G. Geris, Class Notes 3D CV

Nov. 3, 2009

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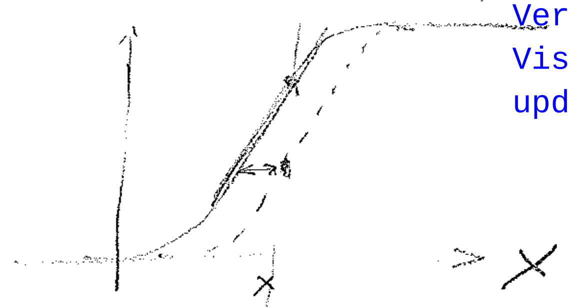
Optical Flow

Notation follows chapter 8 Trucco and Verri "3D Computer Vision" updated 4/11/2012

repetition:

1-D



slope $\frac{\partial E}{\partial x}$

gradient:

$$\frac{\Delta E}{\Delta x} = \frac{\partial E}{\partial x} = \tan\alpha$$

ΔE : observed brightness difference

motion

time? $\Delta x = v \cdot \Delta t$ | v: velocity

What can we measure?

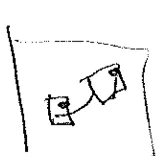
$\frac{\Delta E}{\Delta x}$: spatial image gradient

$E(x, t+\Delta t) - i(x,t)$: change of brightness at x due to motion

Brightness constancy?

$$\left(E\left(x + \frac{dx}{dt}, y + \frac{dy}{dx}, t + \delta t\right) = E(x, y, t) \right)$$

$E(x(t), y(t), t) = \text{constant}$: apparent brightness of moving object remains constant



$$\frac{dE}{dt} = \phi$$

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$E(x,t)$

chain rule

$$\frac{dE(x(t),y(t),t)}{dt} = E_x \frac{dx}{dt} + E_y \frac{dy}{dt} + E_t$$

total derivative

$$\frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} + \frac{\partial E}{\partial t} = 0$$

x, y fct of t

$$\nabla E = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$ image gradient

$$v = \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix}$$: velocity

spatial derivative

$\boxed{\text{Show slides I 60}}$

$$\Rightarrow \boxed{\nabla E \cdot v + E_t = 0}$$

Img brightness constancy equation

partial differentiation wrt time

⇒ ? measure ? : ∇E : image processing

E_t : " " : change of brightness at pixel (x,y)

⇒ solve for v

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vector vector scalar

each pixel: $\nabla E \cdot \mathbf{v} = -E_t$

Can we really easily determine $\mathbf{v}$?

$\rightarrow$ vote ?

$E_x u + E_y v = -E_t$ | 1 equation, 2 unknowns

$$\Rightarrow \quad u + \frac{E_y}{E_x} v = -\frac{E_t}{E_x}$$

$$u = -v \frac{E_y}{E_x} - \frac{E_t}{E_x}$$

| linear equation

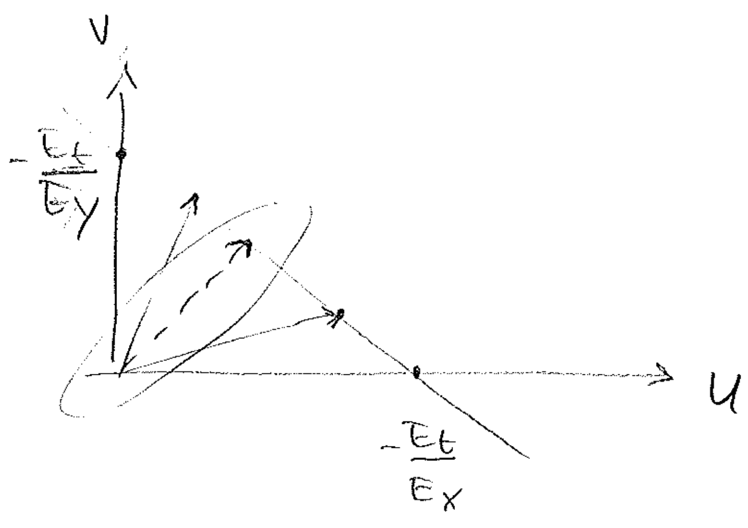
$v = -\frac{E_t}{E_y} - \frac{E_x}{E_y} u$

$v = \emptyset$

$\Rightarrow u = -\frac{E_t}{E_x}$

$u = \emptyset$

$\Rightarrow v = -\frac{E_t}{E_y}$



multiple solutions $\rightarrow$ which to pick ?

