is that we want to demonstrate the behavior of the proposed model for spare parts with different costs and distinct characteristics. According to the unit cost
database of U.S. Department of Transportation, Research and Innovative Technology Administration (15) and the LED vendors Dialight (http://www.dialight.com) and Ecolux (http://www.ecolux-led.com), typical average unit cost for LEDs is selected as $2-3, and they are parts of signals and dynamic message signs basically. Power supplies, on the other hand, have unit costs that differ for various ITS equipments, and their range is from $30 to $350 (See http://www.daktronics.com for more details). In our base case scenario, depending on the severity of the failures or conditions causing those failures, there are five different failure rate scenarios given in an ascending order, where type 1 (u=1) represents the lowest failure rate (best case), and type 5 (u=5) is the highest failure rate (worst case). The idea is that the demand for LEDs starts lower than the demand for power supplies, but increases more rapidly as the failure rate increases. Cost figures are selected as dollars for power supplies and LEDs. For the base case scenario, the following values are chosen:

- ε is 0.1, ζ is 0.01, N is 40 and \( B_u^{(1)} \) is 200 for LEDs and 1000 for power supplies.
- Number of deliveries in the time interval (i.e., a month) is \( n = 4 \).
- Amounts of initial safety stock, \( m^{(1)} \) and \( m^{(2)} \) are 1 and 3 units, respectively.
- Surplus costs are \( q^{(1)} = 5 \) / unit, \( q^{(2)} = 1 \) / unit, and shortage costs are \( q^{-1} = 100 \) / unit, \( q^{-2} = 10 \) / unit.
- \( \delta^{(1)}, \delta^{(2)}, \gamma^{(1)} \) and \( \gamma^{(2)} \) are calculated for each spare part and scenario separately.
- Cost of adjustment function is selected as \( f(x) = 2x \).
- Costs of storage for each spare part are 5/unit and 1/unit, respectively.
- Total storage capacity is 100 units.
- Spaces occupied by commodities are 1/unit.
- Expected total delivery and expected total demand values are taken as

\[
D^1 = [1, 3, 5, 7, 9] \quad C^1 = [5, 10, 15, 20, 25] \\
D^2 = [0, 5, 10, 20, 30] \quad C^2 = [20, 30, 40, 50, 60]
\]

- Probabilities of the discrete supports of demand and delivery values are

\[
p = [0.08, 0.25, 0.34, 0.25, 0.08].
\]

The results are given in Table 1. To satisfy the spare part demand 90% of the time, initial safety stock must be at least more than 30% of the total expected demand for the power supplies. That is, having more than 30% of the spare parts in the inventory before the failures will prevent disruption 90% of the time. For the LEDs, the initial stock must be more than 40% of the total expected demand. This indicates the importance of correctly determining the initial safety stocks in the inventory as having this percentage of stocks will prevent disruption 90% of the time. When the total expected demand is equal to the initial safety stock, the model finds the optimal additional safety stock value as zero. Moreover, it may not be possible to make 4 deliveries in a given period of time, thus additional amount of safety stock for spare parts has to increase.
<table>
<thead>
<tr>
<th>Demand</th>
<th>u=1</th>
<th>u=2</th>
<th>u=3</th>
<th>u=4</th>
<th>u=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Stock (Power supply)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Optimal Additional Stock (Power supply)</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Total Initial Stock (Power supply)</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Total Expected Demand (Power supply)</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>Proportion of Initial Safety Stock to Total Expected Demand (Power supply)</td>
<td>20.0%</td>
<td>30.0%</td>
<td>26.7%</td>
<td>30.0%</td>
<td>28.0%</td>
</tr>
<tr>
<td>Initial Stock (LED)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Optimal Additional Stock (LED)</td>
<td>2</td>
<td>8</td>
<td>12</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>Total Initial Stock (LED)</td>
<td>5</td>
<td>11</td>
<td>15</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>Total Expected Demand (LED)</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>Proportion of Initial Safety Stock to Total Expected Demand (LED)</td>
<td>40.0%</td>
<td>36.7%</td>
<td>37.5%</td>
<td>36.0%</td>
<td>36.7%</td>
</tr>
<tr>
<td>Total Cost</td>
<td></td>
<td></td>
<td></td>
<td>$855</td>
<td></td>
</tr>
</tbody>
</table>

It is reported in (3) that with regard to planning for knock-downs, lightning, floods, tornados, snow storms, major power failures, and other unforeseen events, some allowance needs to be made in terms of spare parts. For instance, this report states that one ITS system in Virginia with over 1000 roadside devices suffers from approximately one knockdown per year. Lightning is extremely variable and, despite the best attempts towards protection, electromagnetic pulses (EMP) can damage the electronics, even when the devices are not directly hit. In fact, most damage to equipment is not caused by direct lightning strikes, but by induced voltages on conductors from nearby strikes. That is, ITS devices are often electronically sensitive devices placed in open areas on top of electrically conducting metal poles. To make things worse, these devices are often connected to both power and communication systems via long-conducting copper wires. There have been examples of ITS camera installations in Florida where all of the PTZ units (pan-tilt-zoom cameras) were rendered inoperative by a single storm where the damage came through the power supply (3). Flooding too can cause major problems to ITS devices. In Bombay, India, the controller bases are four feet high to protect them from the monsoons (3). Obviously, these risks to ITS components will vary significantly in different geographies and climates across the US.

