



# Multi-View Geometry: Find Corresponding Points (New book: Ch7.4, 7.5, 7.6 Old book: 11.3-11.5)

Guido Gerig  
CS-GY 6643, Spring 2017  
[gerig@nyu.edu](mailto:gerig@nyu.edu)

Credit for materials: Trevor Darrell, Berkeley, C280, Marc Pollefeys,  
UNC/ETH-Z, CS6320 S012, Andrew Zisserman, MVG Book

# Excellent Website:

## <http://vision.middlebury.edu/stereo/>

[vision.middlebury.edu](http://vision.middlebury.edu)

[stereo](#) • [mview](#) • [MRF](#) • [flow](#) • [color](#)

**Stereo**

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[Daniel Scharstein](#) • [Richard Szeliski](#)

Welcome to the Middlebury Stereo Vision Page, formerly located at [www.middlebury.edu/stereo](http://www.middlebury.edu/stereo). This website accompanies our taxonomy and comparison of two-frame stereo correspondence algorithms [1]. It contains:

- An [on-line evaluation](#) of current algorithms
- Many [stereo datasets](#) with ground-truth disparities
- Our [stereo correspondence software](#)
- An [on-line submission script](#) that allows you to evaluate your stereo algorithm in our framework

#### How to cite the materials on this website:

We grant permission to use and publish all images and numerical results on this website. If you report performance results, we request that you cite our paper [1]. Instructions on how to cite our datasets are listed on the [datasets page](#). If you want to cite this website, please use the URL "[vision.middlebury.edu/stereo/](http://vision.middlebury.edu/stereo/)".

#### References:

- [1] D. Scharstein and R. Szeliski. [A taxonomy and evaluation of dense two-frame stereo correspondence algorithms](#). *International Journal of Computer Vision*, 47(1/2/3):7-42, April-June 2002.  
[Microsoft Research Technical Report MSR-TR-2001-81](#), November 2001.



# Stereo reconstruction: main steps

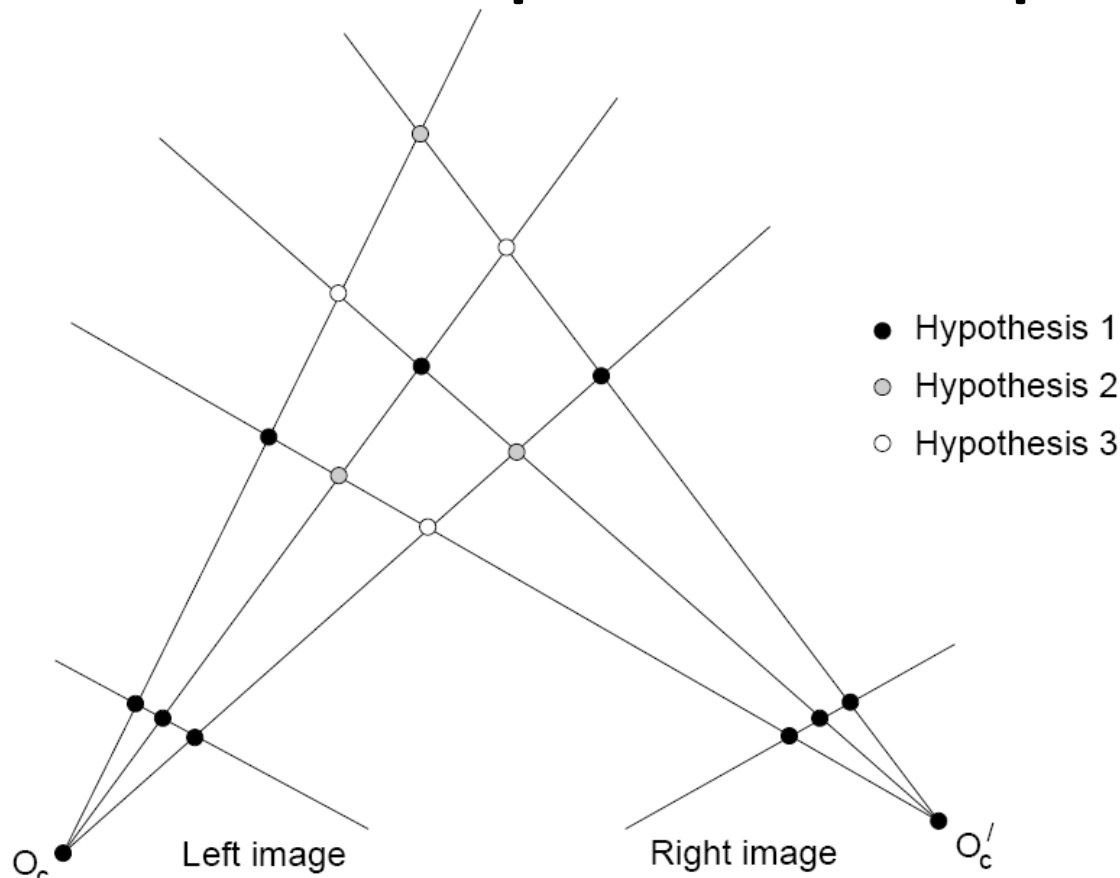
- Calibrate cameras
- Rectify images
- Compute disparity
- Estimate depth

# Stereo reconstruction: main steps

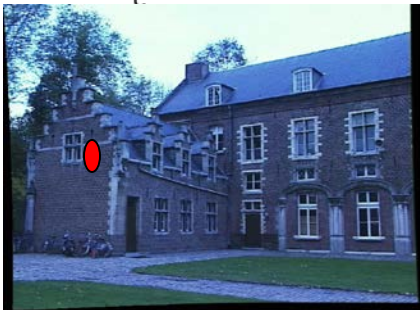
- Calibrate cameras
- Rectify images
- Compute disparity
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# Correspondence problem



Multiple match hypotheses satisfy epipolar constraint, but which is correct?



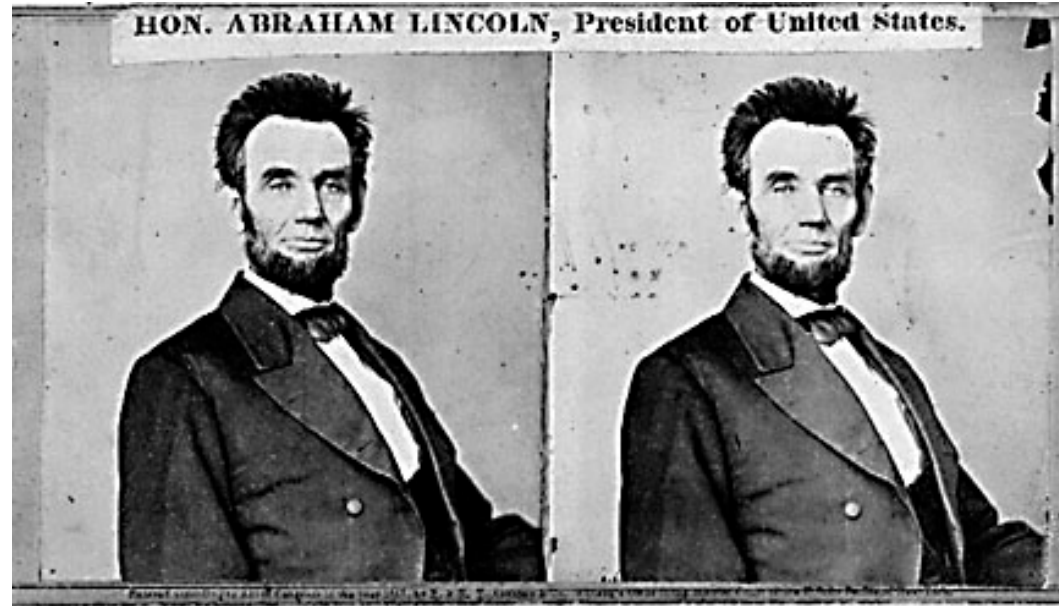
# Correspondence problem

- Beyond the hard constraint of epipolar geometry, there are “soft” constraints to help identify corresponding points
  - Similarity
  - Uniqueness
  - Ordering
  - Disparity gradient

# Correspondence problem

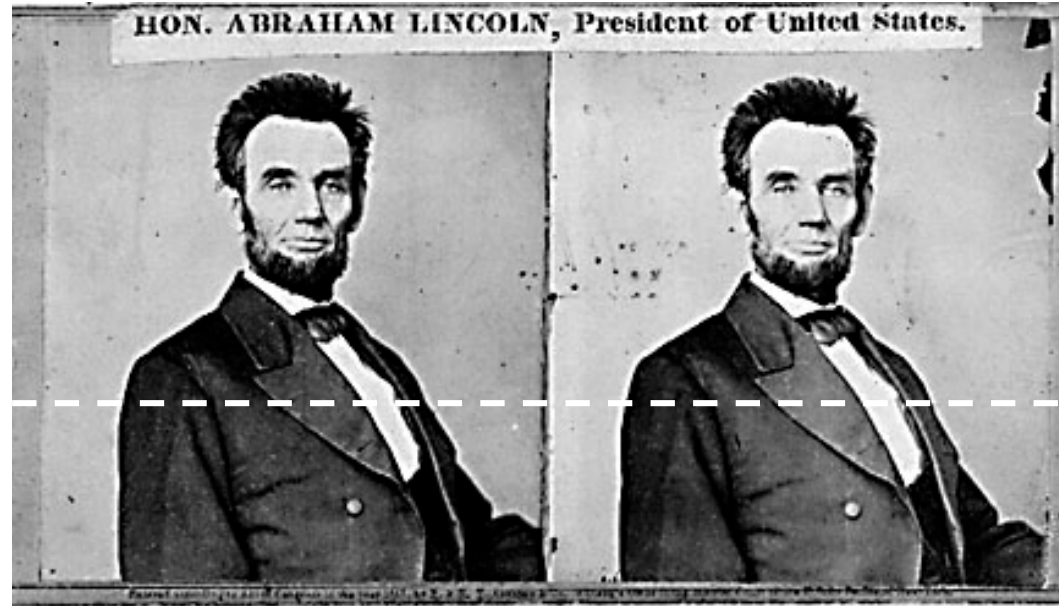
- Beyond the hard constraint of epipolar geometry, there are “soft” constraints to help identify corresponding points
  - Similarity
  - Uniqueness
  - Ordering
  - Disparity gradient
- To find matches in the image pair, we will assume
  - Most scene points visible from both views
  - Image regions for the matches are similar in appearance

# Your basic stereo algorithm



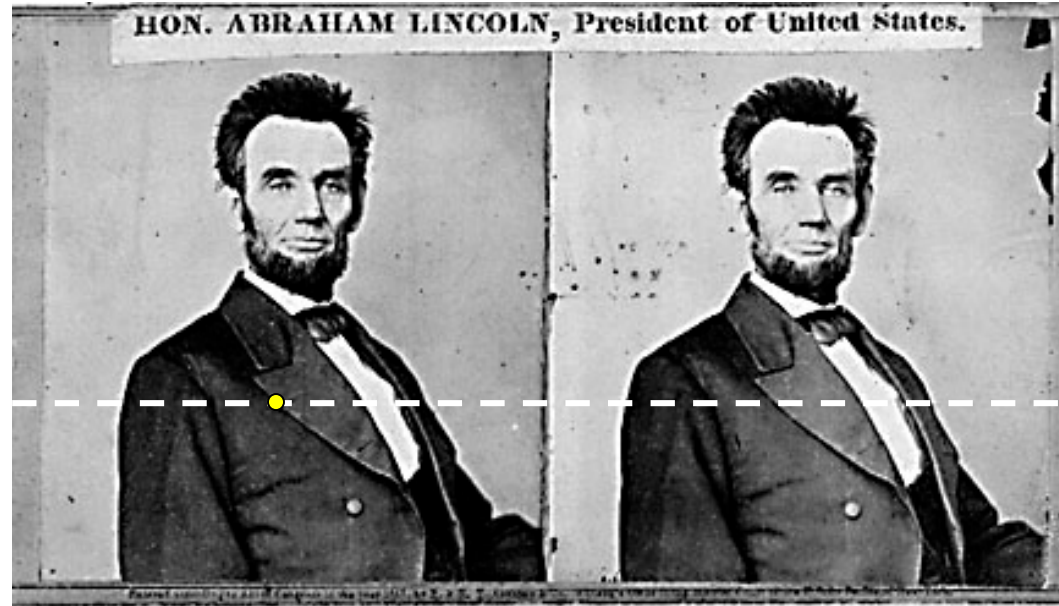


# Your basic stereo algorithm



For each epipolar line:

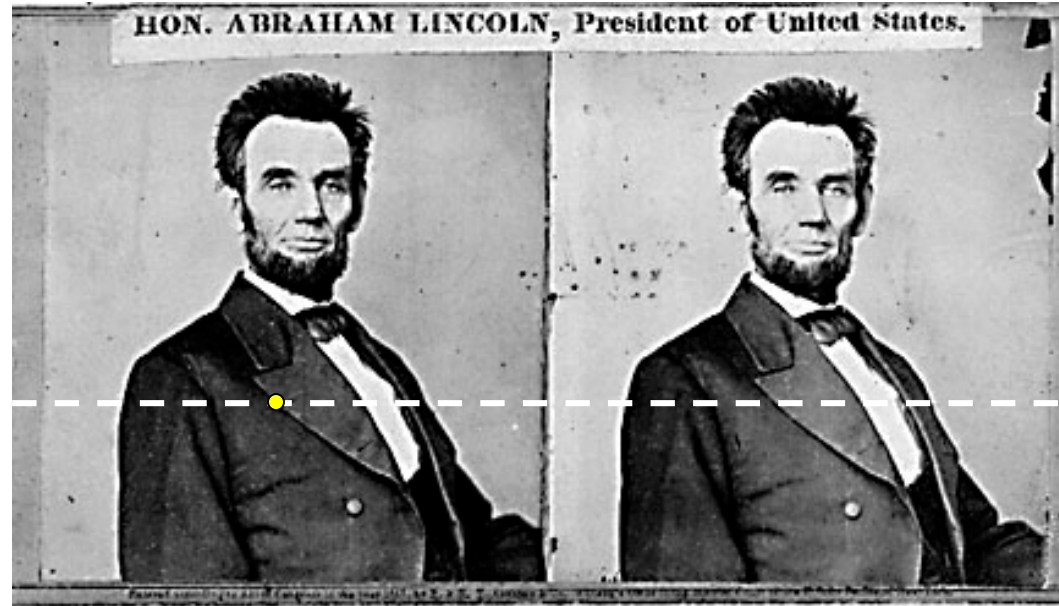
# Your basic stereo algorithm



For each epipolar line:

For each pixel in the left image

# Your basic stereo algorithm

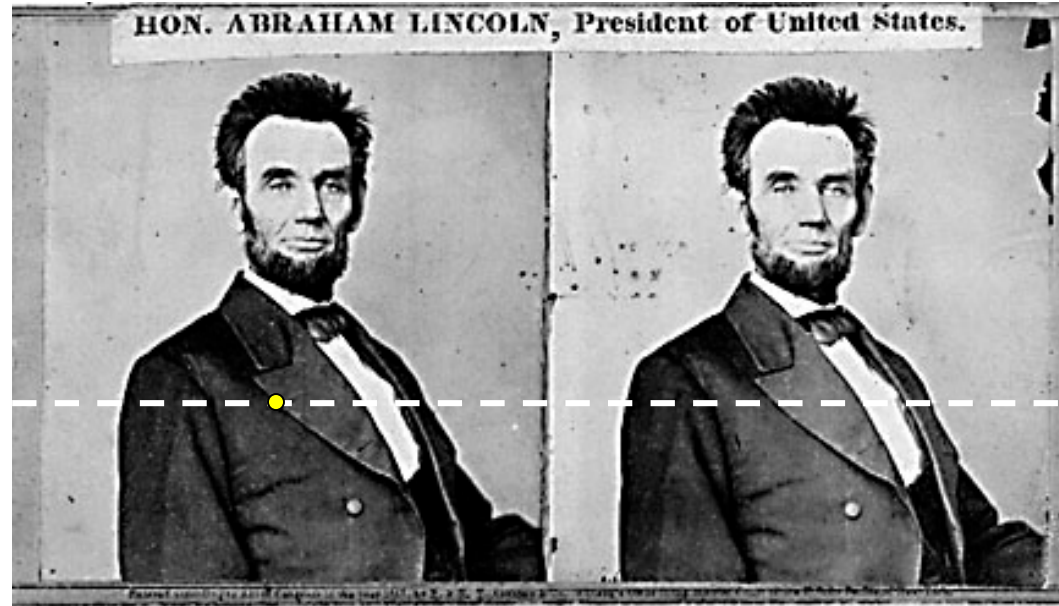


For each epipolar line:

For each pixel in the left image

- compare with every pixel on same epipolar line in right image

# Your basic stereo algorithm

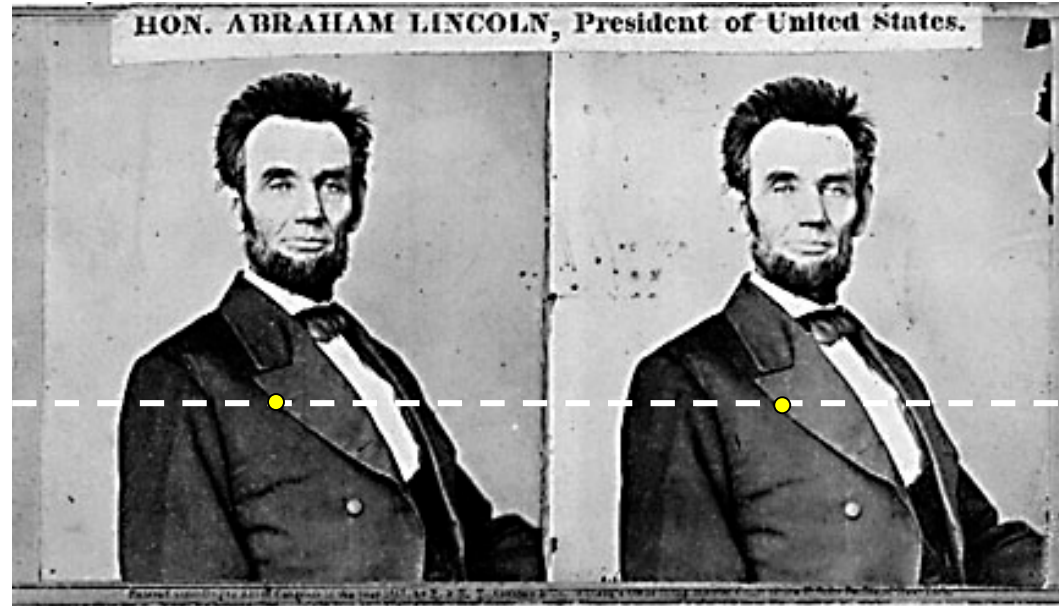


For each epipolar line:

For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

# Your basic stereo algorithm

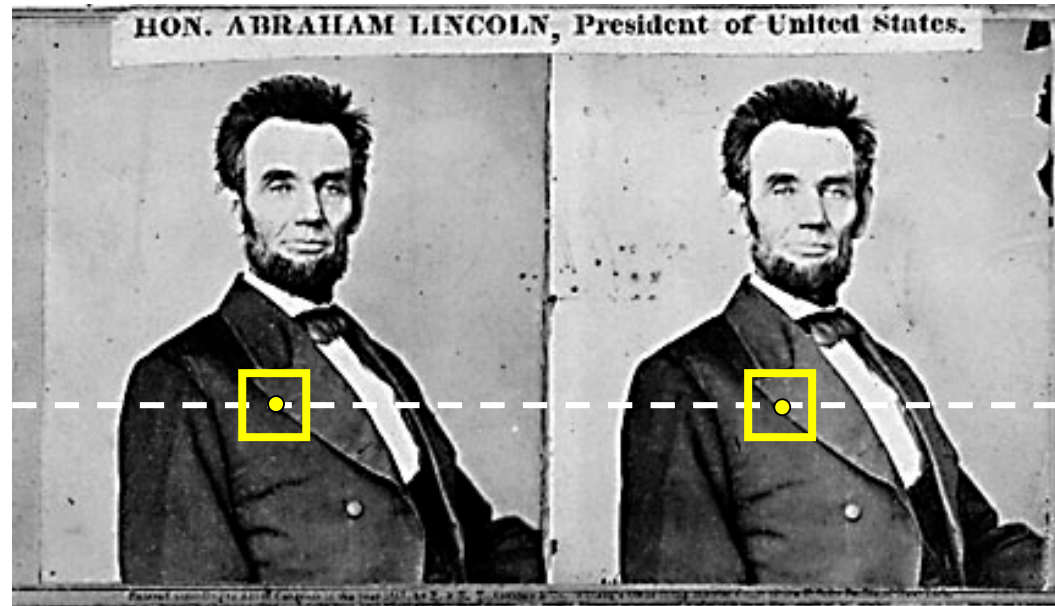


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For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

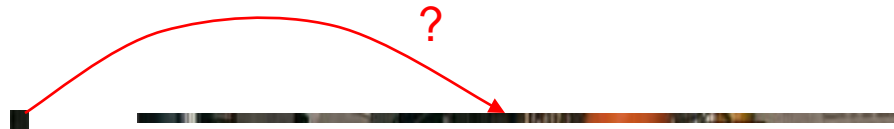
Improvement: match *windows*

- This should look familiar...
- E.g. SSD, correlation etc.

# Stereo matching

- Search is limited to epipolar line (1D)
- Look for “most similar pixel”

```
for x=1:w,  
  for y=1:h,  
    bestdist=inf;  
    for i=-dr:0,  
      if (dist(pix(x,y),pix(x+i,y))<bestdist)  
        d(x,y)=i; best=sim(pix(x,y),pix(x+i,y)); end  
      end  
    end  
  end  
end
```

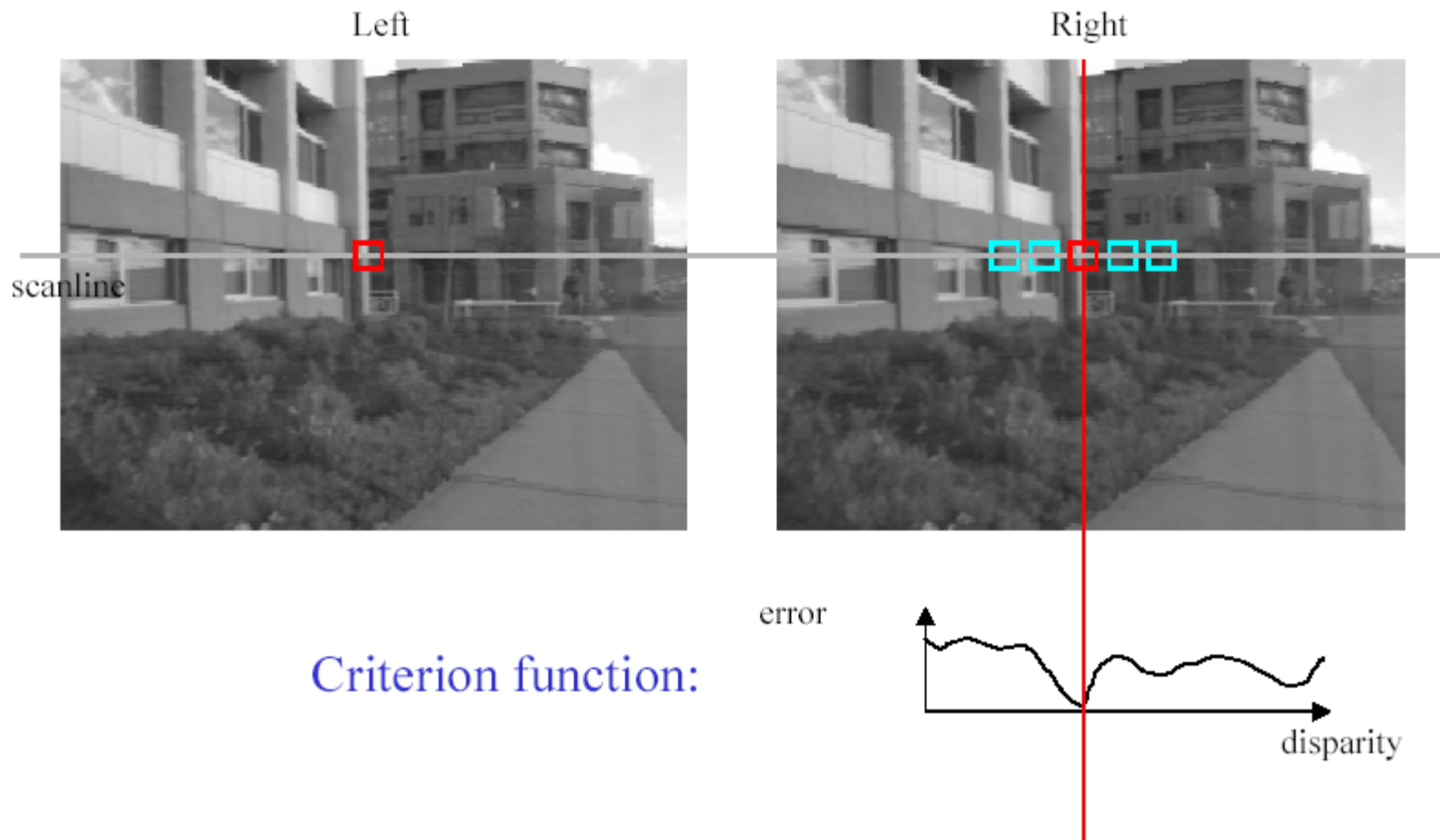


# Stereo matching algorithms

- Match Pixels in Conjugate Epipolar Lines
  - Assume brightness constancy
  - This is a tough problem
  - Numerous approaches
    - dynamic programming [Baker 81, Ohta 85]
    - smoothness functionals
    - more images (trinocular, N-ocular) [Okutomi 93]
    - graph cuts [Boykov 00]
  - A good survey and evaluation:
    - <http://vision.middlebury.edu/stereo/>

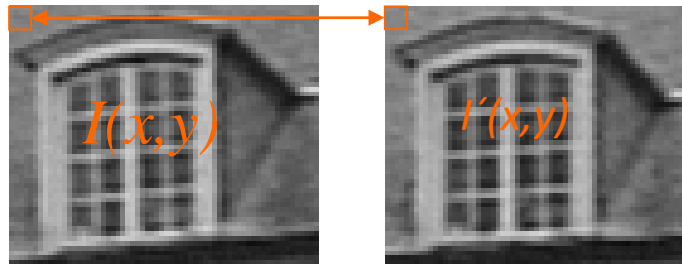


# Correspondence using Discrete Search



# Comparing image regions

Compare intensities pixel-by-pixel



## Similarity measures

Census

$$C_I(i, j) = (I(x + i, y + j) > I(x, y))$$

125	126	125
127	128	130
129	132	135

→

0	0	0
0		1
1	1	1

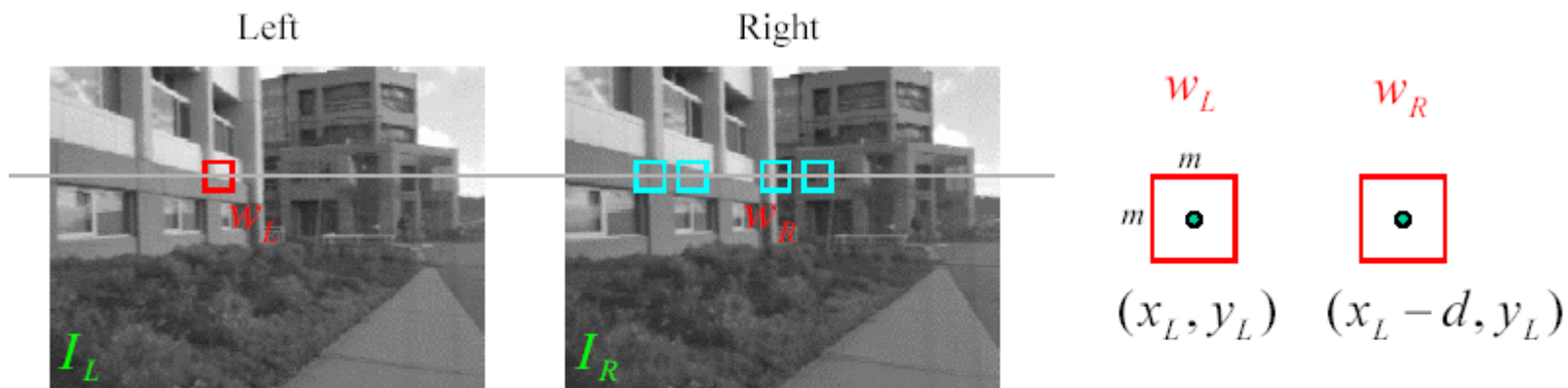
→

[00001111]

only compare bit signature

(Real-time chip from TYZX based on Census)

# Sum of Squared Differences (SSD)



$w_L$  and  $w_R$  are corresponding  $m$  by  $m$  windows of pixels.

