A Time Allocation Theory Based Methodology for Valuation of Travel Time Reliability

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ABSTRACT
Over the last few decades, the value of reliability has been recognized as a significant factor in drivers’ choice behavior. To measure reliability valuation, numerous studies have utilized the empirical scheduling-delay formulation developed by Small (1), which was originally based on classical microeconomic time allocation theory. In this paper we revisit the general time allocation model which was first introduced by Becker (2) and then formalized using time and goods constraints by De Serpa (3). We add schedule delay parameters in the constraints and provide an analytical derivation of a non-linear utility function. Next, we relax the constant marginal utility assumption following Blayac and Causse (4), which is considered an economical restriction that forces a single value of reliability estimation. Finally we give formulations for value of reliability calculation and present an empirical analysis for departure time choice using revealed preference data from New Jersey Turnpike (NJTPK) traveler survey. The results of the analysis show that travelers departing right before the peak congestion periods (i.e. during pre-peak) have a higher average value of reliability compared to the people departing at post-peak periods.
INTRODUCTION

There is a growing interest in the valuation of travel time reliability as a method of understanding driver choice behavior. Drivers often consider the potential savings in their average travel times when making transportation related choices about their route choice or departure time choice. Over the last few decades a rapidly developing body of research focusing on the measurement of the value of reliability showed that reliability associated with the available travel options also play an important role in travelers’ decision making. The predictability of travel times is a crucial component to this decision making, especially when a driver is considering travel options for work trips or airport trips with strict time constraints. The recent progress in the valuation of reliability research has refocused policymakers’ attention on the question of how to enhance the reliability of existing transportation facilities and to consider reliability as a significant factor when assessing the benefits of proposed projects. Recently, European countries such as Sweden and the Netherlands have put more emphasis on reliability improvements in their governmental transportation policy programs (5).

Methodologically, there are two mainstream approaches in the literature for determining the valuation of travel time reliability. The first approach is generally known as the mean-variance approach, in which travel time reliability is linked with a statistical measure of the travel time distribution. In this group of models reliability is directly included in the utility function as a parameter. Statistical measure is usually selected as the variance of the travel time although there are several studies that also consider different measures such as the shortened righthand range (i.e. 90th-75th percentile) of travel time distribution (6). Two major deficiencies of the mean-variance approach are 1) the results are independent of the shape of the travel time distribution, and 2) including reliability as a term in the utility function is not consistent with the classical economic theory that defines the utility for the outcome of an activity (i.e. trip) (7). A more widely used alternative approach for the valuation of reliability is the scheduling approach. The theory behind the scheduling approach has its roots in the microeconomic time allocation theory. In this approach the utility, which is a function of departure time, is defined over being late or early relative to a preferred arrival time (i.e. the outcome of a trip). The drawback of this approach is the difficulty in measuring the actual departure time and preferred arrival time of the users (7). Recently, several studies focused on the relationship between the two approaches and it has been shown that under certain strong assumptions the mean-variance approach becomes equivalent to the scheduling approach (8).

From an econometric point of view scheduling models were originally built upon time allocation models. Small (1) was the first one to show that departure time has an effect on utility and the value of travel time savings when that value is determined within the time allocation model developed by Vickrey (9). The well-established empirical scheduling approach presented in Small’s paper (1) is generally accepted as the base model and has been used in several studies focused on the valuation of travel time savings as well as the valuation of travel time reliability. In this study we revisit the general time allocation model that was first introduced by Becker (2) and then formalized, in terms of its utility using time and goods constraints, by De Serpa (3). We add schedule delay parameters in the constraints, provide an analytical derivation of the utility function, and then give formulations for the value of travel time reliability calculation.

Linear utility functions have been dominantly used for the value of reliability and the value of time calculations. Our assumption of linearity in the parameters results in constant marginal utilities, which itself is an economical restriction, since a single value of travel time reliability can be obtained using the ratio of marginal utilities. Moreover nonlinear form of functions can be more
appealing because they can address possible relationships between different orders of explanatory
variables that are used to measure the indirect utility (4). We challenge the problem of the single
value of reliability estimation by relaxing the constant marginal utility assumption in the original
time allocation model following Blayac and Causse (4) and Ozbay and Yannaz-Tuzel (10). Next
we present a nonlinear utility function and formulate the user-specific value of travel time
reliability. We finally provide an empirical analysis for departure time choice in the existence of
time of day pricing. In particular we investigate the differences in valuation of travel time
reliability for short time periods right before and after peak periods (i.e. pre-peak, post-peak) and
interpret the results using the provided methodology.

LITERATURE REVIEW

The traveler’s marginal value of travel time variability, or what is commonly referred to in
literature as the value of reliability, plays a significant role in choice behavior, as demonstrated by
several empirical studies (11-14). The seminal study carried out by Small (1) extended the classical
microeconomic time allocation theory by including departure time as a variable in the utility
function and using constraints to link departure time with work hours and wage rate to obtain a
value of time that is dependent on work schedule. The author first analytically showed the effect
of departure time on utility using envelope theorem and then provided a “strict utility” function
using empirical observations. The cost function using the scheduling considerations provided in
this study is as follows:

\[ C = \alpha T + \beta SDE + \gamma SDL + \phi DIL \]  

(1)

Where \( T \) is the total travel time, SDE and SDL are the schedule delays in terms of time
when traveler arrives early or late respectively. The last term \( DIL \) is a dummy variable that
accounts for the extra cost for the late arrival that takes a value of 1 if the traveler arrives after
his/her preferred arrival time. For practical applications SDE is defined as \( \text{Max}(0, \text{PAT} - (th+T)) \) and
SDL is defined as \( \text{Max}(0, (th+T) - \text{PAT}) \) if \( th \) is the departure time and PAT is the preferred arrival
time.