Therefore, our model should also incorporate these worst-case scenarios. This can be achieved by changing the relevant parameters to be able to make this type of worst-case analysis. Five demand scenarios are analyzed as before. All other parameters being kept same as the base case (other than these values: \( m^{(1)} \) and \( m^{(2)} \) are 4 and 10 units,
respectively, $N$ is selected as 100, and $B_u^{(i)}$ is 1000 for LED's and 5000 for power supplies), we expect an increase in the total delivery and demand values for spare parts as:

$$D^1 = [3, 5, 7, 9, 11] \quad C^1 = [16, 20, 24, 28, 32]$$

$$D^2 = [5, 10, 15, 20, 25] \quad C^2 = [35, 45, 55, 65, 75]$$

With these values, we change the mean, variance and the covariance matrices of the distribution of the spare parts. Especially for LEDs, the distribution is changed substantially to account for the severity of the lightning affects. As observed from Table 2, to satisfy the needs for the spare parts 90% of the time, initial stock must be more than 40% of the total demand for power supplies. That is, having more than 40% of the spare parts in the inventory before the failure will prevent disruption 90% of the time. However, for LEDs, the initial stock must be almost 50% of the total demand. Of course, 50% safety stock in the inventories may seem to be high for ITS operations, however these are the worst case scenarios as mentioned before. Traffic operations and maintenance personnel of DOT should be aware of this risk, and plan the spare parts inventory accordingly. For instance, for a high risk roadway or region in terms of ITS equipment failure, the spare part inventory can be handled via this type of high risk scenario.

**TABLE 2 Results for the Two Spare Parts Analysis with High Demand**

<table>
<thead>
<tr>
<th>Demand</th>
<th>u=1</th>
<th>u=2</th>
<th>u=3</th>
<th>u=4</th>
<th>u=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Stock (Power supply)</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Optimal Additional Stock (Power supply)</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Total Initial Stock (Power supply)</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>Total Expected Demand (Power supply)</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>Proportion of Initial Safety Stock to Total Expected Demand (Power supply)</td>
<td>37.5%</td>
<td>40.0%</td>
<td>37.5%</td>
<td>39.3%</td>
<td>40.6%</td>
</tr>
<tr>
<td>Initial Stock (LED)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Optimal Additional Stock (LED)</td>
<td>7</td>
<td>11</td>
<td>17</td>
<td>21</td>
<td>27</td>
</tr>
<tr>
<td>Total Initial Stock (LED)</td>
<td>17</td>
<td>21</td>
<td>27</td>
<td>31</td>
<td>37</td>
</tr>
<tr>
<td>Total Expected Demand (LED)</td>
<td>35</td>
<td>45</td>
<td>55</td>
<td>65</td>
<td>75</td>
</tr>
<tr>
<td>Proportion of Initial Safety Stock to Total Expected Demand (LED)</td>
<td>48.6%</td>
<td>46.7%</td>
<td>49.1%</td>
<td>47.7%</td>
<td>49.3%</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$2256</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As the failure rates of ITS components increases due to unexpected or unforeseen events, the demand for spare parts increases in a short amount of time to ensure...
efficiency of ITS operations. Moreover, in the analysis, the demand for LEDs increases more rapidly than is the demand for the power supplies. The model reacts to this scenario by increasing the additional safety stock values more rapidly for LEDs (Figure 2). The most severe condition of the high failure rate case having the largest demand requires the largest safety stocks for both spare parts.

FIGURE 2 Changes in the Additional Safety Stock Values versus Changes in the Demand Scenarios

Software Implementation of the Spare Inventory management System within RITSIMS

Preliminary user interface screenshots proposed for inventory management using RITSIMS are shown in Figure 3 and Figure 4 where an inventory check is being performed for a Variable Message Sign (VMS) when there are broken LEDs, and another check is being performed for a Closed Circuit Television System (CCTV) with a failed power supply. There are two demand scenarios used in the module: low (best case) and high (worst case) demands. Each demand has five different failure rates (which will actually be obtained from the spare part failure data via RITSIMS at the end of the implementation stage). With this tool, traffic operations personnel from DOT not only can check the availability of the relevant inventory, but also corresponding figures will help them to create a safety stock for ITS spare parts for future use.
FIGURE 3 Spare Parts Inventory Module
FIGURE 4 Spare Part Stocks Module for LEDs with High Demand (FIGURE 4.a) and for Power Supplies with Low Demand (FIGURE 4.b) for Different Failure Rates

CONCLUSIONS
The spare parts inventory management for DOT’s represents a very complex problem due to the difficulties of ITS equipment failures and malfunctions caused by the lack of technical knowledge, inadequate inspection during installation and improper maintenance practices. To the best of our knowledge, many of the DOTs do not have a spare part inventory management model to assist inspectors, ITS design, traffic operations and maintenance personnel. To overcome the aforementioned problems, this paper proposes a
robust spare parts inventory management model that is being integrated within RITSIMS (1). This integrated tool is expected to assist the DOT personnel to improve the efficiency of their activities related to the maintenance and repair of various ITS equipment. Case studies show that the proposed stochastic inventory management model can handle various levels of demand uncertainty, including worst case scenarios, by adjusting planning of safety stock levels. It is expected that this system will help create cost-effective solutions to many ITS related maintenance and repair problems by:

- monitoring the performance of equipment,
- increasing the useful life of the equipment through improved and timely maintenance actions,
- keeping inventory costs lower, and
- effectively scheduling maintenance activities.

It is evident that a future study, namely the integration of the failure, inventory and cost data obtained from RITSIMS with the proposed inventory management model will result in an improvement for the inspection and maintenance of ITS equipment. Addition of new functionalities related to the warranty, factory testing information, as well as the record of spare part costs are also possible future works that can improve the proposed model. Application of a prototype for stressful conditions when unexpected events occur, and using different distributions for spare parts are also of interest to further develop and validate our model.

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REFERENCES