We define the window function :

$$W_m(x, y) = \{u, v \mid x - \frac{m}{2} \leq u \leq x + \frac{m}{2}, y - \frac{m}{2} \leq v \leq y + \frac{m}{2}\}$$

The SSD cost measures the intensity difference as a function of disparity :

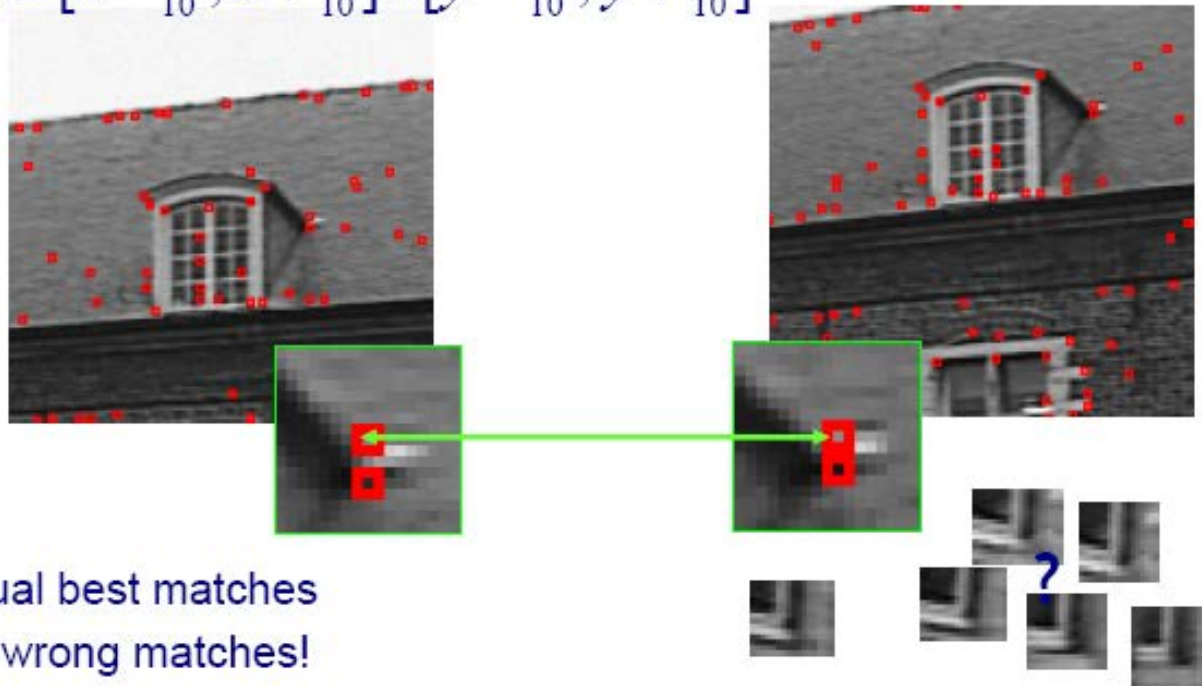
$$C_r(x, y, d) = \sum_{(u,v) \in W_m(x,y)} [I_L(u, v) - I_R(u - d, v)]^2$$

# Example

## Feature Matching

Evaluate NCC for all features with similar coordinates

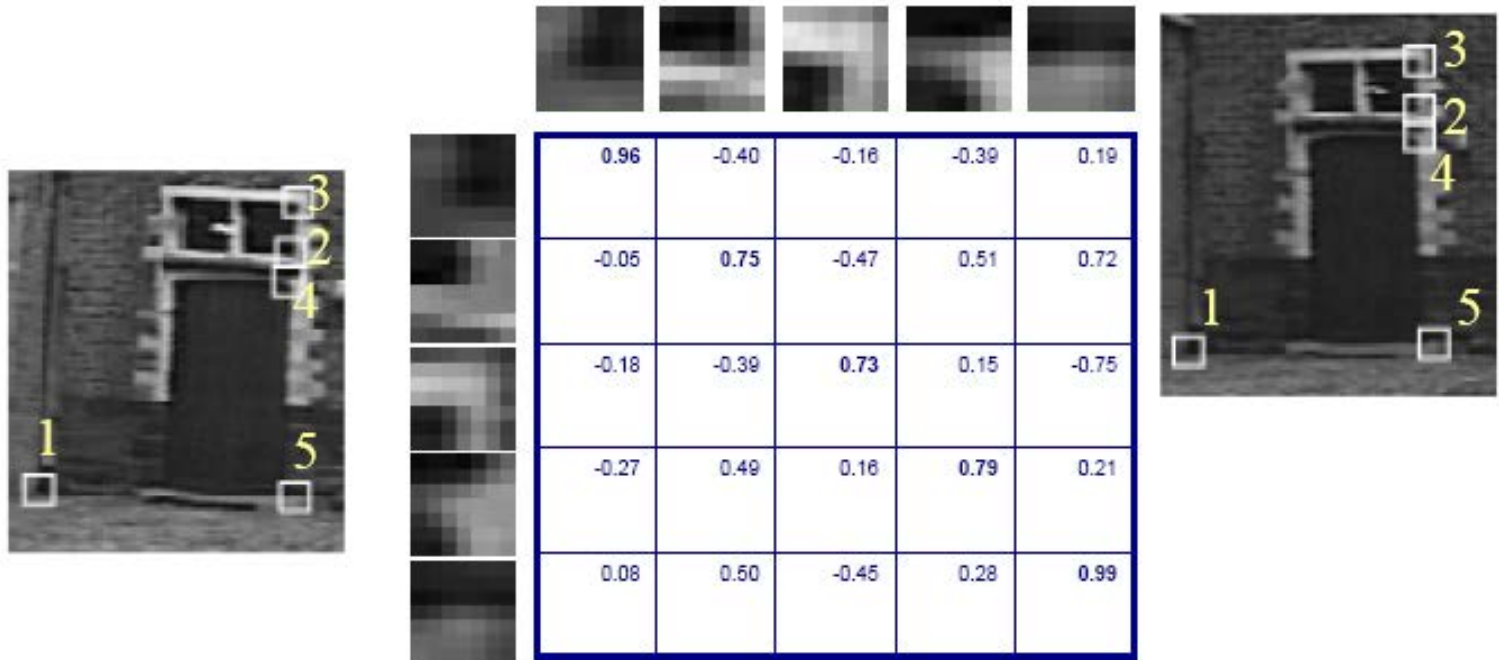
$$\text{e.g. } (x', y') \in \left[ x - \frac{w}{10}, x + \frac{w}{10} \right] \times \left[ y - \frac{h}{10}, y + \frac{h}{10} \right]$$



Keep mutual best matches  
Still many wrong matches!

# Example ctd

## Feature Example



Gives satisfying results  
for small image motions

# Example image pair – parallel cameras



# First image

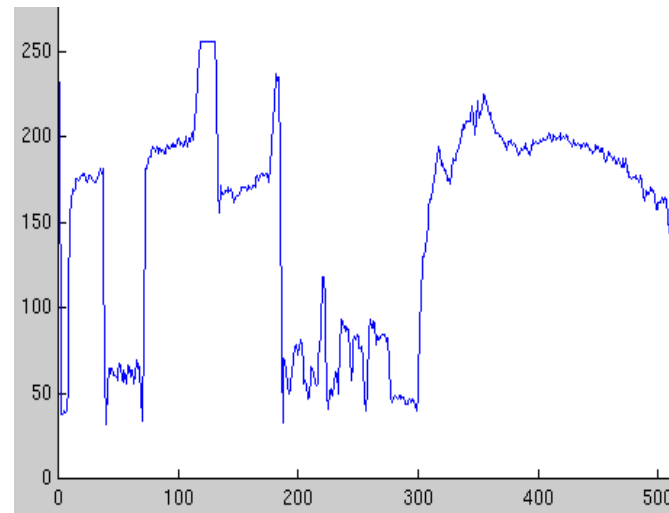
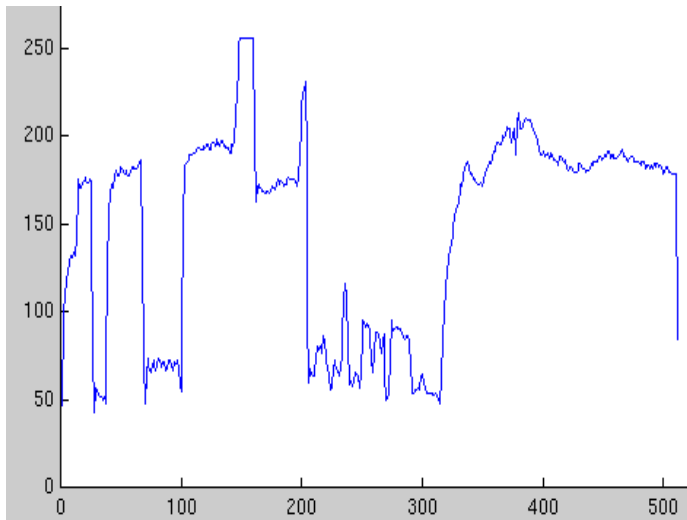
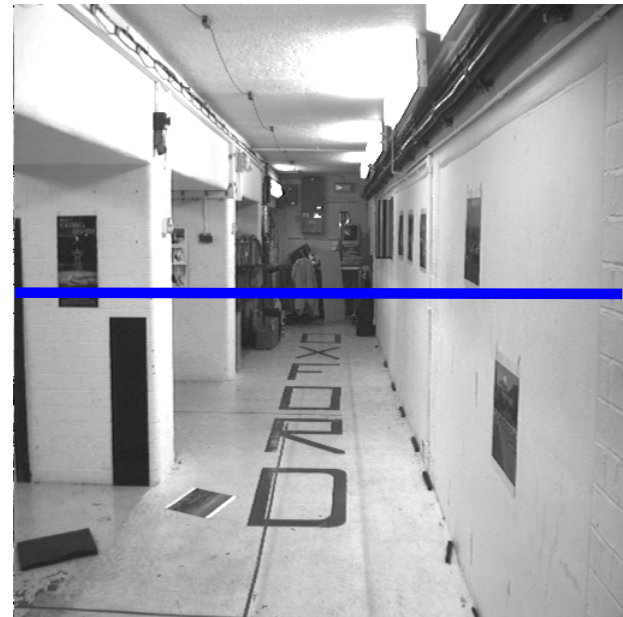


# Second image





# Intensity profiles



- Clear correspondence between intensities, but also noise and ambiguity

# Dense correspondence algorithm

Parallel camera example – epipolar lines are corresponding raster



# Dense correspondence algorithm

Parallel camera example – epipolar lines are corresponding rasters



# Dense correspondence algorithm

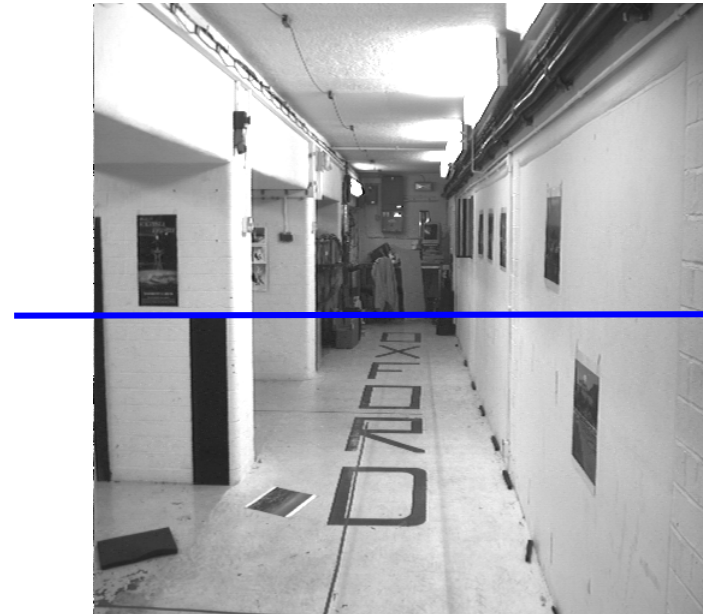
Parallel camera example – epipolar lines are corresponding rasters



epipolar  
line

# Dense correspondence algorithm

Parallel camera example – epipolar lines are corresponding raster



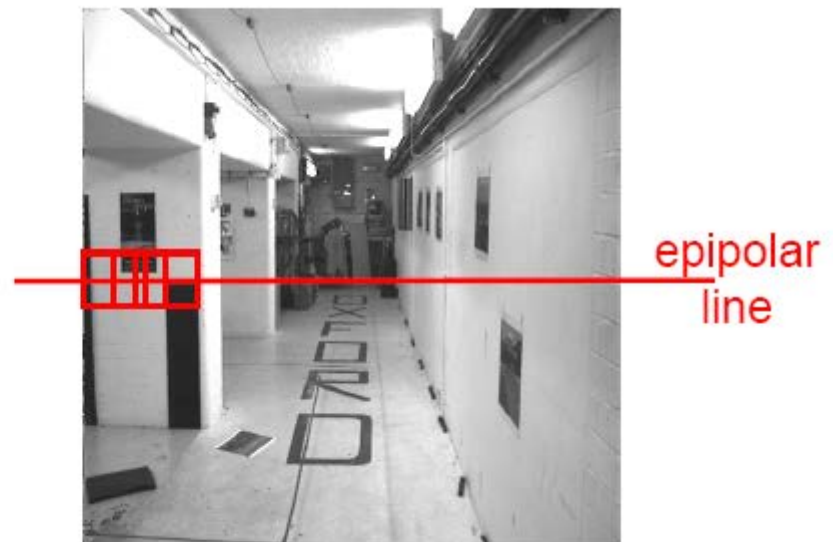
epipolar  
line

Search problem (geometric constraint): for each point in the left image, the corresponding point in the right image lies on the epipolar line (1D ambiguity)