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normal flow: direction of normal !

$$\underbrace{\frac{\nabla E}{|\nabla E|}}_{\mathbf{n}} \cdot \mathbf{v} = -\frac{E_t}{|\nabla E|} \qquad |\nabla E| = \sqrt{E_x^2 + E_y^2}$$

normalized image gradient

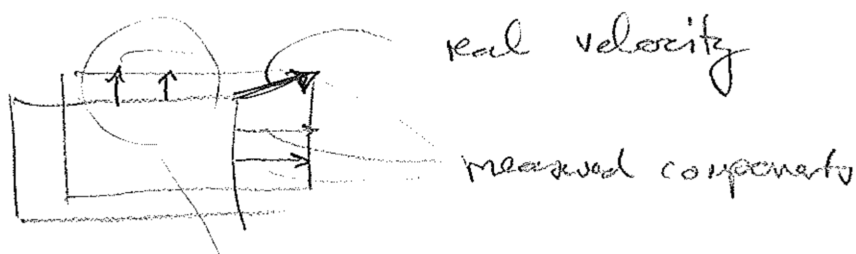
E_t: measured from image

$$\mathbf{n} \cdot \mathbf{v} = -\frac{E_t}{|\nabla E|} = v_n \quad \text{scalar}$$

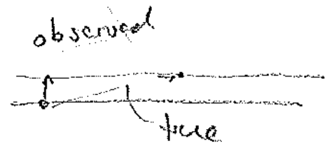
$\mathbf{n}$: normalized image gradient

$\mathbf{v}$: real velocity

v_n: velocity in normal direction !



⇒ aperture problem:



- flow in normal direction: can be determined
- flow parallel to boundary: cannot

($E_x u + E_y v + E_t = 0$

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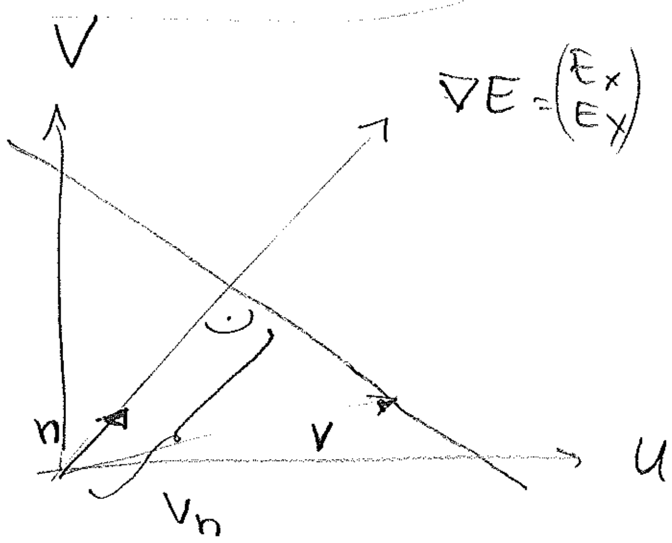


$\mathbf{v}_n = v_n \cdot \mathbf{n}$

normal flow

$\mathbf{v} \cdot \mathbf{n} = v_n$

projection of $\mathbf{v}$ onto direction $\mathbf{n}$

true for all $\mathbf{v}$'s on line

How?

