THE REFLECTANCE MAP AND SHAPE-FROM-SHADING

http://www.cs.jhu.edu/~wolff/course600.461/week9.3/index.htm

REFLECTANCE MODELS

LAMBERTIAN MODEL

$$E = L\,\rho\,\mathrm{COS}\,\theta$$

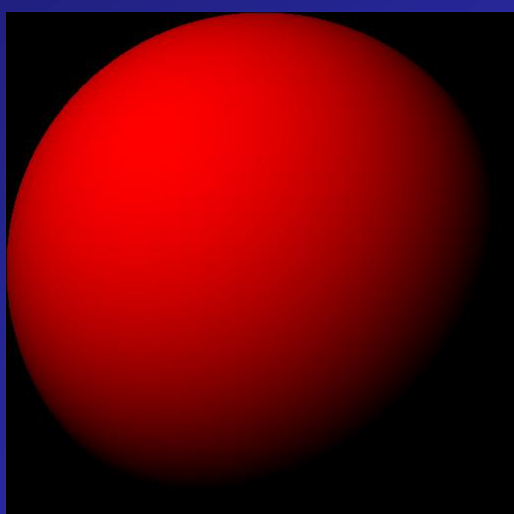
albedo

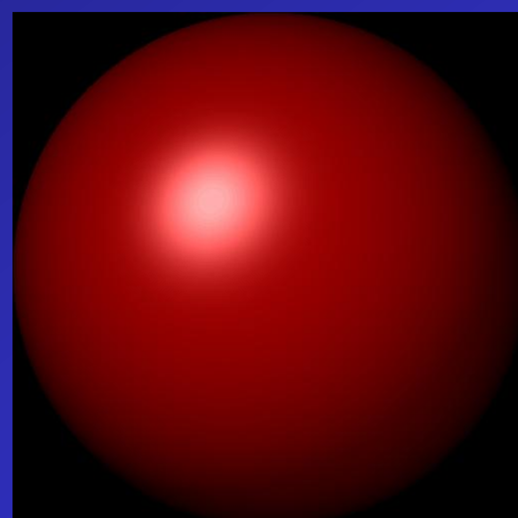
PHONG MODEL

$$E = L\,(a\,\mathrm{COS}\,\theta + b\,\mathrm{COS}^{n}\,\alpha)$$

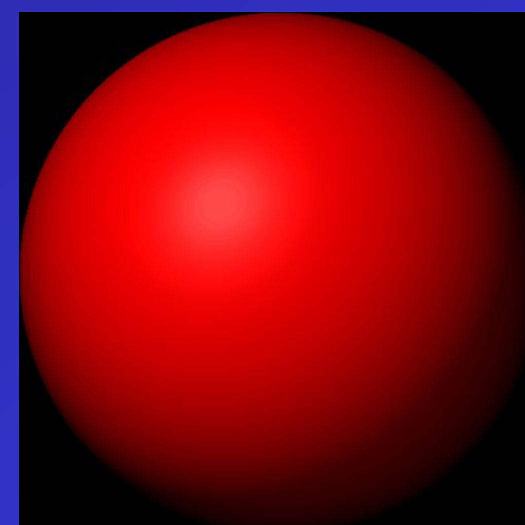
Diffuse albedo

Specular albedo





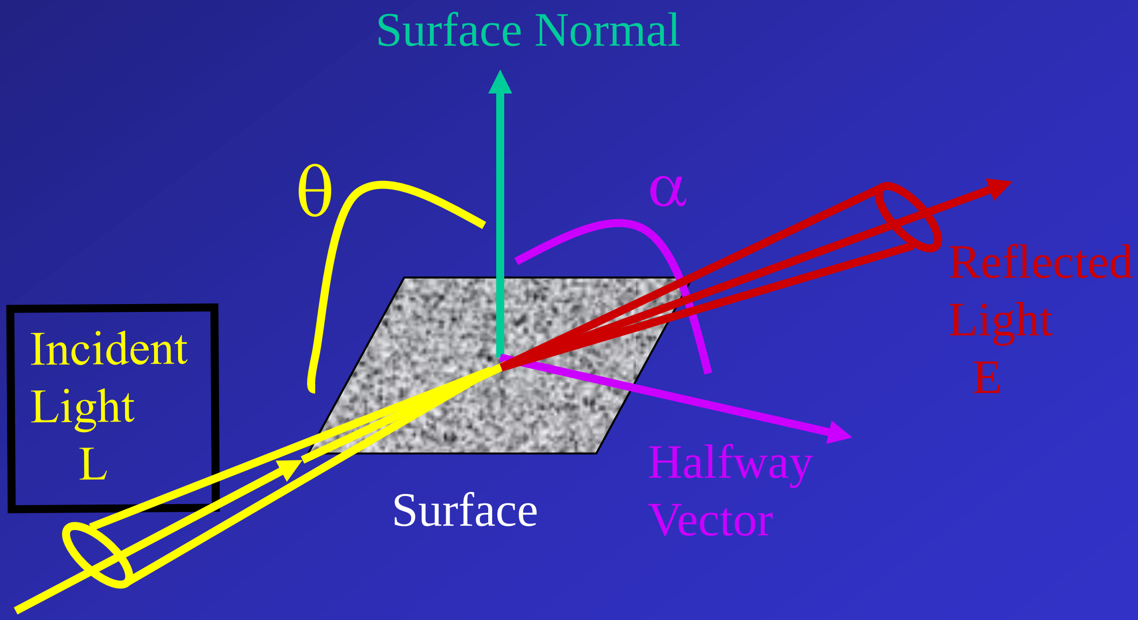