Disambiguating assumption (photometric constraint): the intensity neighbourhood of corresponding points are similar across images

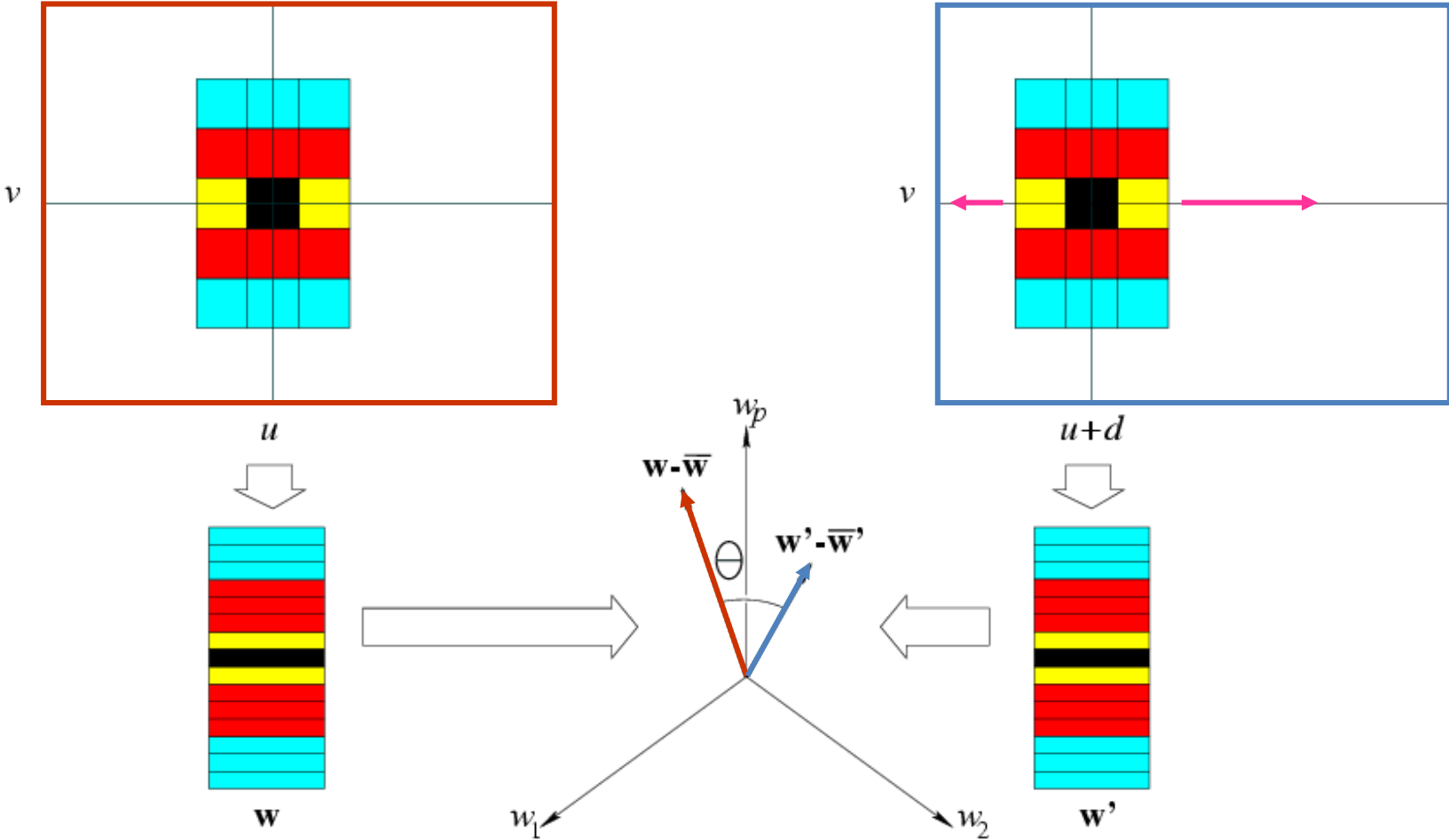
Measure similarity of neighbourhood intensity by cross-correlation

# Correspondence problem



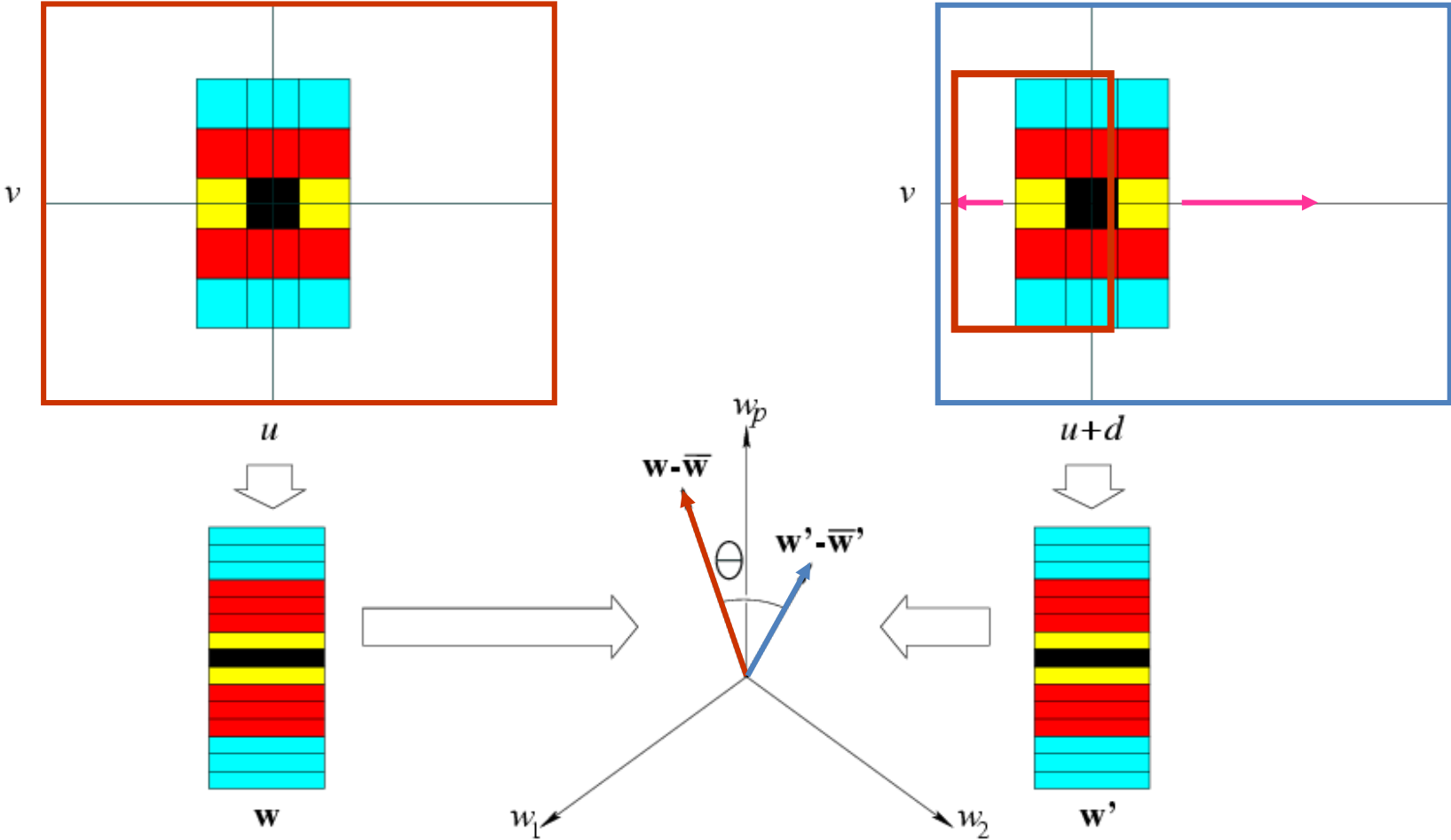
Nighborhood of corresponding points are similar in intensity patterns.

# Correlation Methods (1970--) F&P book new: 7.4, old 11.3



Slide the window along the epipolar line until  $w.w'$  is maximized.

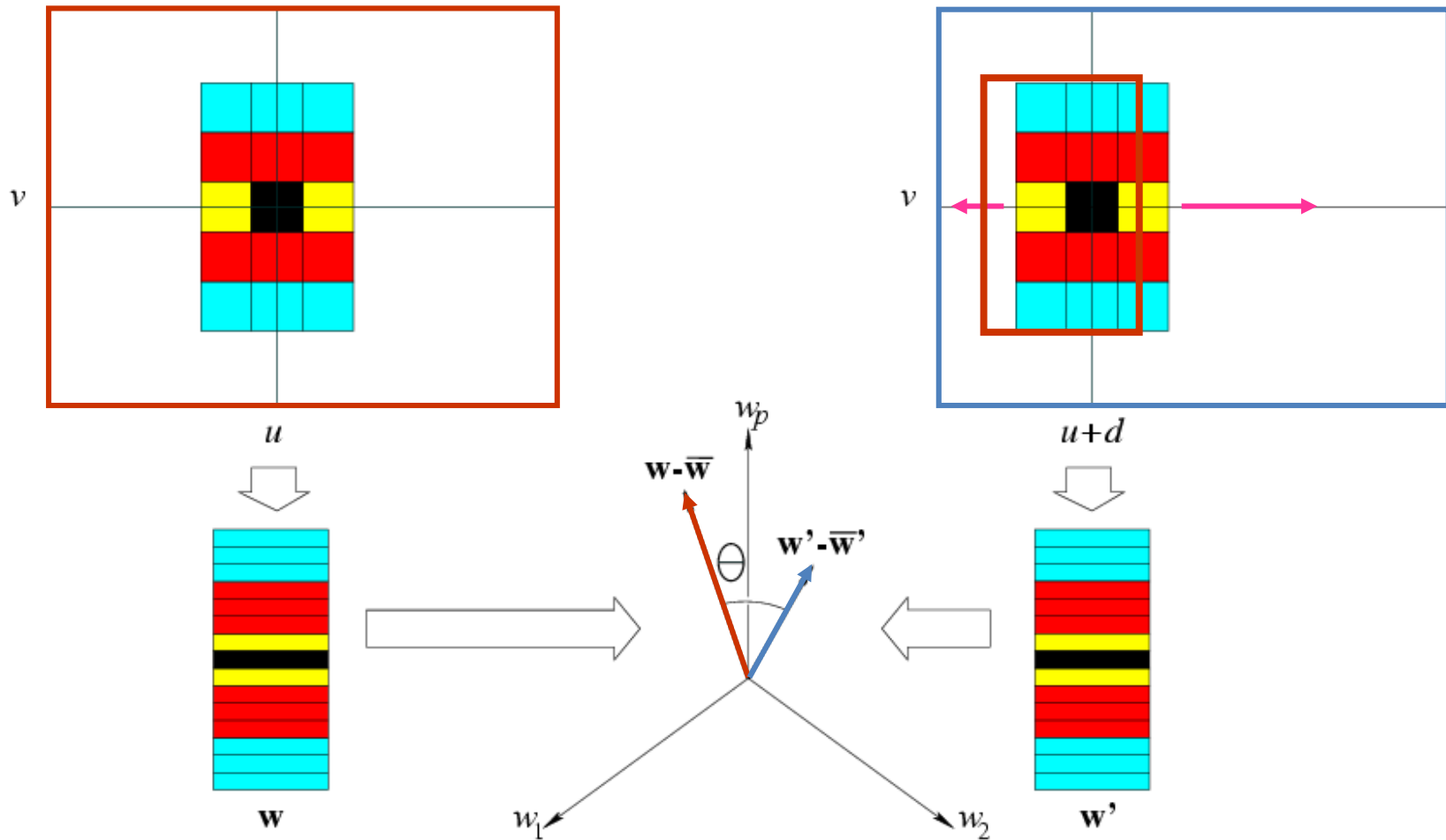
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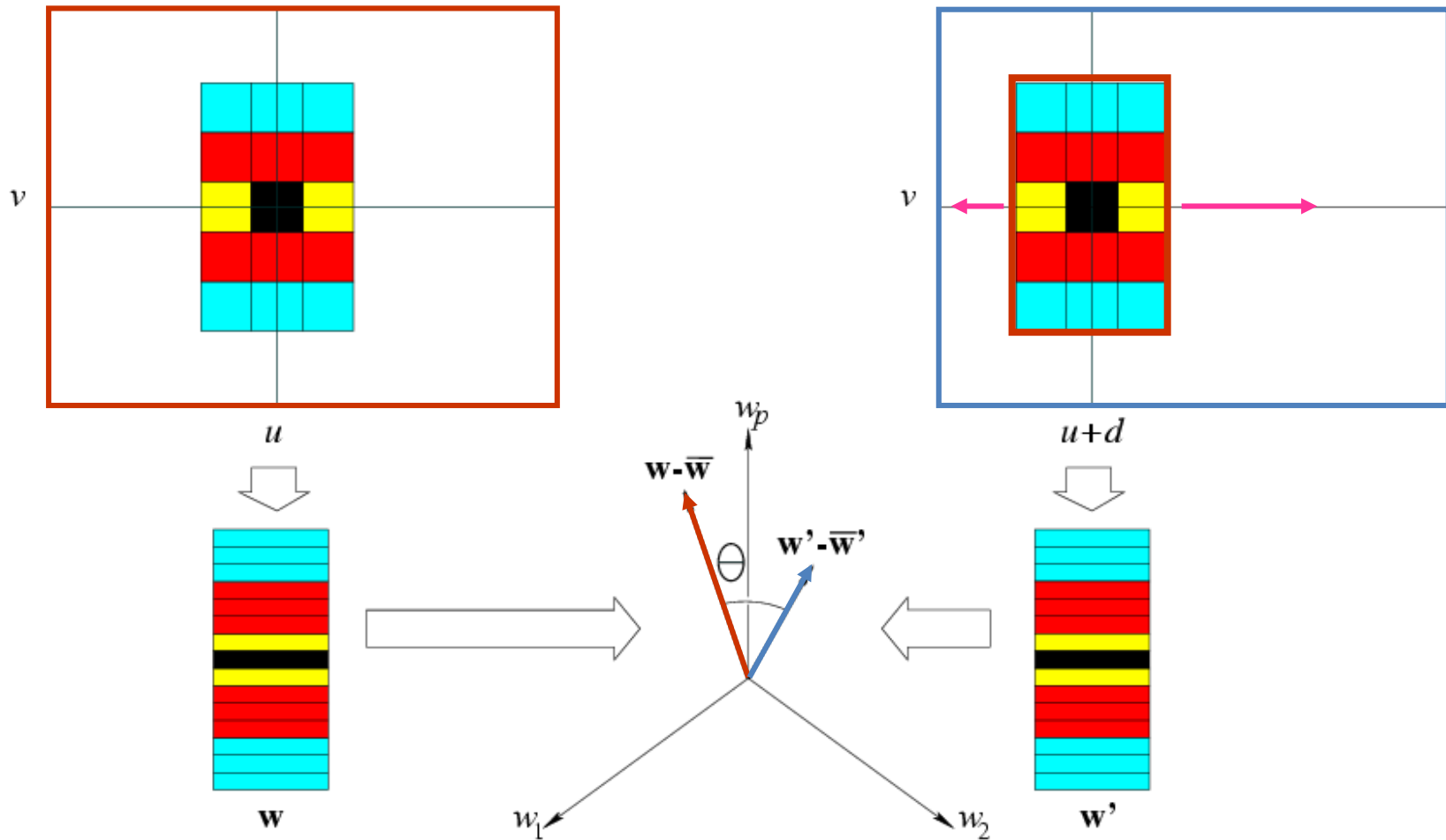