$\mathbf{v}_n = \overbrace{(\mathbf{v}\cdot\mathbf{n})}^{v_n}\,\mathbf{n} = v_n \cdot \frac{\nabla E}{|\nabla E|}$

$= \frac{-E_t}{|\nabla E|} \cdot \frac{\nabla E}{|\nabla E|} = \left(\frac{-E_x E_t}{\nabla E^2}, \frac{-E_y E_t}{\nabla E^2} \right)^T$

Aperture problem:

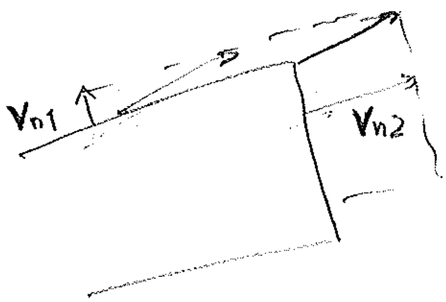
how to deal with this fundamental problem?

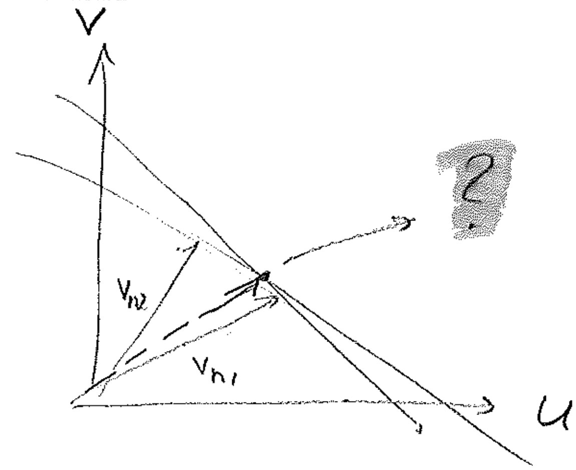
assumptions:

- object of finite size undergoing rigid motion
- velocities vary smoothly
- analysis over larger neighborhood

$\boxed{\text{Slide 27: I}}$

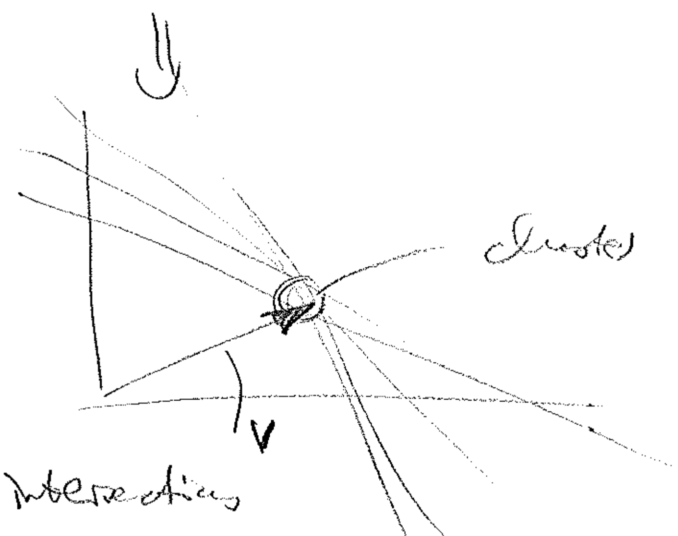
Possible solution: Neighbors





$V_{n1} = V_{n1} \cdot n_1$

$V_{n2} = V_{n2} \cdot n_2$



clustering of intersections

$\Rightarrow$ find $V = \binom{u}{v}$ for sets of points

$\boxed{\text{slide 38 I}}$

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Horn & Schunck:

smoothing:

given noisy estimates of

$(v_x, v_y) \longrightarrow$ apply smoothness constraints

$\boxed{\text{slides 42 ... 52: I}}$

$[((u_x)^2 + u_y^2 + v_x^2 + v_y^2)]$

$$\Rightarrow \iint_{\text{domain } D} (\nabla E \cdot v + E_t)^2 + \lambda^2 \left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_x}{\partial y}\right)^2 + \dots \right] dx\, dy$$

domain D: whole image

$(\nabla E \cdot v + E_t)^2$: close to $\emptyset$, OF constraint

$\lambda^2[\dots]$: smoothness

$\Rightarrow$ variational calculus:
iterative solution via differential equations.

results: slide 54 I

$\boxed{\text{slides 53-55 I}}$

show results in slides

Lucas & Kanade

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Slides II

similar to line fitting,
i.e. find common
solution over
patch !