The framework presented in Small (1) is the pioneering work for later contributions to
scheduling models since it deals with the outcome of the trip from the perspective of a traveler
who dislikes being early or late relative to his/her PAT. Noland and Small (11) extended the above
formulation for the case of stochastic travel times to account for the effects of unexpected traffic
congestion on travel times. They considered travel time \( T \) as a random variable which depends on
the departure time \( th \) and presented the expected utility form of the scheduling function as
following:

\[ E[U(th)] = \alpha E[T(th)] + \beta E[SDE(th)] + \gamma E[SDL(th)] + \phi P_c(th) \]  

(2)

The above formulation uses classical expected utility theory where probabilities are
considered in a linear way. Therefore it is assumed that decisions are made with perfect knowledge
of travel time distributions and the ratio of weights associated with alternative outcomes is equal
to the ratio of the probabilities (14). Noland and Small (11) also showed an optimal departure time
calculation by utility maximization assuming uniform and exponential travel time distributions.
Later studies extended the optimal departure time analysis for Weibull, Gamma (15) and for
general standardized travel time distributions (8).
The second major branch of methodological derivation of the value of reliability has been largely based on Jackson and Jucker (16)'s mean variance approach. In their formulation they define the utility not over the outcomes of the trip (as it is done in the scheduling approach) but over the statistical measures of travel time distribution itself. The model is simple and easy to implement. Utility in mean-variance approach is expressed as follows:

$$U = \bar{T} + \lambda V(T)$$  \hspace{1cm} (3)

Where $\bar{T}$ is the mean travel time, $V(T)$ is the variance of travel time distribution and $\lambda$ is the parameter measuring the effect of variance. Several empirical applications for mean-variance approach are available in the literature (17-21).

Bates et al. (12) and Noland and Polak (22) showed that, if one assumes travelers have the ability to choose the optimal departure time for their trips, and if travel time distribution is independent of departure time, the two approaches become equivalent for certain types of travel time distributions. According to Bates et al. (12) if the travel time distribution does not depend on the departure time (i.e. the mean and variance of the travel time distribution are constant over time) and there is no lateness penalty, the scheduling parameters (i.e. $\beta E[SDE(t_s)] + \gamma E[SDL(t_s)]$) can be approximated by the standard deviation of the travel time. An important contribution by Fosgerau and Karlstrom (8) further improved the theoretical foundation of the equivalency idea when they proved for any standardized travel time distribution that the minimized expected cost of an individual is linear in the mean travel time as well as a scale measure of the travel time distribution (i.e. variance). The authors give a proof for the case where the mean and the scale of travel time distribution are constant over time of day, which in practice is not true. In the same study they also relaxed this assumption and empirically showed it is still possible to approximate the value of travel time variability for the time-varying mean and the standard deviation of travel times. However in this last case they still hold the assumption of time-of-day independent travel time distribution (8).

Borjesson et al. (5) challenged the equivalency idea using stated preference data and empirically showed that the mean-variance utility function, which is derived from the scheduling approach, cannot fully explain the disutility associated with the travel time variability. Their results show that scheduling models yield lower values of reliability compared to mean-variance models. The authors highlighted the lack of addressing the rescheduling possibility in the scheduling models where the cost functions for arriving late or early are exogenously determined and cannot be changed. They also mention the systematical errors due to model assumptions such as considering all the users as utility maximizers or misperception errors about the probabilities which may affect the results. The contribution of Borjesson et al. (5) is important in the sense that it casts reasonable doubt on the practical implications of the theoretical equivalency of the two approaches as previously presented by Fosgerau and Karlstrom (8).

In this study we use microeconomic time allocation models for the derivation of utility functions, which includes scheduling delay parameters. Jara-Díaz (23) provided an excellent summary of the most commonly used time allocation models for the valuation of travel time savings. Most of the studies consider a discrete choice utility model that is assumed to be linearly dependent on travel time and travel cost. The analytical background of these models can also be used to derive valuation of travel time reliability analysis similar to Small (1) and Noland and Small (11). Jara-Díaz (24) investigated leisure and work activities and showed that the value of travel time savings is not equal for different type of activities. Jiang and Morikawa (25) introduced
an additional term, the value of cost savings, in the value of travel time savings formulation by combining the models developed by De Serpa (3) and Evans (26). Blayac and Causee (4) provided a methodology for relaxing the constant marginal utility assumption in the original derivation; Ozbay and Yanmaz-Tuzel (10) extended this analysis by introducing schedule delay terms in the utility function but in their analysis they focused on the value of travel time savings based on early arrivals. In this study we show the analytical derivation of the utility function for the full model with early and late schedule delays and present the formulation for the valuation of travel reliability.

TIME ALLOCATION MODEL FOR VALUE OF RELIABILITY

In the majority of the existing models the variables to be included in the utility model are chosen with empirical observations in mind. For example, Small (1) is the only existing study that derives analytical utility using scheduling constraints, and still the ultimate strict utility function is based on empirical observations. Later studies selected model variables mostly based on an ad-hoc manner such that the variables included in the discrete choice model were picked to obtain the best improvement in the model’s goodness of fit (13). The methodology presented in this paper attempts to explain the analytical relationship of the variables in the utility function and the valuation of travel time reliability considering schedule delays. The variables obtained from this analysis do not depend on any specific route or mode choices but rather a theoretical derivation that considers departure time choice.