a=0.3, b=0.7, n=2



a=0.7, b=0.3, n=0.5

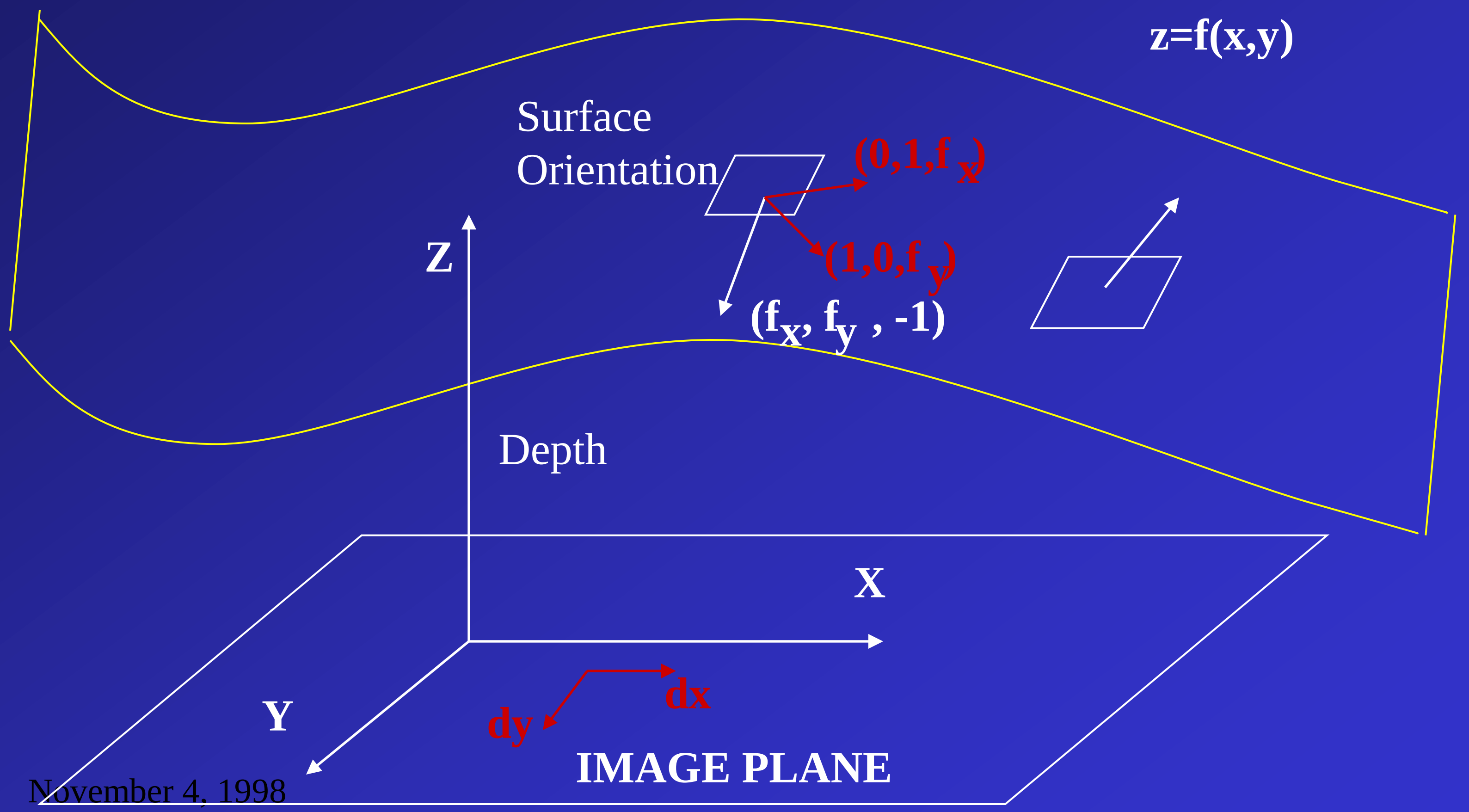
REFLECTANCE MODELS

- Description of how light energy incident on an object is transferred from the object to the camera sensor



REFLECTANCE MAP IS A VIEWER-CENTERED REPRESENTATION OF REFLECTANCE

$$(f_x, f_y, -1) = (0,1,f_x) \times (1,0,f_y)$$



REFLECTANCE MAP IS A VIEWER-CENTERED REPRESENTATION OF REFLECTANCE

$$(f_x, f_y, -1) = (p, q, -1)$$