- patch → single velocity
- Integrate over patch

Energy $E : \sum_{D} (u E_x + v E_y + E_t)^2$

E minimal:

$$\Rightarrow \frac{\partial E}{\partial u} = 0$$

$$\frac{\partial E}{\partial v} = 0$$

⇒ discretization (slide)

⇒ same spirit: smoothing, regularization

next: Structure from Motion:

from flow field in image to
3D motion of object !

Structure from Motion

Object

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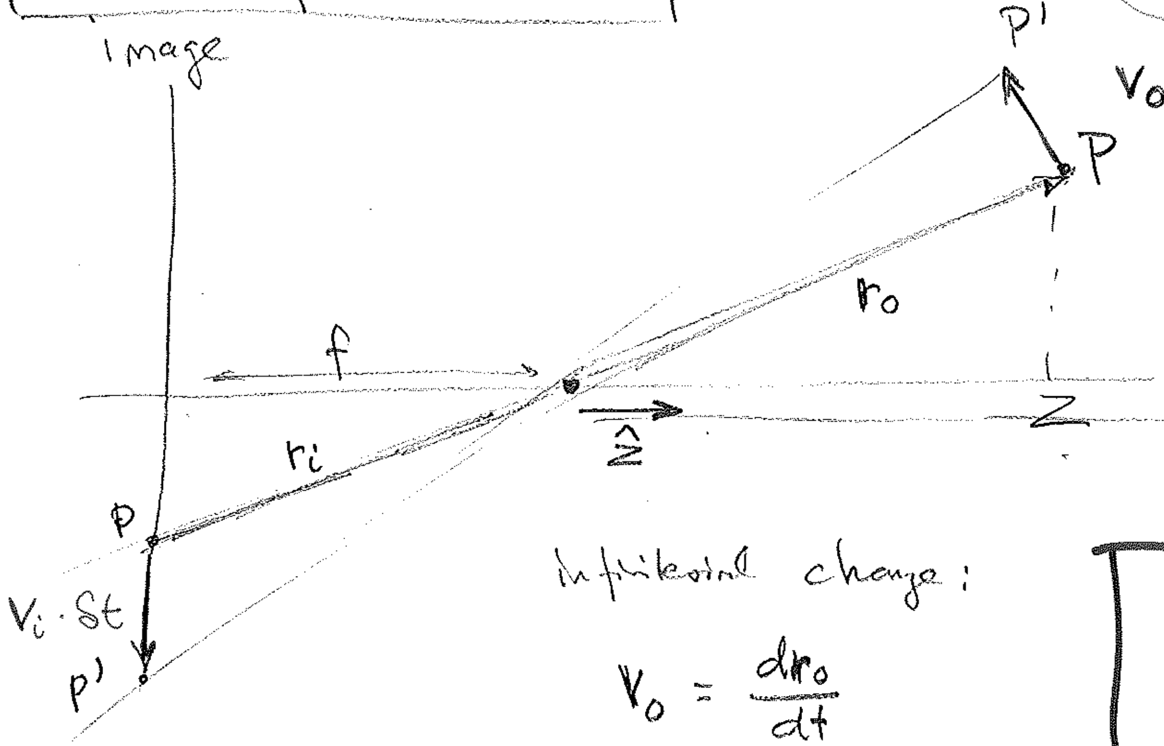
(Horn p. 280)



infinitesimal change:

$$V_o = \frac{dr_o}{dt}$$

$$V_i = \frac{dr_i}{dt}$$

slide 31 II

Projection:

$$\frac{r_i}{f} = \frac{r_o}{Z} = \frac{r_o}{r_o \cdot \underset{\hat{z}}{\hat{r}_z}} \quad \text{unit along } Z$$