First we start with the mathematical formulation of the extended time allocation model that follows from the modifications Ozbay and Yanmaz-Tuzel (10) made to the model originally developed by DeSerpa (3). Ozbay and Yanmaz-Tuzel (10) only focused on the valuation of travel time savings regarding early arrivals and assumed that late arrivals are not desirable at all. The contribution to the extended time allocation model in this study is the inclusion of both schedule delay parameters, SDE and SDL, in the derivation process of the valuation of travel time reliability formulation. The final form of the utility function we obtain here is a more general formulation in the sense that the function in Ozbay and Yanmaz-Tuzel (10) becomes a reduced form when the binary late arrival coefficient (denoted as $l_i$ below) is set to zero. The model is given as follows:

$$\text{Max } V(t, t_i, t_i^{\max}, t_i^{\min}, p, p, T, R, t_{ol}, SDE_i, SDL_i)$$

Subject to

$$px + \sum_{i=1}^{r} d_i p_i = R [\lambda]$$

$$t + \sum_{i=1}^{r} d_i t_i = T [\mu]$$

$$t_i \geq t_i^{\min} \left[ \kappa_i \right]$$

$$e_i(t_{oi} - t_i) = SDE_i \left[ \theta_i \right]$$

$$l_i(-t_{ol} + t_i) = SDL_i \left[ \vartheta_i \right]$$

$$e_i SDE_i \leq SDE_i^{\max} \left[ \alpha_i \right]$$

$$l_i SDL_i \leq SDL_i^{\max} \left[ \psi_i \right]$$

$$d_i, e_i, l_i = (0,1) \{ t, t_i^{\min}, p, p, T, R, SDE_i, SDL_i \} \geq 0$$

where;

$$\text{Max } V(t, t_i, t_i^{\max}, t_i^{\min}, p, p, T, R, t_{ol}, SDE_i, SDL_i)$$

Subject to

$$px + \sum_{i=1}^{r} d_i p_i = R [\lambda]$$

$$t + \sum_{i=1}^{r} d_i t_i = T [\mu]$$

$$t_i \geq t_i^{\min} \left[ \kappa_i \right]$$

$$e_i(t_{oi} - t_i) = SDE_i \left[ \theta_i \right]$$

$$l_i(-t_{ol} + t_i) = SDL_i \left[ \vartheta_i \right]$$

$$e_i SDE_i \leq SDE_i^{\max} \left[ \alpha_i \right]$$

$$l_i SDL_i \leq SDL_i^{\max} \left[ \psi_i \right]$$

$$d_i, e_i, l_i = (0,1) \{ t, t_i^{\min}, p, p, T, R, SDE_i, SDL_i \} \geq 0$$

where;
The indirect utility function $V$ in equation (4a) includes user-specific travel time, time spent for other activities than traveling, cost of travel, cost of other activities, income, total available time, departure time and SDE and SDL. The objective of the model is to maximize the utility function using a total income constraint (4b) stating that the total income is allocated between cost of travel and cost of other goods, a total time constraint (4c) meaning the total available time is allocated to travel time and other activities (i.e. leisure). The third constraint (4d) is the minimum time requirement for each travel choice. Constraints (4e) and (4f) are the scheduling constraints for early and late arrivals which are multiplied by binary factors ensuring that either one of them is included in the utility function. The following two constraints (4g) and (4h) ensure that each individual has a flexibility of maximum early or late arrival regarding their PAT.

The Lagrangian of this problem using the Lagrangian multipliers in the parenthesis for each constraint is given as:

$$L = V + \lambda \left( px + \sum_{i=1}^{r} d_i p_i - R \right) + \mu \left( t + \sum_{i=1}^{r} d_i t_i - T \right) + \sum_{i=1}^{r} d_i \kappa_i \left( t_i^{min} - t_i \right)$$

$$+ \sum_{i=1}^{r} e_i d_i \theta_i \left( -t_i + t_{oi} - SDE_i \right) + \sum_{i=1}^{r} l_i d_i \phi_i \left( -t_i + t_{oi} + SDL_i \right) +$$

$$\sum_{i=1}^{r} e_i \sigma \left( SDE_i - SDE_i^{max} \right) + \sum_{i=1}^{r} l_i \psi \left( SDL_i - SDL_i^{max} \right)$$

$$+ \rho_i SDE_i + \varsigma_i SDL_i$$

(5a)
Applying the envelope theorem, the first order partial derivatives are given as follows:

\[ \frac{\partial V}{\partial R} = \lambda \]  
\[ \frac{\partial V}{\partial T} = \mu \]  
\[ \frac{\partial V}{\partial t_i} = -\kappa_i d_i \]  
\[ \frac{\partial V}{\partial p_i} = -\lambda x \]  
\[ \frac{\partial V}{\partial t_{ai}} = -d_i \lambda \]  
\[ \frac{\partial V}{\partial SDE_i} = e_i d_i \theta_i \]  
\[ \frac{\partial V}{\partial SDL_i} = -l_i d_i \theta_i \]

The total differentials for utility \( dV \) and utility for a specific choice \( dV_i \) are given below in equations (7a) and (7b) respectively. Assuming prices other than travel costs are steady over time, \( \frac{\partial V}{\partial p} \) is set equal to zero and the ultimate form of differential for choice specific utility is given in equation (7c):

\[ dV = \frac{\partial V}{\partial R} dR + \frac{\partial V}{\partial p} dp + \frac{\partial V}{\partial T} dT + \sum_{i=1}^n d_i \frac{\partial V}{\partial t_i} dt_m + \sum_{i=1}^n d_i \frac{\partial V}{\partial SDE_i} dSDE_i + \sum_{i=1}^n d_i \frac{\partial V}{\partial SDL_i} dSDL_i \] (7a)

\[ dV_i = \frac{\partial V}{\partial R} dR + \frac{\partial V}{\partial p} dp + \frac{\partial V}{\partial T} dT + \frac{\partial V}{\partial t_i} dt_m + \frac{\partial V}{\partial SDE_i} dSDE_i + \frac{\partial V}{\partial SDL_i} dSDL_i \] (7b)

\[ dV_i = -\lambda (dp_i - dR) + \mu dT - \kappa_i d_m - \theta_i \left( dt_{ai} - dSDE_i \right) - \theta_i \left( dt_{ai} + dSDL_i \right) \] (7c)