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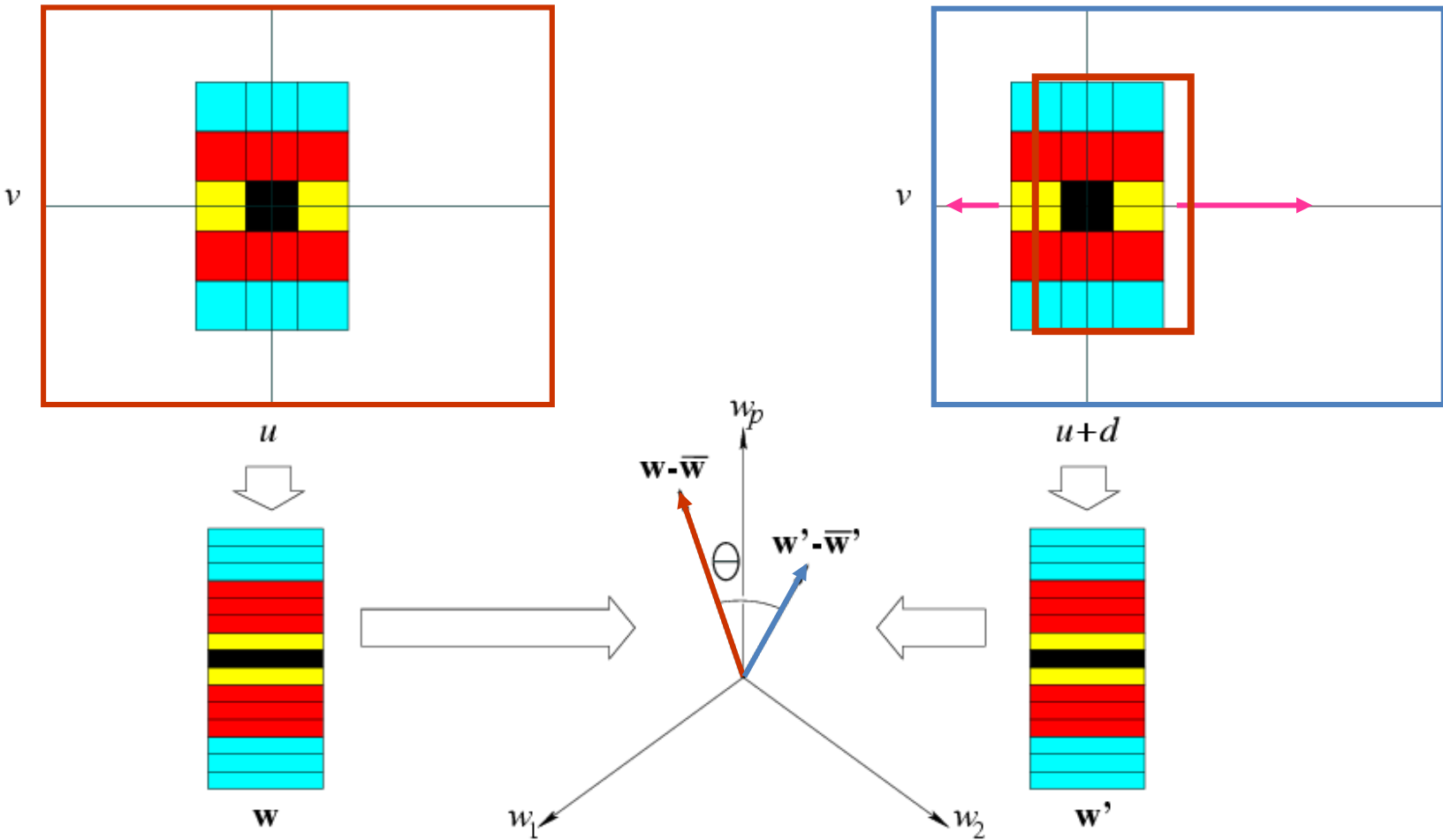
Slide the window along the epipolar line until  $w.w'$  is maximized.

# Correlation Methods (1970--) F&P book new: 7.4, old 11.3



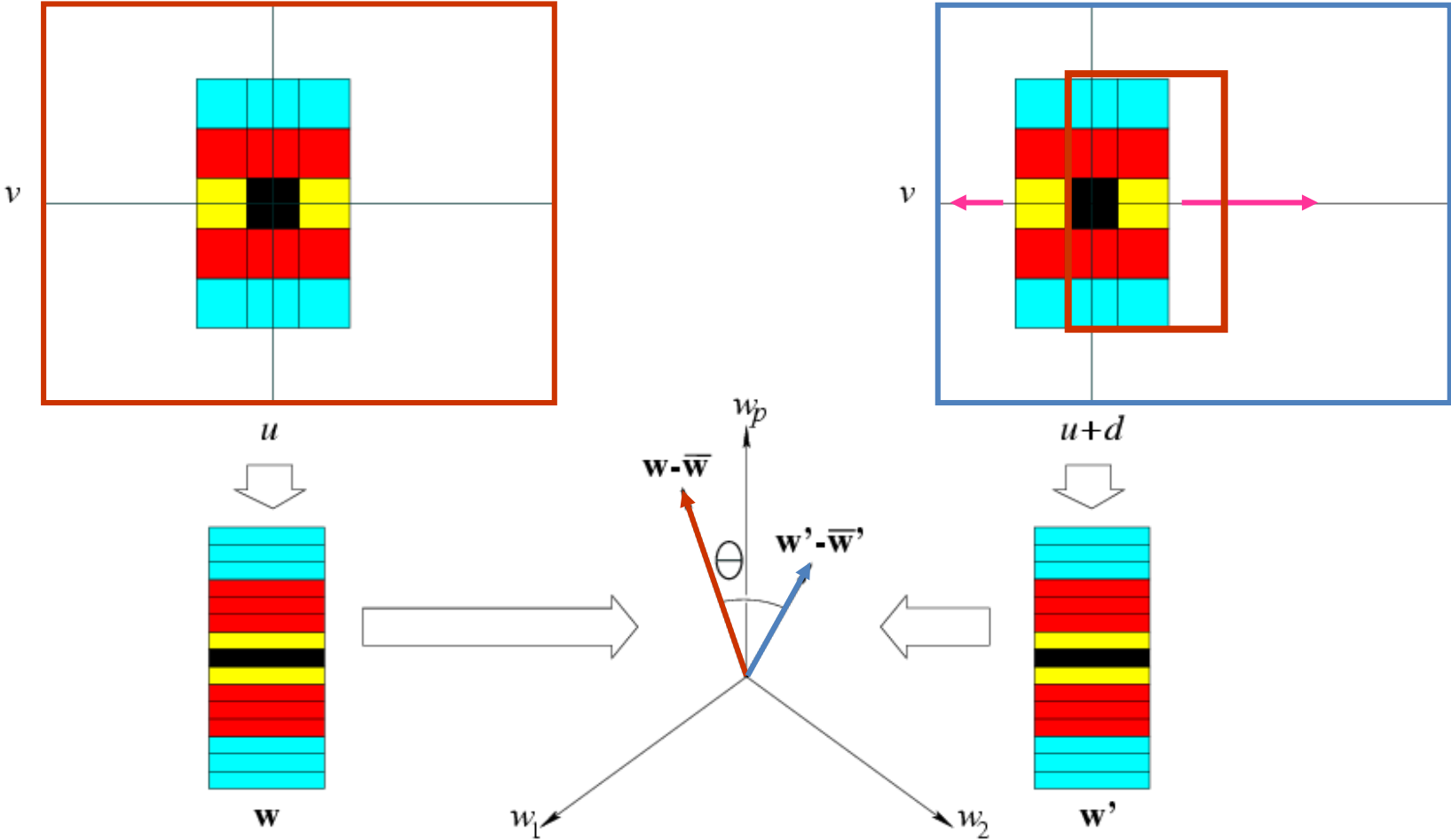
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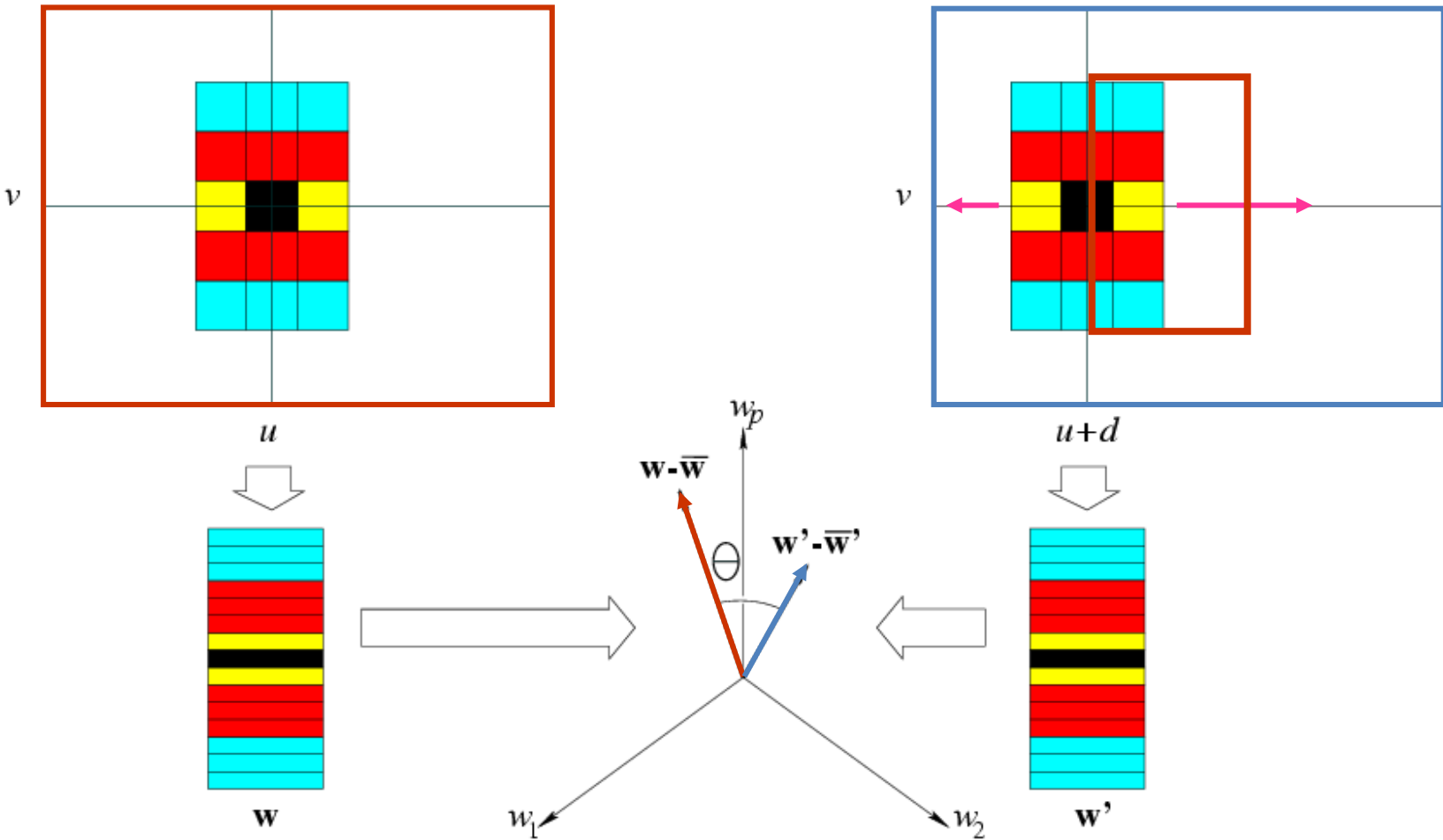
Slide the window along the epipolar line until  $w \cdot w'$  is maximized.