$\hat{r}_z$: normal in Z direction $= \hat{Z}$

$$V_i = \frac{dr_i}{dt} = f \cdot \frac{d}{dt}\left(\frac{r_o}{r_o \cdot \underset{\hat{z}}{\hat{r}_z}}\right)$$

chain rule: $\left(\frac{1}{(fz)}g\right)' = \frac{g'}{fz} + \left(-\frac{f'z}{(fz)^2} \cdot g\right) \ldots$

$$\Rightarrow\Rightarrow \boxed{V_i = f \cdot \frac{\overset{\left(\frac{dr_o}{dt}\right)}{(r_o \times V_o)} \times \hat{Z}}{(r_o \cdot \hat{z})^2}}$$

comes out

perpendicular to r_o and V_o $\Rightarrow$

then perp. to $\hat{z}$

projection to img plane

18. Nov. 2008

Horn

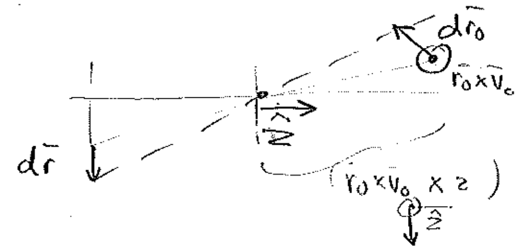
9b

$$\frac{1}{f'}\,\overline{r_i} = \left(\frac{1}{\overline{r_0}\cdot\hat{z}}\right)\cdot\overline{r_0}$$

$$\overline{V_i}\cdot\frac{1}{f'} = \frac{1}{f'}\,\frac{d\overline{r_i}}{dt} = \left(\left(\frac{1}{\overline{r_0}\cdot\hat{z}}\right)\cdot\overline{r_0}\right)'$$

$$\left(\left(\frac{1}{f\cdot z}\right)g\right)' = \frac{g'}{f\cdot z} - \frac{f'\cdot z}{(f\cdot z)^2}\cdot g$$

$$= \frac{\frac{d\overline{r_0}}{dt}}{\overline{r_0}\cdot\hat{z}} - \frac{\frac{d\overline{r_0}}{dt}\cdot z\cdot\overline{r_0}}{(\overline{r_0}\cdot\hat{z})^2}$$



$$= \frac{\overline{V_0}}{\overline{r_0}\cdot\hat{z}} - \frac{\overline{V_0}\cdot\hat{z}\cdot\overline{r_0}}{(\overline{r_0}\cdot\hat{z})^2}$$

Horn p.280

$$= \frac{(\overline{r_0}\cdot\hat{z})\cdot\overline{V_0} - (\overline{V_0}\cdot\hat{z})\,\overline{r_0}}{(\overline{r_0}\cdot\hat{z})^2} \quad ✓$$

$$\underbrace{\overline{V_i}\cdot\frac{1}{f'}}_{\frac{d\overline{r_i}}{dt}} = \boxed{\frac{(\overline{r_0}\times\overline{V_0})\times\hat{z}}{(\overline{r_0}\cdot\hat{z})^2}}$$

$\frac{d\overline{r_0}}{dt}$

perpendicular to $\overline{r_0}$ and $d\overline{r_0}$ ⇒ with $\hat{z}$! in image plane! ✓

$$\overline{V_0} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

$$\overline{V_i} = \begin{bmatrix} V_{ix} \\ V_{iy} \end{bmatrix}$$

sof: $\hat{z} = (0,0,z_n) = (0,0,1)$

$$\Rightarrow \overline{V_i}\cdot\frac{1}{f'} = \left(\frac{r_z V_x - r_x V_z,\; r_z V_y - r_y V_z, 0}{z^2},\; 0\right)$$