In the given functional form (7c), \( \lambda \) represents the marginal utility of having additional income, \( \mu \) is the marginal utility of having additional unit of available time, \( \kappa \) is the marginal utility of decreasing time requirements for travel choice \( i \), \( \theta_i \) and \( \theta_i \) are the marginal utilities of decreasing...
the early and late arrival times by changing the departure time. Either late or early arrivals are
allowed to be possible using binary variables $e_i$ and $l_i$. Constancy of the marginal utilities can be
regarded as an economical limitation of the above model since value of time and value of reliability
calculations are performed by taking the ratio of the partial derivatives or the marginal utilities (i.e.
value of time = $\kappa_i / \lambda$ or value of schedule delay early = $e_i \theta_i / \lambda$) (24). Kato and Onoda (27) show
that when using non-constant marginal utilities the value of travel time savings decreases as travel
time increases. Borjesson et al. (5) used the so called “Step Model”, drawn from Small’s (1)
scheduling model, and the “Slope Model”, which is based on Vickrey (28) and Fosgerau and
Engelson (29). They state that the slope model produces a better model fit compared to the step
model due to the relaxation of the constant marginal utility of time at the origin. In this study we
use Taylor’s expansion theorem around the average point of the marginal utilities,

$$(R, \overline{p}_i, \overline{t}_{i_{min}}, \overline{SDE}, \overline{SDL})$$

with the assumption that each marginal utility parameter varies by all
parameters in the utility function. A similar analysis for a reduced functional form can be found in
Ozbay and Yanmaz-Tuzel (10).

For illustration purposes we present the marginal utility relaxation for the income
parameter. The appendix shows the relaxation formulations for the other parameters. By relaxing
the assumption of constant marginal utility, the first order Taylor expansion is applied for the
multiplier $\theta_i$ which itself refers to the marginal utility of the late arrival for travel option $i$.

Equations (8a) through (8i) represent the derivation for this specific multiplier.

$$\frac{\partial V_i}{\partial (t_o + SDL)} = -l_i \theta_i \approx -l_i$$

$$-\frac{\partial \theta_i}{\partial (p_i - R)} = \frac{\partial^2 V_i}{\partial (t_o + SDL) \partial (p_i - R)} = \frac{\partial \lambda}{\partial (t_o + SDL)} = -c_1$$

$$-\frac{\partial \theta_i}{\partial t_{i_{min}}} = \frac{\partial^2 V_i}{\partial (t_o + SDL) \partial t_{i_{min}}} = \frac{\partial k_i}{\partial (t_o + SDL)} = -c_2$$

$$-\frac{\partial \theta_i}{\partial (t_o - SDL)} = \frac{\partial^2 V_i}{\partial (t_o - SDL)^2} = -c_3$$

$$-\frac{\partial \theta_i}{\partial T} = \frac{\partial^2 V_i}{\partial (t_o + SDL) \partial T} = -\frac{\partial \mu}{\partial (t_o + SDL)} = -c_4$$

$$-l_i \theta_i \approx -c_o - c_1 (p_i - R) (\overline{p}_i - \overline{R}) - c_2 (t_{i_{min}} - \overline{t}_{i_{min}}) - c_3 (t_o + SDL) - (\overline{t}_o + SDL) - c_4 (T - \overline{T})$$

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Equation (8g) shows the ultimate formulation for the marginal utility of late arrival. One can observe that the final form of the marginal utility is a function of price, income, available time, departure time, PAT and the actual amount of late arrival. This functional form enters the utility function if and only if late arrival is experienced, thus \( l_i \neq 1 \). Compared to the available functional forms in the literature, equation (8g) takes a number of different variables into account rather than just considering the amount of late arrival time. Although this brings additional complexity for practical calculations it could also improve the model’s predictive ability by accommodating a number of related factors in a functional form that could possibly affect the valuation of travel reliability. The coefficients associated with the model variables can be calculated using maximum likelihood methods.

Marginal utilities of available price, travel time, income and early arrival are derived using a similar methodology (see Appendix) and the relaxed marginal utilities are obtained as follows:

\[
\lambda_i \approx \lambda_i = \alpha_i(p_i - R) + \alpha_i t_i^{min} + \epsilon_i b_i(t_{oi} - SDE_i) + l_i c_i(t_{oi} + SDL_i) + \sigma_i T_i
\]

where,

\[
c_i = -c_i + c_i(p_i - \tilde{R}) + c_i t_i^{min} + c_i(t_{oi} - SDL_i) + c_i T_i
\]

\[
k_i \approx e_i + \alpha_i(p_i - R) + \epsilon_i t_i^{min} + \epsilon_i b_i(t_{oi} - SDE_i) + l_i c_i(t_{oi} + SDL_i) + \sigma_i T_i
\]

\[
\mu \approx \sigma_i - \sigma_i(p_i - R) - \sigma_i t_i^{min} - \epsilon_i b_i(t_{oi} - SDE_i) - l_i c_i(t_{oi} + SDL_i) + \sigma_i T_i
\]

Replacing the derived relaxed marginal utilities into equation (7c) we obtain the differential for a specific travel option \( i \) as given in equation (10a) below. Integrating the differential utility function with respect to appropriate variables, the indirect utility function for travel option \( i \) is given in Equation (10b).