# Correlation Methods (1970--) F&P book new: 7.4, old 11.3



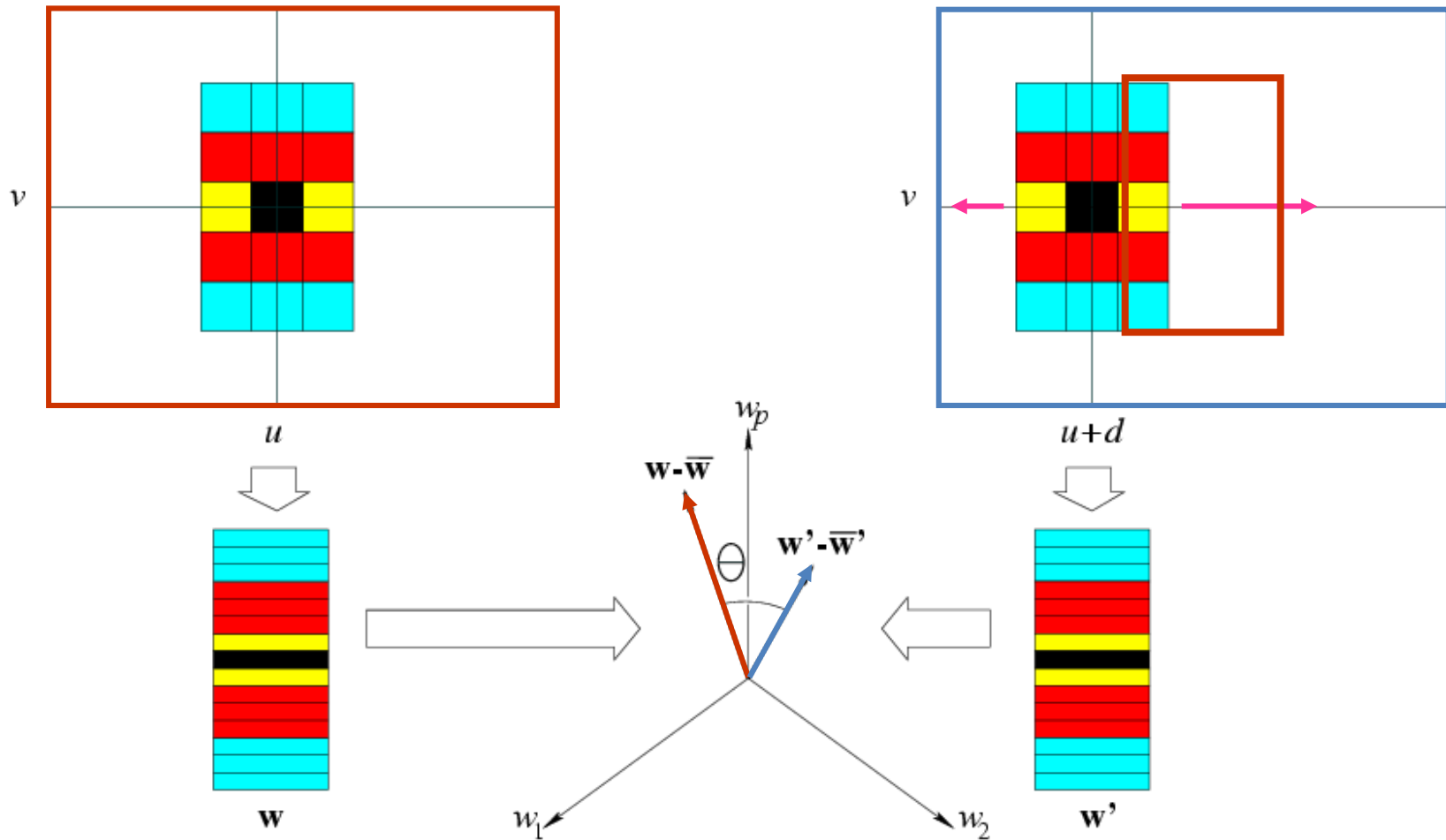
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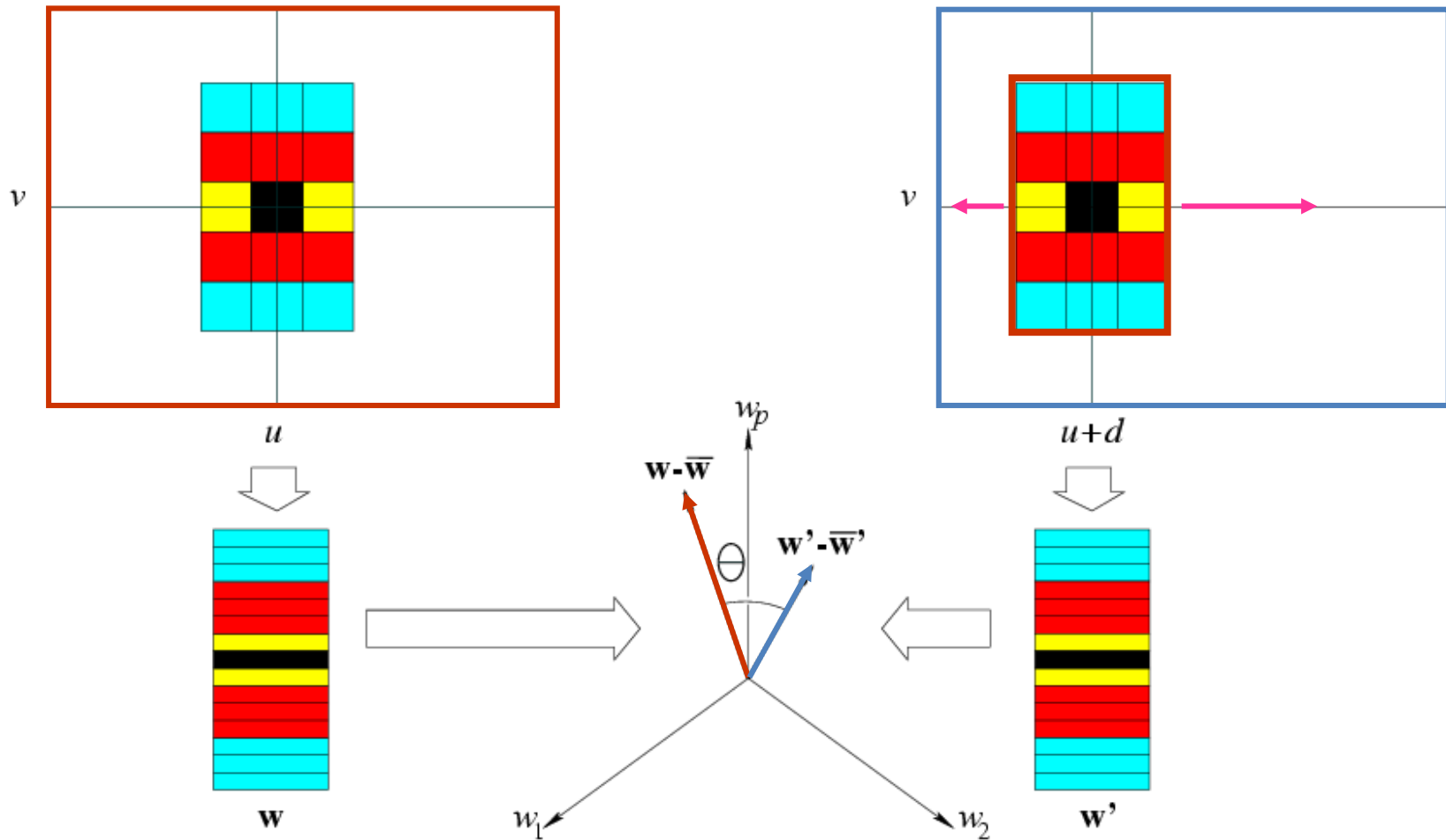
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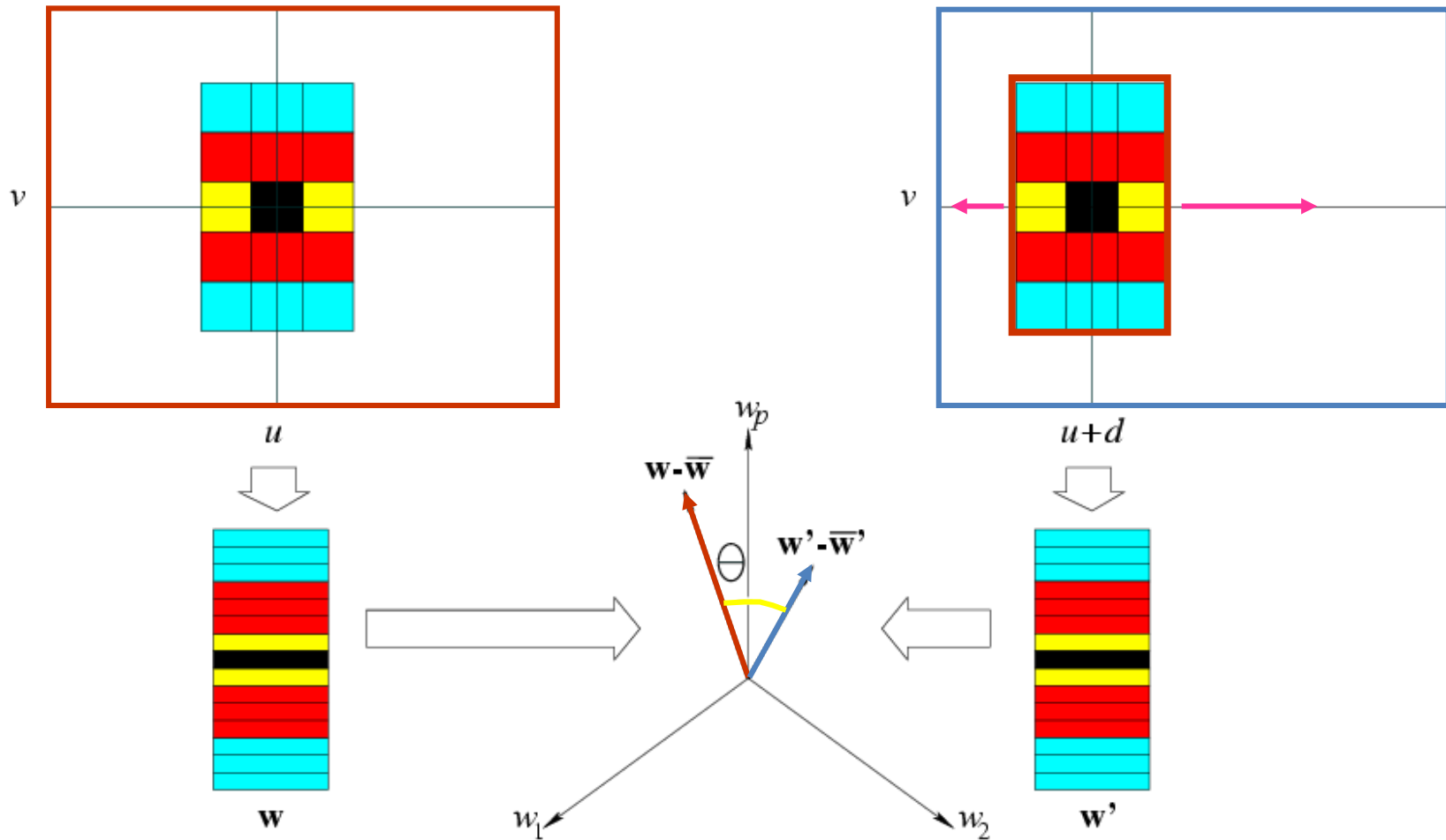
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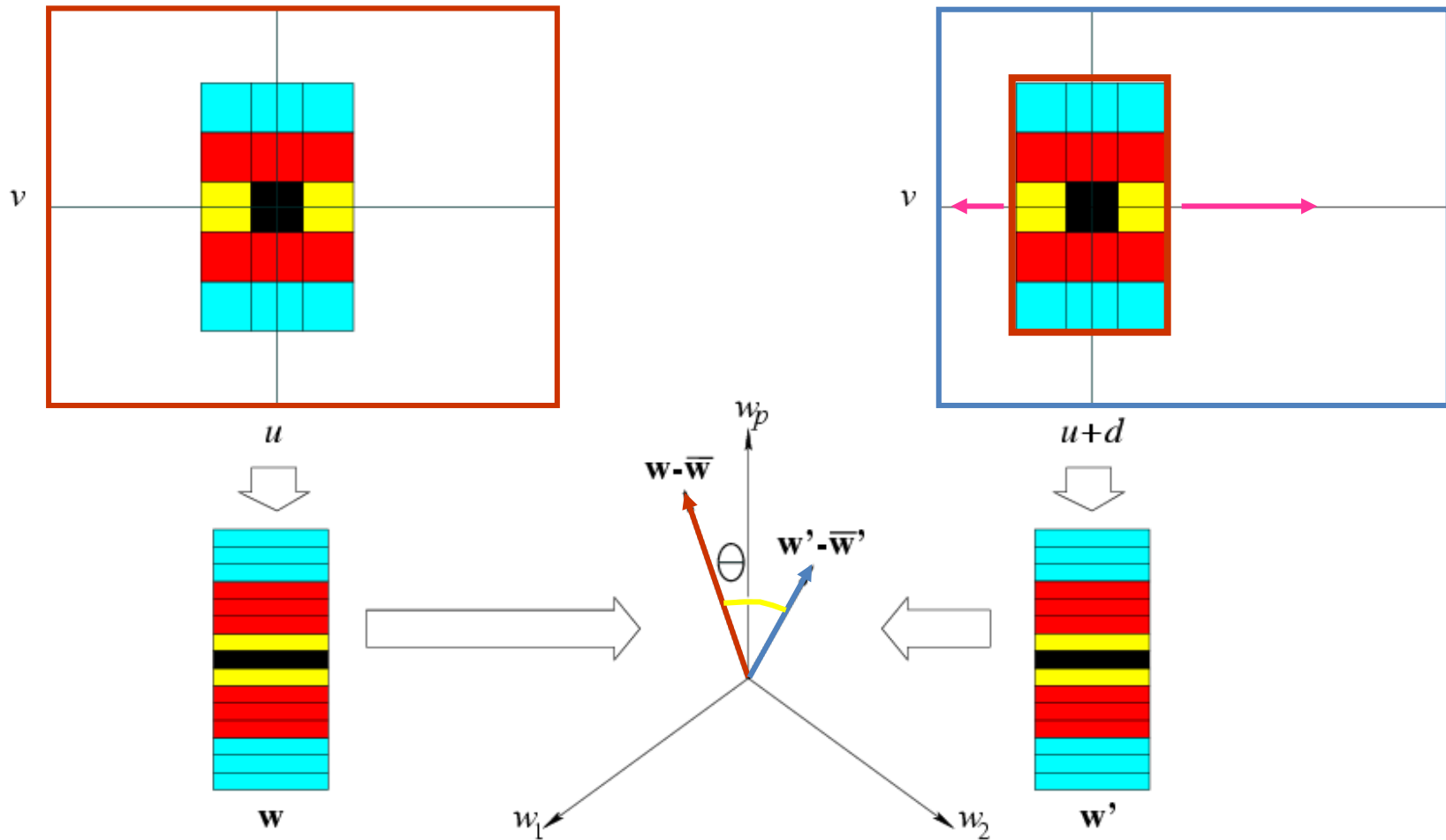


Slide the window along the epipolar line until  $w.w'$  is maximized.