$\frac{r_z}{z} = 1$ $\qquad \frac{r_x\cdot f}{z} = x$

$$\Rightarrow \frac{V_{ix}}{f} = \frac{r_z V_{0x}}{z^2} - \frac{r_x V_{0z}}{z^2}$$

$$\frac{V_{iy}}{f} = \frac{r_z V_{0y}}{z^2} - \frac{r_y V_{0z}}{z^2}$$

$$V_{ix} = \frac{r_z V_{0x}\cdot f}{z^2} - \frac{r_x V_{0z}}{z^2}\cdot f$$

$$= \frac{V_x\cdot f}{z} - \frac{x\cdot V_z}{z}$$

$$V_{iy} = \frac{V_y\cdot f}{z} - \frac{y\cdot V_z}{z}$$

$\vec{}$ slide 25

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(10)

image $\quad$ object

$$\boxed{V_i = f\,\frac{(r_o \times V_o) \times \hat{Z}}{(r_o \cdot \hat{Z})^2}}$$

$$V_o = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$V_i = \begin{pmatrix} V_{ix} \\ V_{iy} \end{pmatrix}$$

do the math:

$$V_i = f\,\frac{\left(\begin{pmatrix} r_{ox} \\ r_{oy} \\ r_{oz} \end{pmatrix} \times \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}\right) \times \begin{pmatrix} Z_x \\ Z_y \\ Z_z \end{pmatrix}}{(r_o \cdot \hat{Z})^2}$$

set $\hat{Z} = \begin{pmatrix} 0 \\ 0 \\ Z_n \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 0 \\ 0 \\ Z_n \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= f\,\frac{\begin{pmatrix} r_{oy} \cdot V_z - r_{oz} V_y \\ -r_{ox} \cdot V_z + r_{oz} V_x \\ r_{ox} V_y - r_{oy} V_x \end{pmatrix} \times \begin{pmatrix} Z_x \\ Z_y \\ Z_z \end{pmatrix}}{(r_o \hat{Z})^2}$$

$P(X, Y, Z)$

$p(x, y)$

$$r_x \quad \frac{X}{Z} = \frac{x}{f}$$

$$\frac{Y}{Z} = \frac{y}{f}$$

$$= f\,\frac{\begin{pmatrix} (-r_{ox} V_z + r_{oz} V_x) Z_n \\ (-r_{oy} V_z + r_{oz} V_y) Z_n \\ 0 \end{pmatrix}}{(r_o \hat{Z})^2} \quad (r_o \hat{Z})^2 \approx Z^2$$

r_o in direction $\hat{Z}$

$\Rightarrow Z$

$$\Rightarrow \frac{V_{ix}}{f} = \frac{r_z V_x}{Z^2} - \frac{r_x V_z}{Z^2}$$

$$\frac{V_{iy}}{f} = \frac{r_z V_y}{Z^2} - \frac{r_y V_z}{Z^2}$$

$\frac{r_z}{Z} = 1$

$\frac{r_x \cdot f}{Z} = x$

$\frac{r_y \cdot f}{Z} = y$

$$\Rightarrow V_{ix} = \frac{V_x \cdot f}{Z} - \frac{x \cdot V_z}{Z}$$

$$V_{iy} = \frac{V_y \cdot f}{Z} - \frac{y \cdot V_z}{Z}$$

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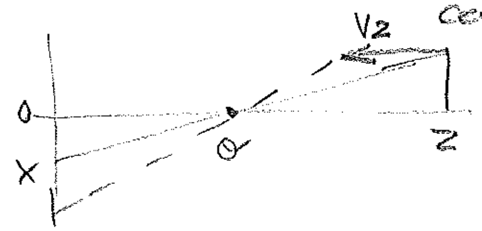
$$\begin{bmatrix} V_{ix} \\ V_{iy} \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix}}_{\text{perspective projection of 3D velocity}} \frac{1}{Z} \begin{bmatrix} V_{ox} \\ V_{oy} \\ V_{oz} \end{bmatrix}$$

Intuition: look at $V_{ix} = \frac{V_X \cdot f}{Z} - \frac{x V_Z}{Z}$

$\frac{V_{ix}}{f} = \frac{V_X}{Z}$ component due to V_X

$\frac{V_{ix}}{x} = -\frac{V_Z}{Z}$ component due to V_Z (however, camera)