\[
dV_i = -\lambda_i dp_i - \mu dt_i^{min} + \epsilon_i \left( dt_{oi} - dSDE_i \right) - \theta e_i \left( dt_{oi} + dSDL_i \right) - \theta \left( dt_{oi} + dSDL_i \right) - \theta l_i \left( dt_{oi} + dSDL_i \right) = \left( -\alpha_i(p_i - R) - \epsilon_i t_i^{min} - \epsilon_i b_i(t_{oi} - SDE_i) - l_i c_i(t_{oi} + SDL_i) - \sigma_i T_i \right) \left( dp_i - dR \right) + \left( -\epsilon_i - \sigma_i(p_i - R) - \sigma_i t_i^{min} - \epsilon_i b_i(t_{oi} - SDE_i) - l_i c_i(t_{oi} + SDL_i) + \sigma_i T_i \right) dt_i^{min}
\]

\[
+ e_i b_i(p_i - R) - b_i t_i^{min} - e_i b_i(t_{oi} - SDE_i) - b_i T_i \left( dt_{oi} - dSDE_i \right) + e_i b_i(p_i - R) - b_i t_i^{min} - e_i b_i(t_{oi} - SDE_i) - b_i T_i \left( dt_{oi} + dSDL_i \right)
\]

\[
+ l_i \left( -\alpha_i(p_i - R) - \epsilon_i t_i^{min} - \epsilon_i b_i(t_{oi} - SDE_i) - c_i T_i \right) \left( dt_{oi} + dSDL_i \right)
\]
The utility of a specific travel choice \( i \), which is considered to be the departure time choice in this study as given in equation (10b), increases with increasing income and decreasing travel time, travel cost, deviation from PAT, and departure time. Please note that the obtained utility formulation reduces to Ozbay and Ynamaz-Tuzel (10) when considering only early arrivals (set \( e_i = 1 \) and \( l_i = 0 \)). Presented form of utility function includes second-order income term which was analytically shown to have a risk of hiding income effects when an alternate policy is tested (30, 31, 32). Blayac and Causse (4) excluded the time and income terms from a similar non-linear utility form mentioning that these terms are independent of the choice of travel mode. Empirical analysis presented in Herriges and Kling (1999) showed that the welfare estimates are not substantially different with or without nonlinear income considerations although these results can be data specific and cannot be easily generalized (31). Provided non-linear formulation in this paper can also help empirically analyze the income and time effects in random utility models.

Finally, the value of travel time reliability can be obtained with the ratio of the partial derivative of the utility function with respect to schedule delay parameters \( (SDE_i, e_i = 1; SDL_i, l_i = 1) \) and the partial derivative of the utility function with respect to travel cost.

\[
\frac{\partial V_i}{\partial SDE_i} = e_i \left( \frac{b_i R + b_i p_i + b_i t_{om} + b_i t_{o} + b_i T - b_i t_{om}}{(2 \alpha_i) R - 2 \alpha_i t_{om} - \alpha_i - 2 \sigma_i T - 2 b_i t_{o} - \alpha_i p_i + b_i SDE_i} \right)
\]

\[
\frac{\partial V_i}{\partial SDL_i} = -l_i \left( \frac{c_i R + c_i p_i + c_i t_{om} + c_i t_{o} + c_i T + c_i p + c_i SDL_i}{(2 \alpha_i) R - 2 \alpha_i t_{om} - \alpha_i - 2 \sigma_i T - 2 b_i t_{o} - \alpha_i p_i + c_i SDL_i} \right)
\]

This next section gives a numerical example using revealed preference data for the presented methodology.

DATA DESCRIPTION

The data used for this case study is from a user survey collected in 2004 to estimate the value of travel time savings for regular New Jersey Turnpike (NJTPK) users. A comprehensive explanation of survey design, data collection method and descriptive data analysis can be found in Ozbay et al. (33). Respondents were asked questions about their most recent trip during morning and afternoon peak and therefore the results reflect a preference about departure time for each individual for a single trip. Trip related questions included origin, destination, travel time, toll, purpose of travel, departure time, deviations from preferred arrival time (i.e. SDE and SDL). The socioeconomic characteristics of the respondents such as income, education, and gender were also included in the final dataset.

For the time period that the data was collected, time-of-day pricing was being implemented throughout the NJTPK. This can be regarded as an effecting factor in the departure time choice. Ozbay et al. (33) also noted that 90% of the travelers using the facility during peak hours and peak
shoulders (pre-peak and post-peak) were driving passenger cars with electronic toll collection transponders.

The original dataset includes 483 observations. For modeling purposes we only used weekday trips for both cash and electronic toll collection transactions; weekend trips were not used for analysis. Data points with missing information about the selected explanatory variables were eliminated and 142 observations were included in the analysis. Each observation is a combination of responses from a single person participated in the survey. Annual income level is almost uniformly distributed over the final set of respondents which reduces the risk of missing income effects with possible non-linear income terms in the utility formulation.

ESTIMATION

This section presents the estimation results for the scheduling model we developed using marginal utility relaxation. The model used in the estimation is a Multinomial Logit (MNL) model. For the dependent variable departure we used time choice, and the options were grouped in pre-peak, post-peak or peak. Pre-peak and post-peak periods were considered as the peak shoulders to reflect the time periods when risk-averse users tend to depart with the intention of avoiding a late or early arrival or to avoid peak period congestion. All parameters included in equation (10b) are used for estimation and the ones yielding the best estimation results were kept in the final model. Essentially similar to all scheduling models the user-specific utility function was defined depending on the result of the trip. More specifically, if a late arrival or early arrival was observed then the valuation of the reliability was calculated for an individual. For the interaction of the independent variables included in the final model, multicollinearity tests were conducted and no statistically significant problems were observed.

Estimation results for the MNL departure time choice model taking peak period departure choice as the base case is summarized in Table 1. For both models second-order time and toll parameters were observed to have an effect on the overall utility similar to previous studies (10, 33).