Normalized Correlation: minimize  $\theta$  instead.



# Correlation Methods (1970--) F&P book new: 7.4, old 11.3



Slide the window along the epipolar line until  $w \cdot w'$  is maximized.

Normalized Correlation: minimize  $\theta$  instead.  $\leftrightarrow$  Minimize  $|w - w'|$ .<sup>2</sup>

# Cross-correlation of neighbourhood regions



- left and right windows encoded as vectors  $w$  and  $w'$
- zero-mean vectors  $(w - \bar{w})$  and  $(w' - \bar{w}')$
- Normalized cross-correlation:

$$C(d) = \frac{1}{\|w - \bar{w}\|} \frac{1}{\|w' - \bar{w}'\|} [(w - \bar{w}) \cdot (w' - \bar{w}')],$$

- Advantage: Invariant to intensity differences: Invariant to affine intensity transformation  $I' = \alpha I + \mu$

# Cross-correlation of neighbourhood regions



epipolar  
line

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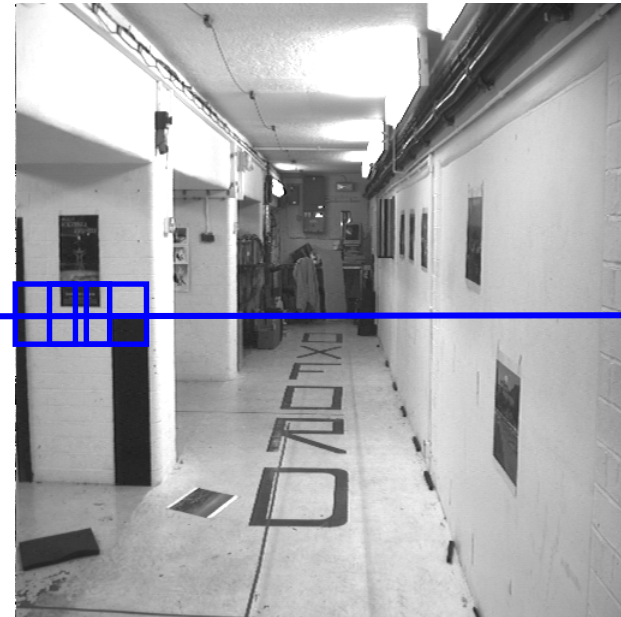
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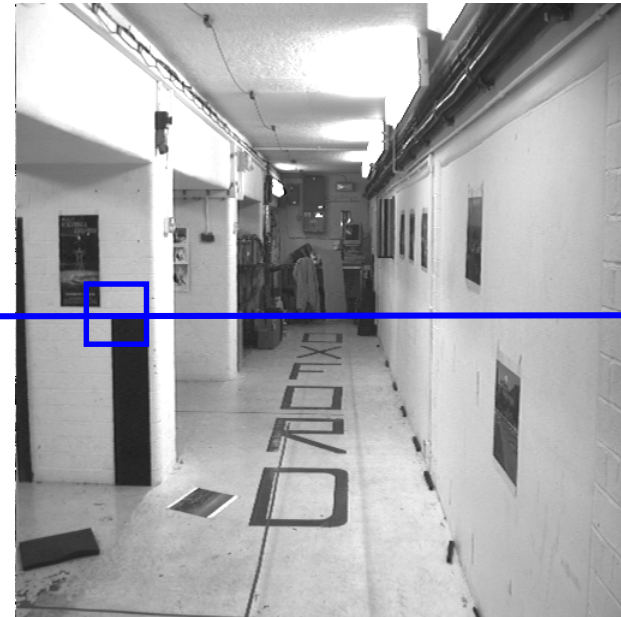
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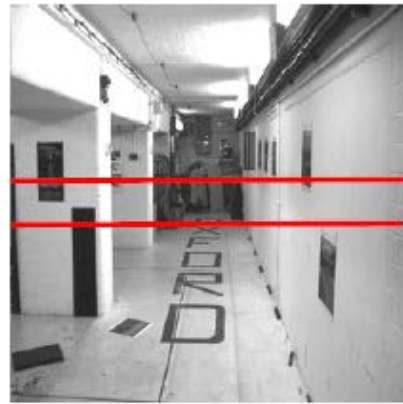
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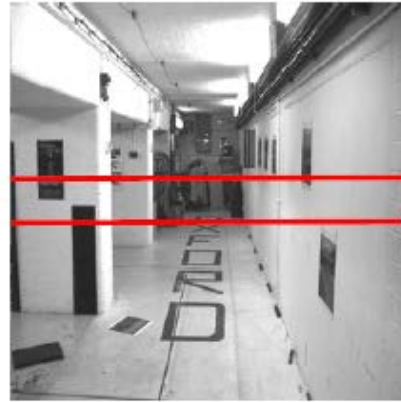
# Correlation-based window matching



left image band (x)



# Correlation-based window matching

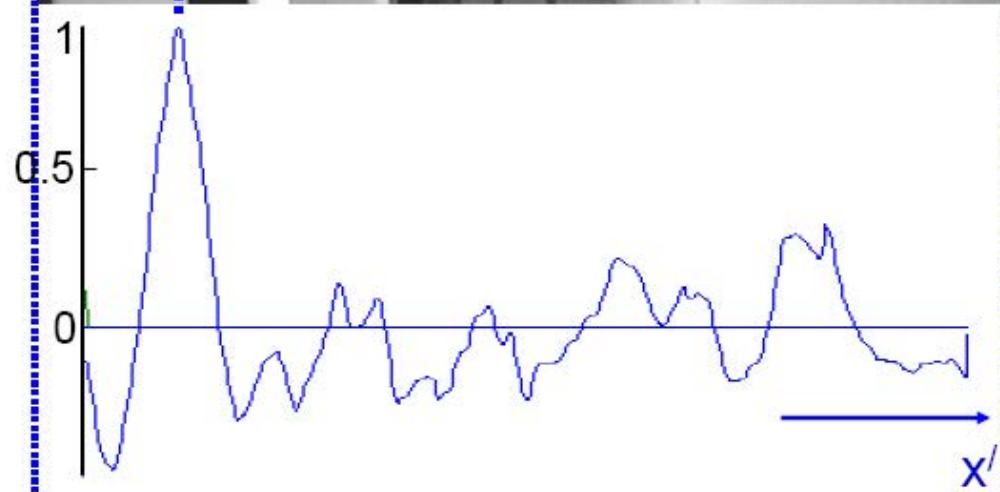


left image band ( $x$ )

right image band ( $x'$ )



# Correlation-based window matching



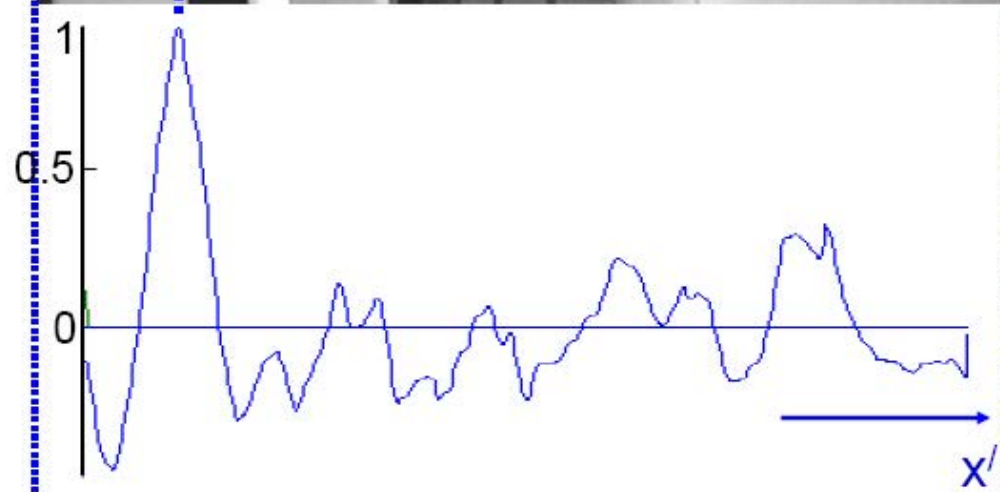
left image band ( $x$ )

right image band ( $x'$ )

cross  
correlation

disparity =  $x' - x$

# Correlation-based window matching



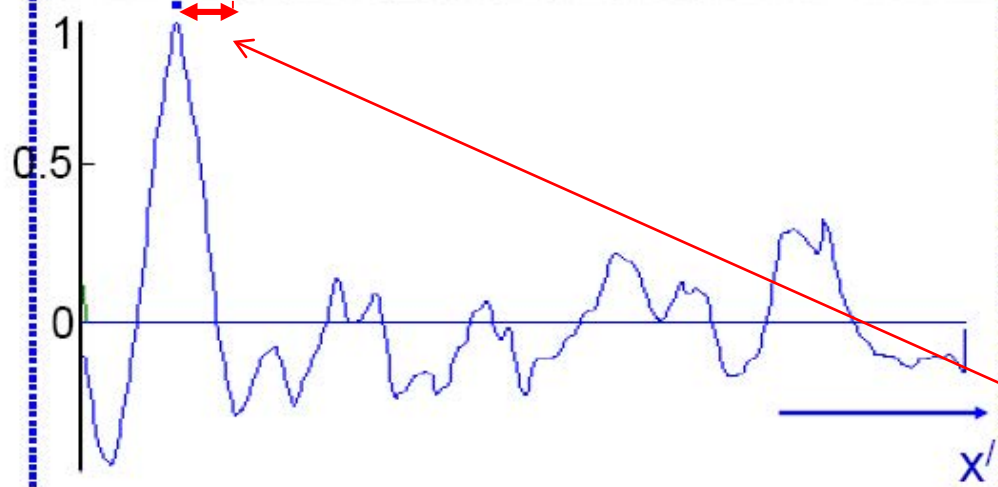
left image band ( $x$ )

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# Correlation-based window matching



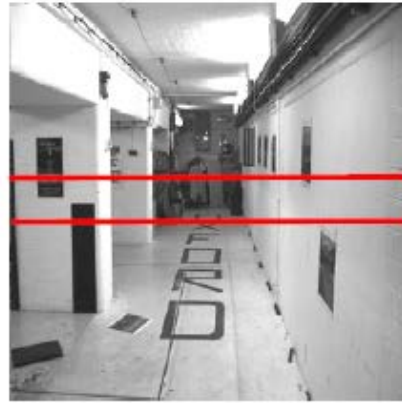
left image band ( $x$ )

right image band ( $x'$ )

cross  
correlation

disparity =  $x' - x$

# Textureless regions



target region

left image band (x)