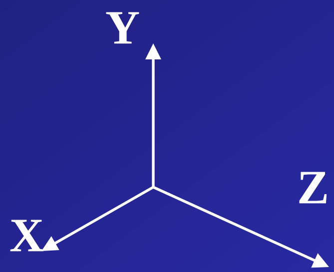
p, q comprise a gradient or gradient space representation for local surface orientation.

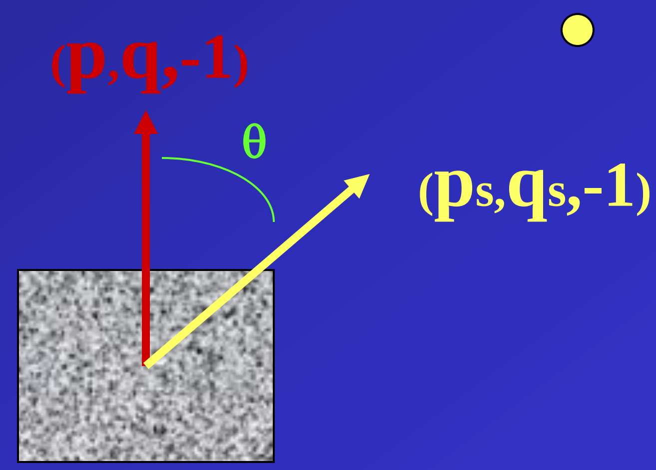
Reflectance map expresses the reflectance of a material directly in terms of viewer-centered representation of local surface orientation.

LAMBERTIAN REFLECTANCE MAP

LAMBERTIAN MODEL

$$E = L \rho \, COS \, \theta$$





$$COS\theta = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s{}^2 + q_s{}^2}}$$