Table 1: Estimation Results for the MNL model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-Peak(^a)</th>
<th></th>
<th>Post-Peak(^b)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>P&gt;</td>
<td>z</td>
<td></td>
</tr>
<tr>
<td>Toll (S)</td>
<td>-1.140</td>
<td>0.205</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Travel Time (hr)</td>
<td>-0.088</td>
<td>0.088</td>
<td>-0.092</td>
<td>0.054</td>
</tr>
<tr>
<td>Early Arrival (hr)</td>
<td>-0.214</td>
<td>0.040</td>
<td>-0.036</td>
<td>0.051</td>
</tr>
<tr>
<td>Late Arrival (hr)</td>
<td>-0.325</td>
<td>0.058</td>
<td>-0.089</td>
<td>0.203</td>
</tr>
<tr>
<td>Toll(^2)</td>
<td>-0.210</td>
<td>0.109</td>
<td>-0.029</td>
<td>0.163</td>
</tr>
<tr>
<td>Time(^2)</td>
<td>-1.753</td>
<td>0.120</td>
<td>-1.642</td>
<td>0.105</td>
</tr>
<tr>
<td>(Early Arrival)x(Income)</td>
<td>-0.036</td>
<td>0.029</td>
<td>-0.002</td>
<td>0.159</td>
</tr>
<tr>
<td>(Late Arrival)x (Toll)</td>
<td>-0.056</td>
<td>0.143</td>
<td>-1.495</td>
<td>0.048</td>
</tr>
<tr>
<td>(Late Arrival)x (Income)</td>
<td>-0.016</td>
<td>0.226</td>
<td>-0.006</td>
<td>0.217</td>
</tr>
<tr>
<td>Pseudo, (r^2)</td>
<td>0.200</td>
<td></td>
<td>0.190</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Pre-peak: 6-7 AM and 3:30-4:30PM.
\(^b\)Post-peak: 9-10 AM and 6:30-7:30 PM
Peak: 7-9 AM and 4:30-6:30 PM.
The results given in Table 1 have some intuitive implications. The effects of SDE and SDL in utility function were considerably higher for the pre-peak departure case, which means that the travelers had a high tendency to depart before the peak period to avoid possible schedule delays. For the post-peak case the effect of the toll on the utility was found to be insignificant so the toll parameter was not included as an independent variable. The effect of the toll was included in the post-peak function using interaction variables. As expected due to almost uniformly distributed income levels in the final dataset neither first-order nor second-order income term was found to be significant and thus not included in the empirical utility formulation. Using Equation (10c) user-specific value of reliability was calculated for each respondent. The mean values are shown in Table 2. According to this calculation, the pre-peak period had higher means for the value of the reliability compared to the post-peak period. This can be explained by the fact that peak-period NJTPK travelers with tight schedules were willing to pay higher amounts for a more reliable travel option.

### Table 2: Value of Reliability Estimations

<table>
<thead>
<tr>
<th>Value of Reliability (Reliability Ratio)</th>
<th>Mean VSDE</th>
<th>Mean VSDL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Peak</td>
<td>$10.3 (0.6)</td>
<td>$31.4 (1.85)</td>
</tr>
<tr>
<td>Post-Peak</td>
<td>$8.45 (0.53)</td>
<td>$13.7 (0.86)</td>
</tr>
</tbody>
</table>

To measure the consistency of the mean VOR, estimations for NJTPK users were obtained in this study by using the existing literature’s reliability ratio, which is defined as the ratio of the value of reliability to the value of travel time. We did not estimate the value of time in this study however Ozbay and Yanmaz-Tuzel (10) provided estimations using the same dataset for pre-peak and post-peak periods, as $17/hr and $16/hr respectively. According to this, reliability ratios are given in parenthesis for each of the estimations in Table 2. The calculated ratios are within the range of previous numbers, which vary from 0.1 to 2.51 (35).

It should be noted that, although the dataset represents the actual behavior of the respondents regarding their most recent trip, the systematic limitations of stated preference surveys still hold for the presented analysis. For example, Borjesson et al. (5) mentions the “focus bias” of the respondent’s unconscious overstating or understating by simply forgetting the important characteristics of a trip. A combination of stated-preference and revealed-preference data could give a better explanation of the driver behavior by using the advantages of the tractability of the former and the realism of the latter (35).

### CONCLUSION

We have presented a time-allocation model-based methodology for the valuation of travel time reliability. The previous literature on the value of reliability estimations using scheduling delays are built upon Small’s study (1), which provided a utility formulation based on empirical analysis. We provided an analytical approach to determine the parameters to be included in the utility function. One important feature of the developed methodology is the relaxation of the constant marginal utility assumption in the original model, which is regarded as an economical restriction (4, 5, 10, 27). Ultimately we obtained a non-linear utility function that is used to estimate the valuation of travel times by calculating the ratio of the marginal utility of schedule delay to the
marginal utility of travel cost. The unique feature of the obtained formulation is such that, while measuring the value assigned to the reliability, it takes into account not only the travel time deviations from preferred arrival time but also other affecting variables. Moreover, the resulting value of the reliability is user-specific rather than a single value calculated using constant marginal utilities for the entire population.

We tested the analytical VOR functions using the dataset collected for NJTPK users in the presence of time-of-day pricing. The estimation results using a MNL model for departure time choice showed that a higher VOR is observed for users who decide to depart at pre-peak periods compared to the ones who depart at post-peak periods. The difference in VORs can be explained with the risk-averse behavior of the early departures at pre-peak period who value reliability more than post-peak departures.

In this paper we showed a numerical example based on observed single trips per respondent. The presented methodology can be further tested for empirical evidence using emerging data collection technologies such as GPS-probe vehicles. Observations for a certain time-period for similar users can give more reasonable explanations about the relationship between reliability and departure time choice behavior. Another important research venue is to test the equivalency of scheduling and mean-variance approaches using the provided functional form.