8.2 Trucco

now: $\Omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$ angular velocity of 3D motion

(slide 6 III) $T = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}$ translational component of 3D motion

$$V = T + \Omega \times P$$

| P: 3D point in camera reference frame

$\Rightarrow$ $V_x = T_x + \omega_y Z - \omega_z Y$

$V_y = T_y + \omega_z X - \omega_x Z$ $\Rightarrow$ $[V] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & Z & -Y \\ 0 & 1 & 0 & -Z & 0 & X \\ 0 & 0 & 1 & Y & -X & 0 \end{array}\right] \begin{bmatrix} T \\ \Omega \end{bmatrix}$

(slide 7) $V_Z = T_Z + \omega_x Y - \omega_y X$

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continue;

slide 9 III

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \underbrace{\underset{\text{persp. proj.}}{\overset{2\times 3}{\begin{bmatrix} & & \end{bmatrix}}} \frac{1}{Z} \underset{\text{3D velocity}}{\overset{3\times 6}{\begin{bmatrix} & & & & & \end{bmatrix}}}}_{H \; [2\times 6]} \begin{bmatrix} T \\ \Omega \end{bmatrix}$$

(slide 11)

$$\frac{1}{Z}\left[\begin{array}{ccc|ccc} f & 0 & -x & -xY & (Zf+xX) & -fY \\ 0 & f & -y & (-fZ-yY) & -yY & fY \end{array}\right]$$

$$\underbrace{\qquad\qquad}_{\text{Transl.}} \qquad \underbrace{\qquad\qquad\qquad}_{\text{rotation}}$$

$$\Rightarrow \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \underbrace{\begin{bmatrix} v_x \\ v_y \end{bmatrix}_{\text{transl.}}}_{\frac{1}{Z}\begin{bmatrix} ft_x - xt_z \\ ft_y - yt_z \end{bmatrix}} + \underbrace{\begin{bmatrix} v_x \\ v_y \end{bmatrix}_{\text{angular}}}_{}$$

(slide 11 II)
(Trucco p. 184)

$$-\frac{xy}{f}\omega_x + \left(f + \frac{x^2}{f}\right)\omega_y - y\omega_z$$

$$-f\omega_x - \frac{y^2}{f}\omega_x - \frac{yx}{f}\omega_y + y\omega_z$$

$$= f(T, Z) \qquad\qquad f(\Omega, x, y)$$

$$\neq f(Z)\,!$$

motion field that depends on angular velocity does not carry information on depth!

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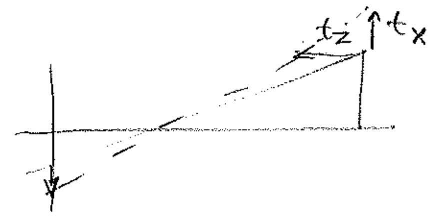
Pure translation



$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \frac{1}{Z} \begin{bmatrix} fT_x - xT_Z \\ fT_y - yT_Z \end{bmatrix}$$

introduce: $p_0 = (x_0, y_0)^T$

Choose p0 so that vx and vy get 0: point which does not move.

let: $x_0 T_Z = f T_X \Rightarrow x_0 = \frac{fT_x}{T_Z}$

$y_0 T_Z = f T_y \Rightarrow y_0 = \frac{fT_y}{T_Z}$

plug in

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \frac{1}{Z} \begin{bmatrix} x_0 T_Z - x T_Z \\ y_0 T_Z - y T_Z \end{bmatrix} = \frac{T_Z}{Z} \begin{bmatrix} x_0 - x \\ y_0 - y \end{bmatrix}$$

motion field of pure translation is radial!