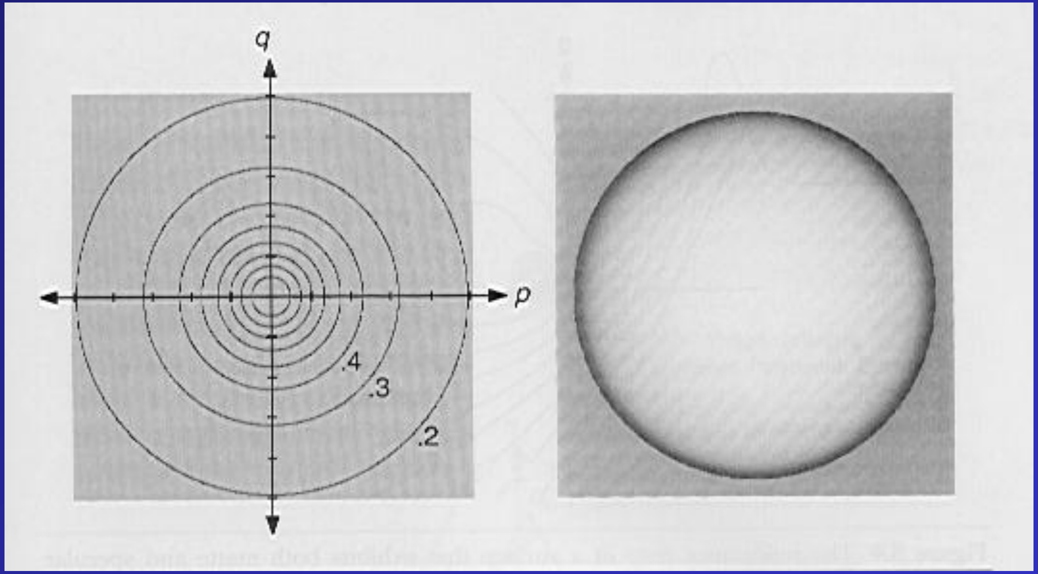
LAMBERTIAN REFLECTANCE MAP

$$E = L\rho \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s^{\ 2} + q_s^{\ 2}}}$$

Grouping L and ρ as a constant , local surface orientations that produce equivalent intensities under the Lambertian reflectance map are quadratic conic section contours in gradient space.

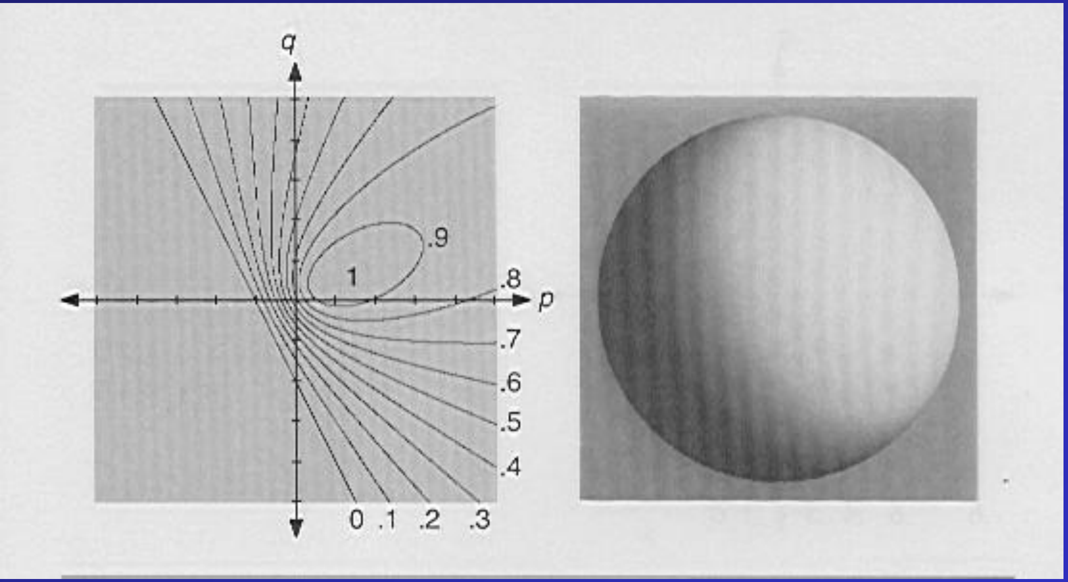
$$I = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s^{\ 2} + q_s^{\ 2}}}$$