**DISCLAIMER**
The contents of this paper only reflect views of the authors who are responsible for the facts and accuracy of the data and results presented herein.

**APPENDIX**

**A1. Marginal Utility of Departure Time and Early Arrival**
The multiplier, $\theta$, in Equation (4c), refers to the marginal utility of departure time. By relaxing the assumption of constant marginal utility, first order Taylor expansion is applied for the multiplier, $\theta$. Equations (A1.a) through (A1.h) represent the derivation for this specific multiplier.

\[
\frac{\partial V_i}{\partial \left( t_i - SDE_i \right)} = -e \theta_i \approx -e \left( b_i + \frac{\partial \theta_i}{\partial (p_i - R)} \left( (p_i - R) - (\overline{p}_i - \overline{R}) \right) + \frac{\partial \theta_i}{\partial \min(t_i)} \left( (t_i - \min(t_i)) + \frac{\partial \theta_i}{\partial T} (T - \overline{T}) \right) + \frac{\partial \theta_i}{\partial (\tau_i - SDE_i)} \left( (t_i - SDE_i) - (\overline{t}_i - \overline{SDE}_i) \right) \right)
\]  
(A.1a)

\[
\frac{\partial \theta_i}{\partial (p_i - R)} = \frac{\partial \lambda}{\partial (t_i - SDE_i)} = \frac{\partial \lambda}{\partial (t_i - SDE_i)} = -b_1
\]  
(A.1b)

\[
\frac{\partial \theta_i}{\partial \min(t_i)} = \frac{\partial \lambda}{\partial (t_i - SDE_i)} = \frac{\partial \lambda}{\partial (t_i - SDE_i)} = -b_2
\]  
(A.1c)

\[
\frac{\partial \theta_i}{\partial (t_i - SDE_i)} = \frac{\partial \lambda}{\partial \tau_i} = \frac{\partial \lambda}{\partial \tau_i} = -b_3
\]  
(A.1d)

\[
\frac{\partial \theta_i}{\partial T} = \frac{\partial \lambda}{\partial (t_i - SDE_i)} = \frac{\partial \lambda}{\partial (t_i - SDE_i)} = -b_4
\]  
(A.1e)
The multiplier, \( \mu \), in Equation (4c), refers to the marginal utility of available time. Even if, the available time is fixed within a given period of time, the distribution of time between work/leisure activities and travel will affect the individual’s utility. By relaxing the assumption of constant marginal utility, first order Taylor expansion is applied for the multiplier, \( \mu \). Equations (A.2a) through (A.2i) represent the derivation for this specific multiplier.

\[
\frac{\partial V}{\partial T} = \mu \approx \left[ \sigma_o + \frac{\partial \mu}{\partial (p_i - R)} ((p_i - R) - (\bar{p}_i - \bar{R})) + \frac{\partial \mu}{\partial t_{min}^i} (t_{min}^i - \bar{t}_{min}^i) + \frac{\partial \mu}{\partial T} (T - \bar{T}) \right]
\]

(A.2a)

\[
\frac{\partial \mu}{\partial (p_i - R)} = \frac{\partial^2 V}{\partial T \partial (p_i - R)} = \frac{\partial \lambda}{\partial T} = \sigma_i
\]

(A.2b)

\[
\frac{\partial \mu}{\partial t_{min}^i} = \frac{\partial^2 V}{\partial T \partial t_{min}^i} = \frac{\partial \kappa_i}{\partial T} = \sigma_2
\]

(A.2c)

\[
\frac{\partial \mu}{\partial T} = \frac{\partial^2 V}{\partial T^2} = -\sigma_3
\]

(A.2d)

\[
\frac{\partial \mu}{\partial (t_{min}^i - SDE)} = \frac{\partial^2 V}{\partial T \partial (t_{min}^i - SDE)} = \frac{\partial \theta_i}{\partial T} = -b_i
\]

(A.2e)

\[
\frac{\partial \mu}{\partial (t_{min}^i + SDL)} = \frac{\partial^2 V}{\partial T \partial (t_{min}^i + SDL)} = \frac{\partial \phi_i}{\partial T} = -c_i
\]

(A.2f)

\[
\mu \approx \left[ \sigma_o + \sigma_i ((p_i - R) - (\bar{p}_i - \bar{R})) + (\sigma_2) (t_{min}^i - \bar{t}_{min}^i) - \sigma_3 (T - \bar{T}) \right]
\]

\[
- e b_i \left( t_{min}^i - SDE_i \right) - \sigma_i \left( t_{min}^i + SDL_i \right) - \left( t_{min}^i + SDL_i \right)
\]

(A.2g)

where,

\[
\sigma_o = -\sigma_o + \sigma_i (p_i - R) + \sigma_2 t_{min}^i + e b_i \left( t_{min}^i + SDL_i \right) + l e_i \left( t_{min}^i + SDL_i \right) - \sigma_i \bar{T}
\]

(A.2h)
A3. Marginal Utility of Travel Time

The multiplier, \( k_i \), in Equation (4c), refers to the marginal utility of minimal travel time. By relaxing the assumption of constant marginal utility, first order Taylor expansion is applied for the multiplier, \( k_i \). Equations (A.3a) through (A.3i) represent the derivation for this specific multiplier.

\[
\frac{\partial V}{\partial t_i^\text{min}} = -k_i \approx - \left[ e_i + \frac{\partial k_i}{\partial (p_i - R)} \left( (p_i - R) - (\bar{p}_i - \bar{R}) \right) + \frac{\partial k_i}{\partial t_i^\text{min}} \left( t_i^\text{min} - \bar{t}_i^\text{min} \right) + \frac{\partial k_i}{\partial T} \left( T - \bar{T} \right) \right]
\]