(slide 12 III)

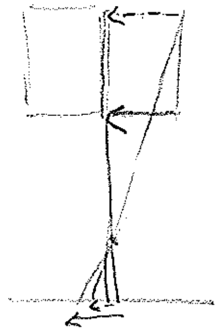
$p_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ focus of expansion

vanishing point

cases:

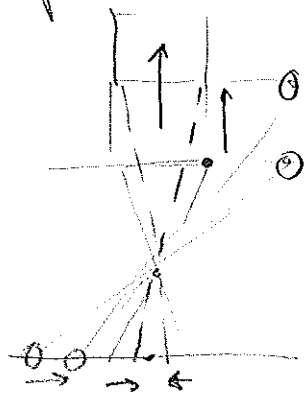
a) $T_Z = \emptyset \Rightarrow \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \frac{f}{Z} \begin{bmatrix} T_x \\ T_y \end{bmatrix}$

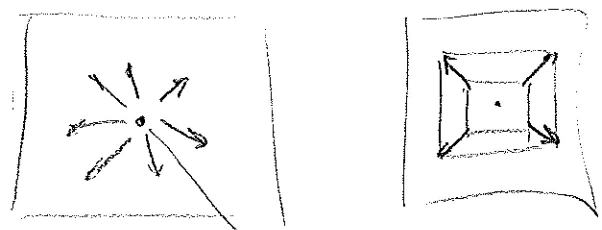
- all motion vectors parallel
- amount V inverse proportional to depth



b) pure T_z : $T_x, T_y = 0$

$\Rightarrow x_0, y_0 = (0,0)^T$



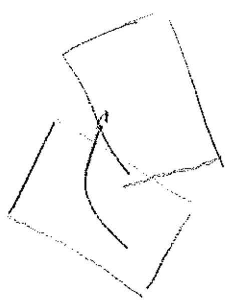


c) <u>moving plane</u> (Trucco p.187)

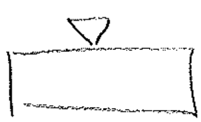
slide 13

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = f(x, y, xy, x^2, y^2)$$

quadratic polynomial



pls. look up!



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final idea: epipolar constraint

Moving plane

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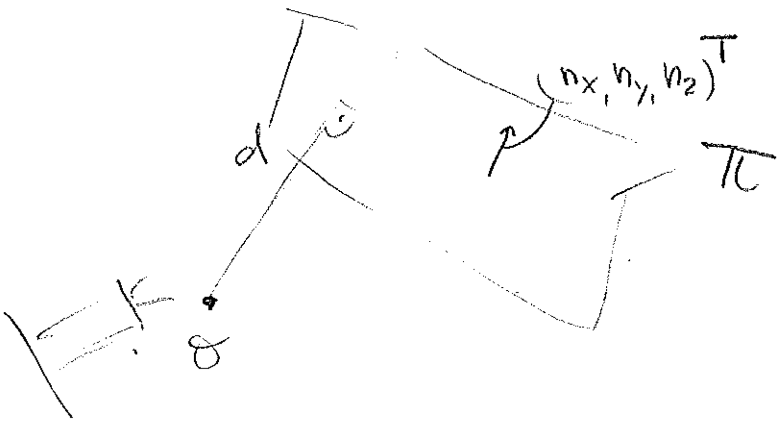

$$\bar{n}^t \cdot P = d$$

moving with $T \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$ and $\omega = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$

$\Rightarrow \bar{n}(t)$, $T(t)$

$\bar{n}^t \cdot \underbrace{P}_{\frac{Z \cdot p}{f}} = d$ | $p = f \cdot \frac{P}{Z}$

$$\Rightarrow \frac{(n_x \cdot x + n_y \cdot y + n_z \cdot f)}{f} \cdot Z = d$$

$\Rightarrow$ solve for Z

$\Rightarrow$ plug into $\begin{bmatrix} v_x \\ v_y \end{bmatrix} = H \begin{bmatrix} T \\ \Omega \end{bmatrix}$

p. 187 Trucco

$\Rightarrow v_x = \frac{1}{fd}(x^2, xy, fx, fy, f^2)$

$v_y = \frac{1}{fd}(xy, y^2, fy, fx, f^2)$

$\Rightarrow$ motion field of a moving planar surface is quadratic polynomial in (x, y, f)

$\Rightarrow$ p. 187/188: motion field not unique?