LAMBERTIAN REFLECTANCE MAP



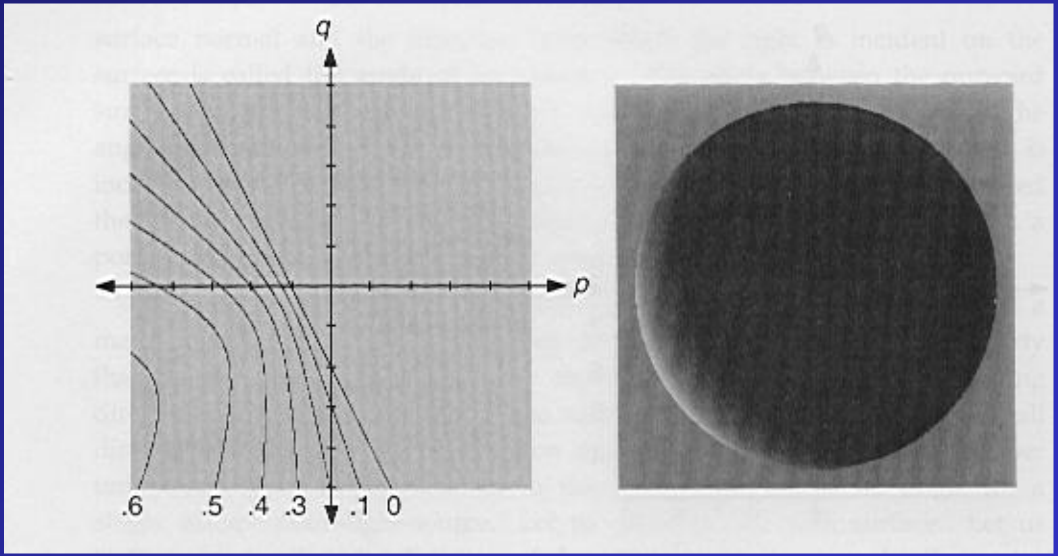
$p_s=0 \quad q_s=0$

LAMBERTIAN REFLECTANCE MAP



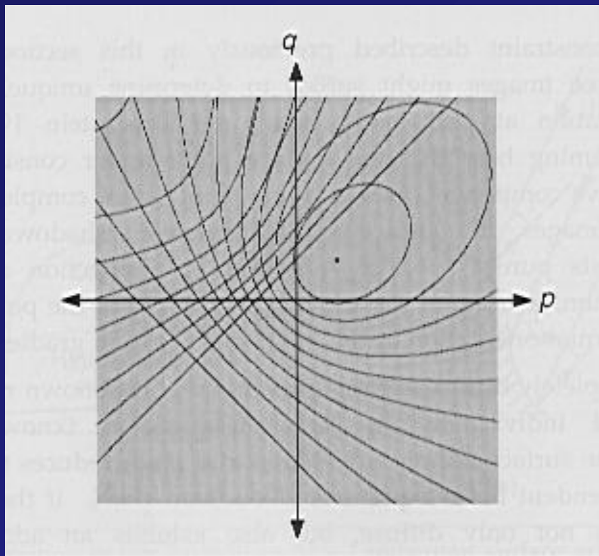
p_s=0.7 q_s=0.3

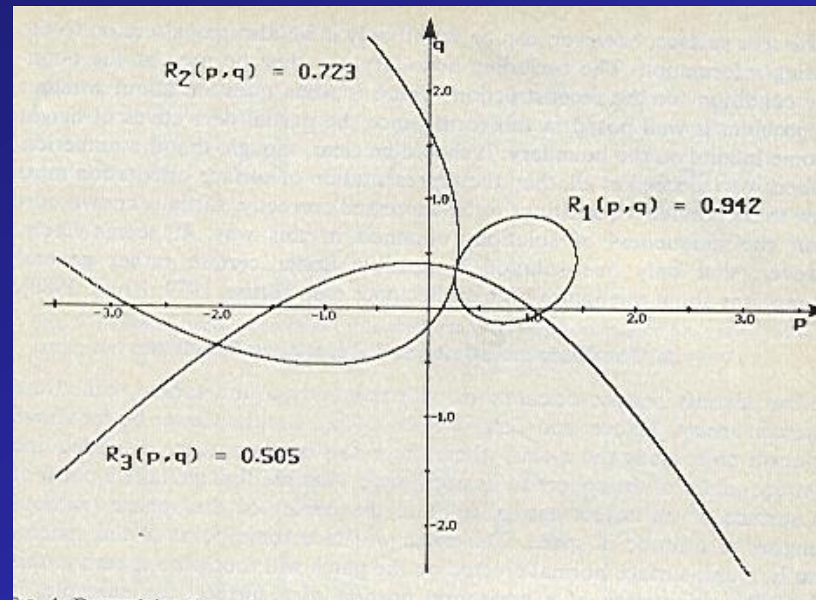
LAMBERTIAN REFLECTANCE MAP



$p_s = -2 \quad q_s = -1$

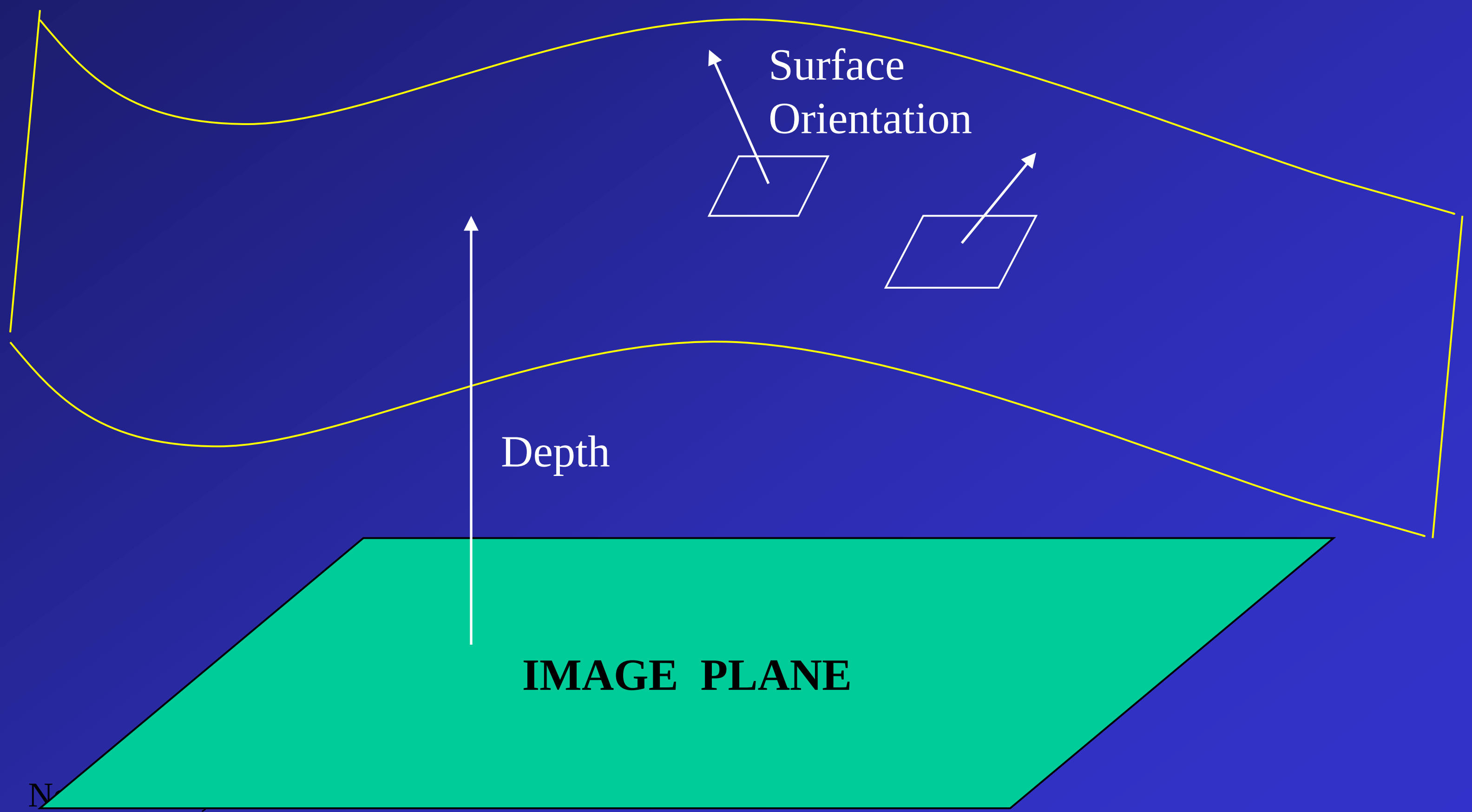
PHOTOMETRIC STEREO





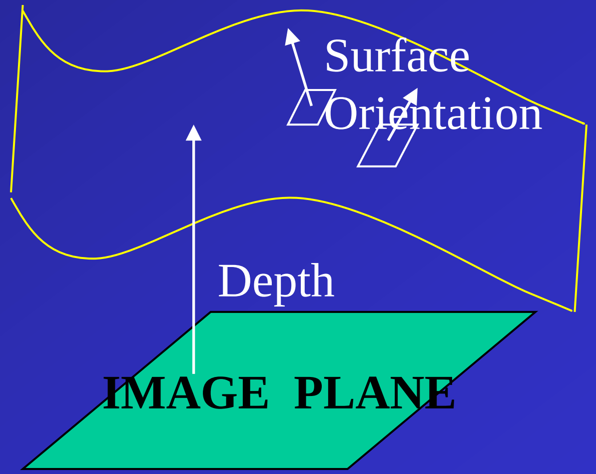