(A.3a)

\[
- \frac{\partial k_i}{\partial (p_i - R)} = \frac{\partial^3 V}{\partial t_i^\text{min}^2 \partial (p_i - R)} = - \frac{\partial \lambda}{\partial t_i^\text{min}} = -\alpha_2
\]

(A.3b)

\[
- \frac{\partial k_i}{\partial t_i^\text{min}} = \frac{\partial^3 V}{\partial t_i^\text{min}^2} = -\epsilon_i
\]

(A.3c)

\[
- \frac{\partial k_i}{\partial T} = \frac{\partial^3 V}{\partial T \partial t_i^\text{min}} = - \frac{\partial \theta}{\partial t_i^\text{min}} = -\sigma_2
\]

(A.3d)

\[
\frac{\partial k_i}{\partial (t_i - SDE_i)} = \frac{\partial^3 V}{\partial t_i^\text{min} \partial (t_i - SDE_i)} = \frac{\partial \theta}{\partial t_i^\text{min}} = b_2
\]

(A.3e)

\[
\frac{\partial k_i}{\partial (t_i + SDL_i)} = \frac{\partial^3 V}{\partial t_i^\text{min} \partial (t_i + SDL_i)} = \frac{\partial \varphi}{\partial t_i^\text{min}} = c_2
\]

(A.3f)

\[
-k_i \approx - \left[ e_i + \alpha_2 \left( (p_i - R) - (\bar{p}_i - \bar{R}) \right) + (\epsilon_i) \left( t_i^\text{min} - \bar{t}_i^\text{min} \right) + \sigma_2 \left( T - \bar{T} \right) \right]
\]

(A.3g)

\[
\approx \hat{e}_i + \alpha_2 (p_i - R) + \epsilon_i t_i^\text{min} + \epsilon_i b_2 (t_i - SDE_i) + \epsilon_i c_2 (t_i + SDL_i) + \sigma_2 T
\]

(A.3h)

where,

\[
\hat{e}_i = e_i - \alpha_2 (p_i - R) - \epsilon_i t_i^\text{min} - \epsilon_i b_2 (t_i - SDE_i) - \epsilon_i c_2 (t_i + SDL_i) - \sigma_2 T
\]

(A.3i)

A4. Marginal Utility of Income

The multiplier, \( \lambda \), in Equation (4c), refers to the marginal utility of minimal travel time. By relaxing the assumption of constant marginal utility, first order Taylor expansion is applied for the multiplier, \( \lambda \). Equations (A.4a) through (A.4i) represent the derivation for this specific multiplier.
\[
\frac{\partial V}{\partial (p_i - R)} = -\lambda \approx -\left( \alpha_e + \frac{\partial \lambda}{\partial (p_i - R)} \left( (p_i - R) - \bar{(p_i - R)} \right) + \frac{\partial \alpha}{\partial \alpha} t_{\text{min}} - \bar{t}_{\text{min}} \right) + \frac{\partial \lambda}{\partial \alpha} \left( T - \bar{T} \right) \\
+ e \lambda \left( (p_i - SDE_r) - \bar{(p_i - SDE_r)} \right) + I \left( (p_i + SDL_i) - \bar{(p_i + SDL_i)} \right) \\
+ \frac{\partial \lambda}{\partial \alpha} \left( t_{\text{SDL}} - \bar{t}_{\text{SDL}} \right) + \frac{\partial \lambda}{\partial \alpha} \left( T - \bar{T} \right)
\] (A.4a)

\[
- \frac{\partial \lambda}{\partial (p_i - R)} = \frac{\partial^2 V}{\partial (p_i - R)^2} = -\alpha_e
\] (A.4b)

\[
- \frac{\partial \lambda}{\partial t_{\text{min}}} = \frac{\partial^2 V}{\partial t_{\text{min}}^2} = \frac{\partial \alpha}{\partial \alpha} = -\alpha_e
\] (A.4c)

\[
- \frac{\partial \lambda}{\partial T} = \frac{\partial^2 V}{\partial T^2} = \frac{\partial \mu}{\partial (p_i - R)} = \sigma_e
\] (A.4d)

\[
\frac{\partial \lambda}{\partial (p_i - SDE_r)} = \frac{\partial^2 V}{\partial (p_i - R)(p_i - SDE_r)} = \frac{\partial \theta}{\partial (p_i - R)} = -b_e
\] (A.4e)

\[
\frac{\partial \lambda}{\partial (p_i + SDL_i)} = \frac{\partial^2 V}{\partial (p_i - R)(p_i + SDL_i)} = \frac{\partial \theta}{\partial (p_i - R)} = -c_e
\] (A.4f)

\[
\lambda \approx \left( \alpha_e + \alpha_e \left( (p_i - R) - \bar{(p_i - R)} \right) + \left( \alpha_e \right) \left( t_{\text{min}} - \bar{t}_{\text{min}} \right) + \sigma_e \left( T - \bar{T} \right) \right)
\] (A.4g)

\[
\lambda \approx \left( \alpha_e + \alpha_e \left( (p_i - R) - \bar{(p_i - R)} \right) + \left( \alpha_e \right) \left( t_{\text{min}} - \bar{t}_{\text{min}} \right) + \sigma_e \left( T - \bar{T} \right) \right)
\] (A.4h)

where,

\[
- \alpha_e = \alpha_e - \alpha_e \left( (p_i - R) - \bar{(p_i - R)} \right) + \left( \alpha_e \right) \left( t_{\text{min}} - \bar{t}_{\text{min}} \right) + \sigma_e \left( T - \bar{T} \right)
\] (A.4i)

1 REFERENCES


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