Derivation of local surface normal at each pixel creates the derived normal map.

NORMAL MAP vs. DEPTH MAP



NORMAL MAP vs. DEPTH MAP

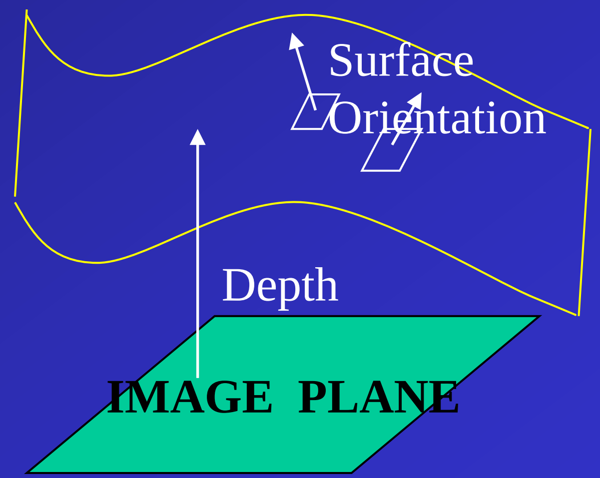
- Can determine Depth Map from Normal Map by integrating over gradients p,q across the image.
- Not all Normal Maps have a unique Depth Map. This happens when Depth Map produces different results depending upon image plane direction used to sum over gradients.
- Particularly a problem when there are errors in the Normal Map.



NORMAL MAP vs. DEPTH MAP

- A Normal Map that produces a unique Depth Map independent of image plane direction used to sum over gradients is called integrable.
- Integrability is enforced when the following condition holds:

$$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$$



NORMAL MAP vs. DEPTH MAP

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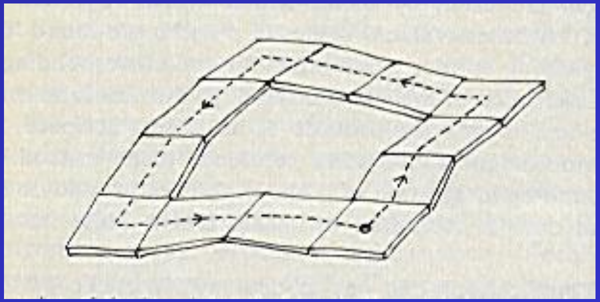
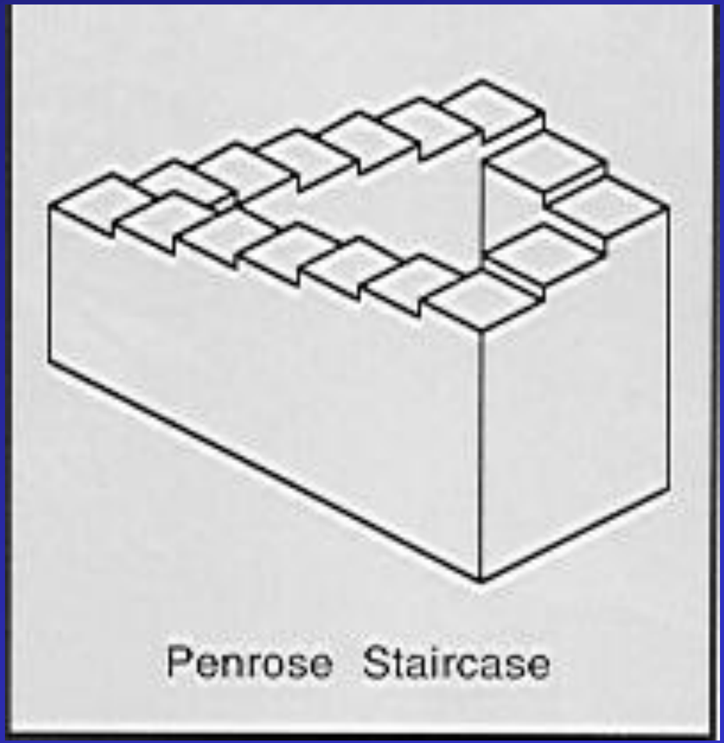
GREEN'S THEOREM

$$\iint (\partial p / \partial y - \partial q / \partial x) dxdy = \oint (pdx + qdy)$$



NORMAL MAP vs. DEPTH MAP

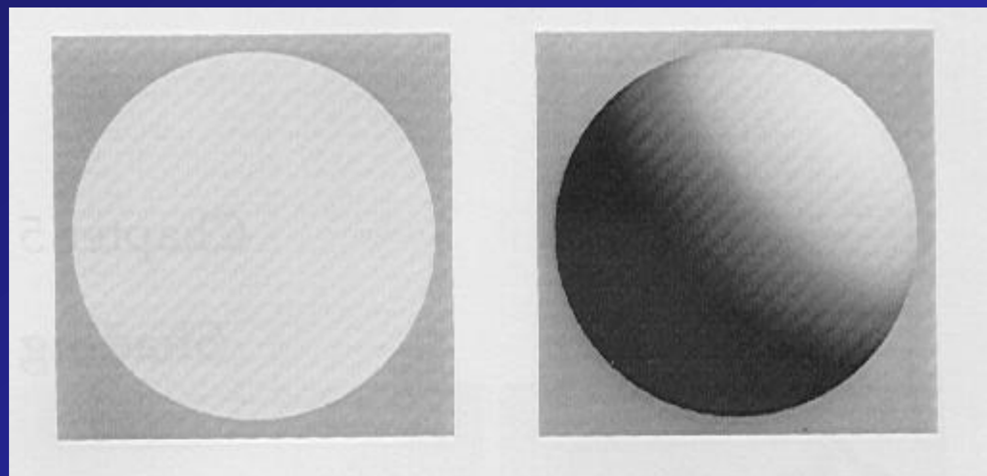
VIOLATION OF INTEGRABILITY



SHAPE FROM SHADING

CONSTANT INTENSITY

SHADING FROM LAMBERTIAN REFLECTANCE



From a monocular view with a single distant light source of known incident orientation upon an object with known reflectance map, solve for the normal map.

SHAPE FROM SHADING

- Formulate as solving the Image Irradiance equation for surface orientation variables p,q:

$$\mathbf{I(x,y) \quad = \quad R(p,q)}$$

- Since this is underconstrained we can't solve this equation directly
- What do we do ??.

SHAPE FROM SHADING
(Calculus of Variations Approach)

- First Attempt: Minimize error in agreement with Image Irradiance Equation over the region of interest:

$$\iint_{object} (I(x,y) - R(p,q))^2 \, dxdy$$

SHAPE FROM SHADING (Calculus of Variations Approach)

- Better Attempt: Regularize the Minimization of error in agreement with Image Irradiance Equation over the region of interest:

$$\iint_{object} {p_x}^2 + {p_y}^2 + {q^2}_x + {q^2}_y + \lambda(I(x,y) - R(p,q))^2\, dxdy$$