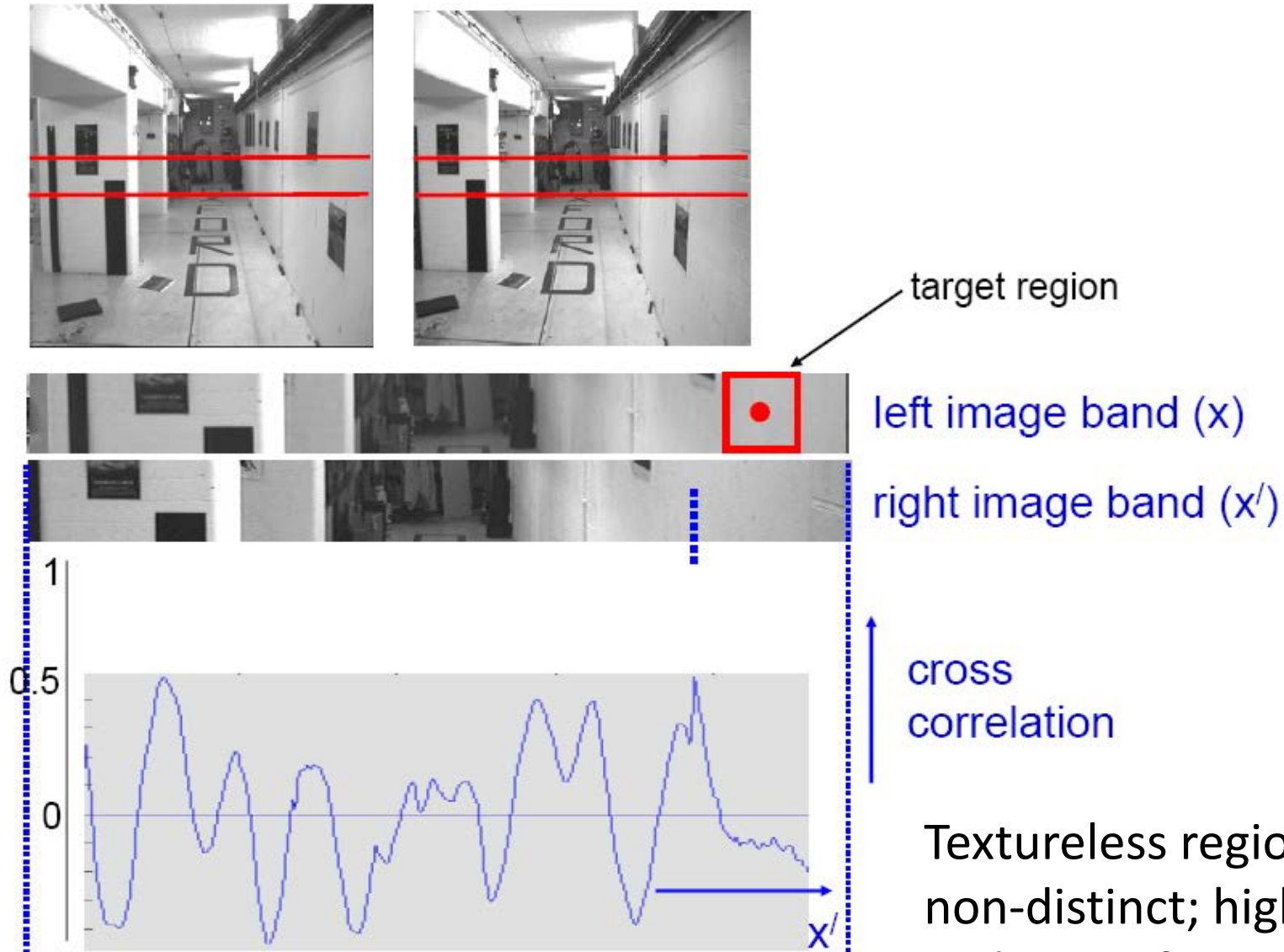
# Textureless regions



target region

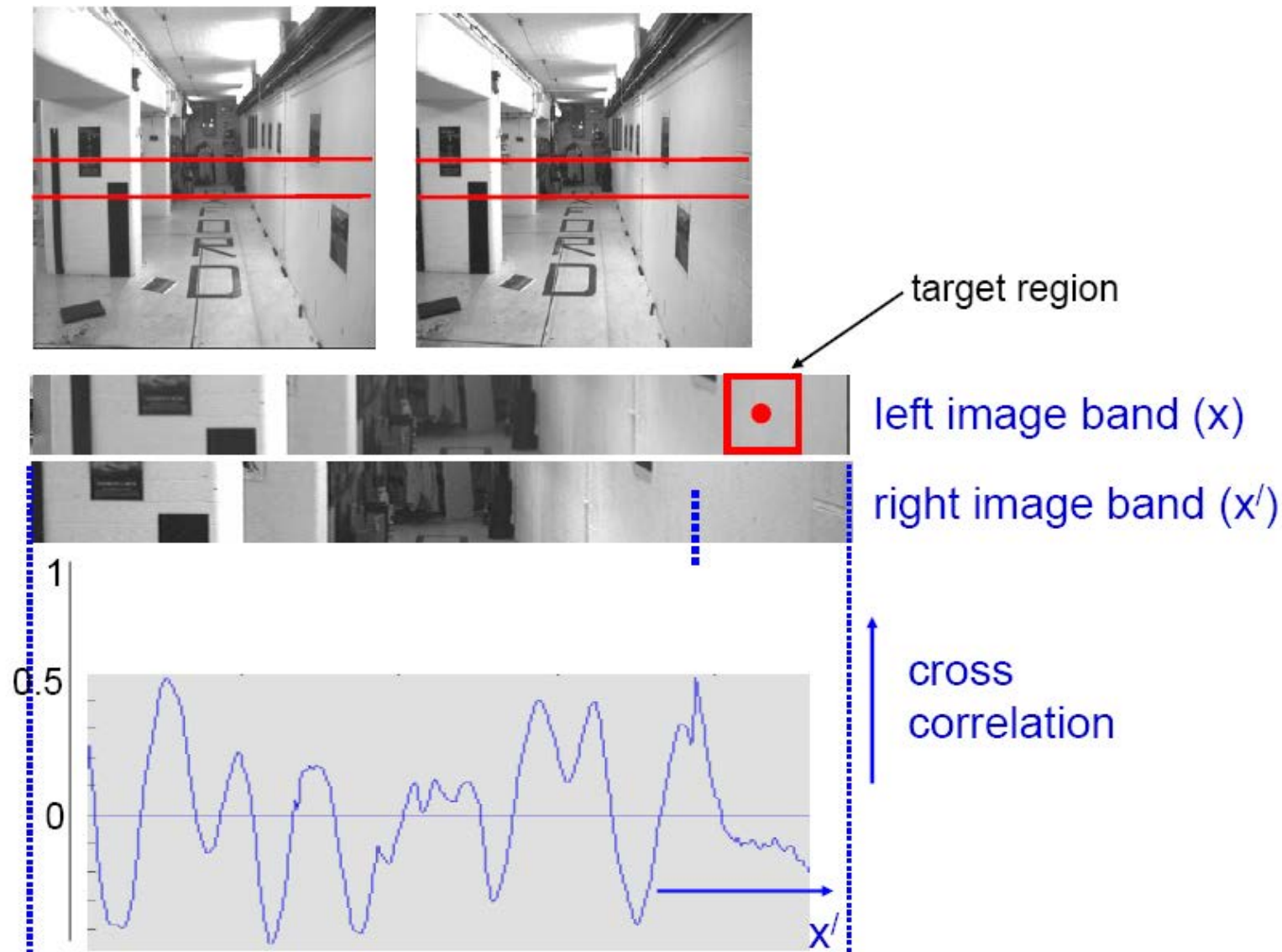
left image band (x)

# Textureless regions

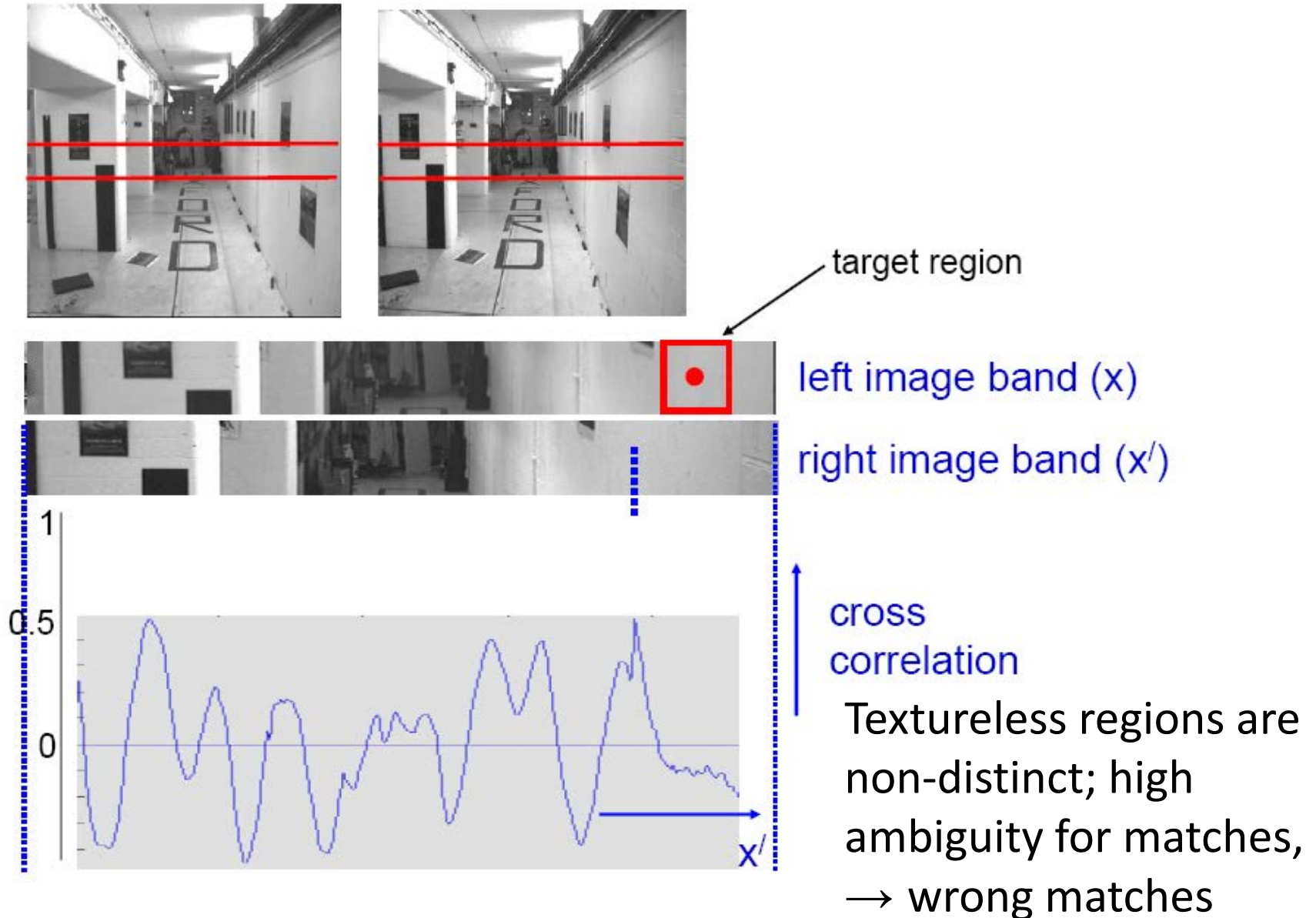




# Textureless regions



# Textureless regions

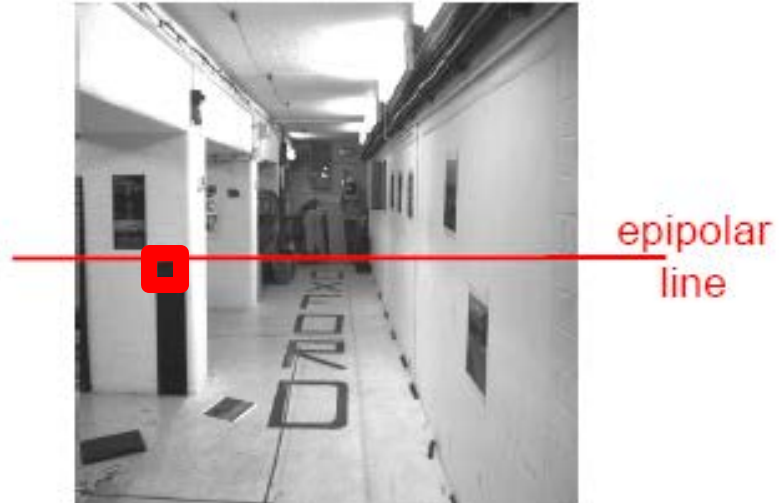




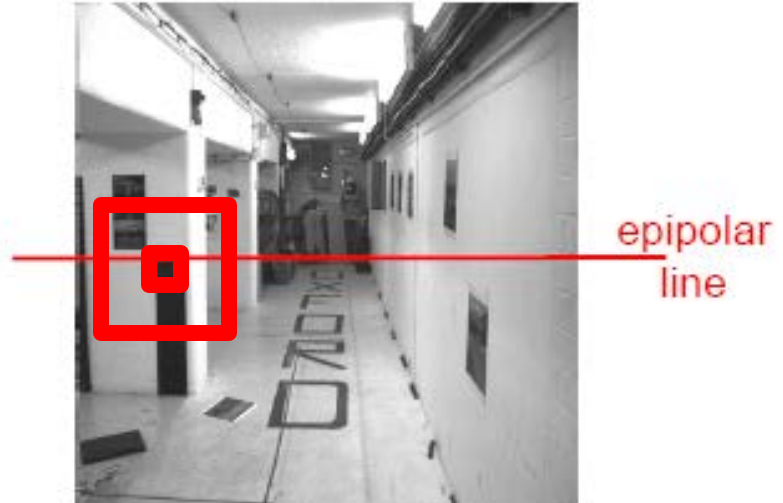
# Effect of window size



# Effect of window size



# Effect of window size



# Problems with window matching

Patch too small?

Patch too large?

*Can try variable patch size [Okutomi and Kanade],  
or arbitrary window shapes [Veksler and Zabih]*

# Effect of window size

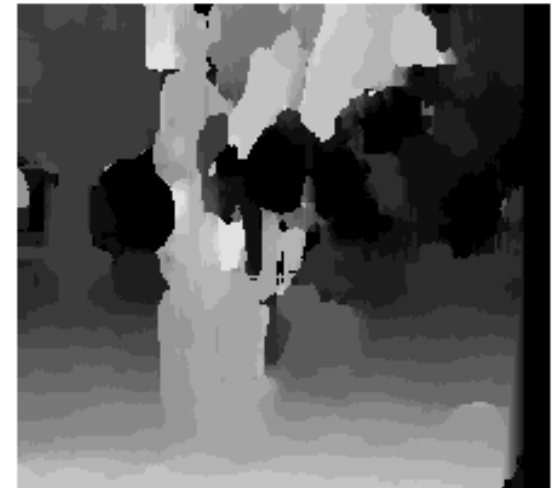


Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

# Effect of window size



$W = 3$



$W = 20$

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

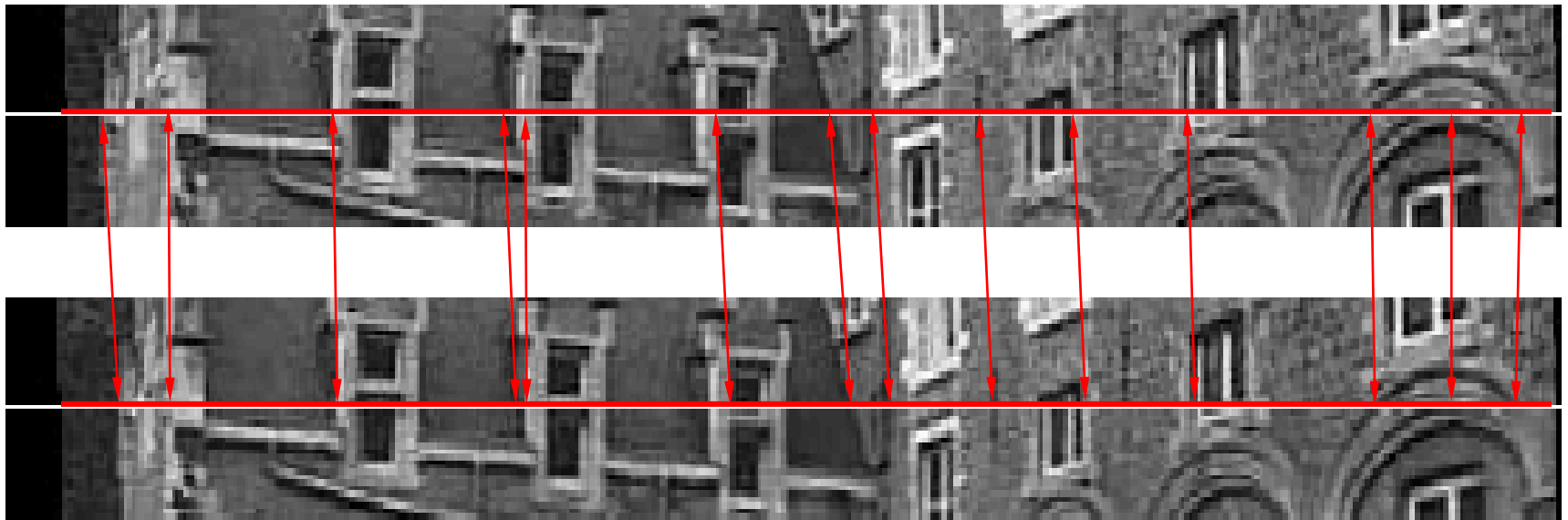
# Problems?

- Ordering
- Occlusion
- Foreshortening

## Solutions:

- Formulate Constraints
- Use more than two views
- Smart solutions vs. “brute force” searches with statistics

# Exploiting scene constraints

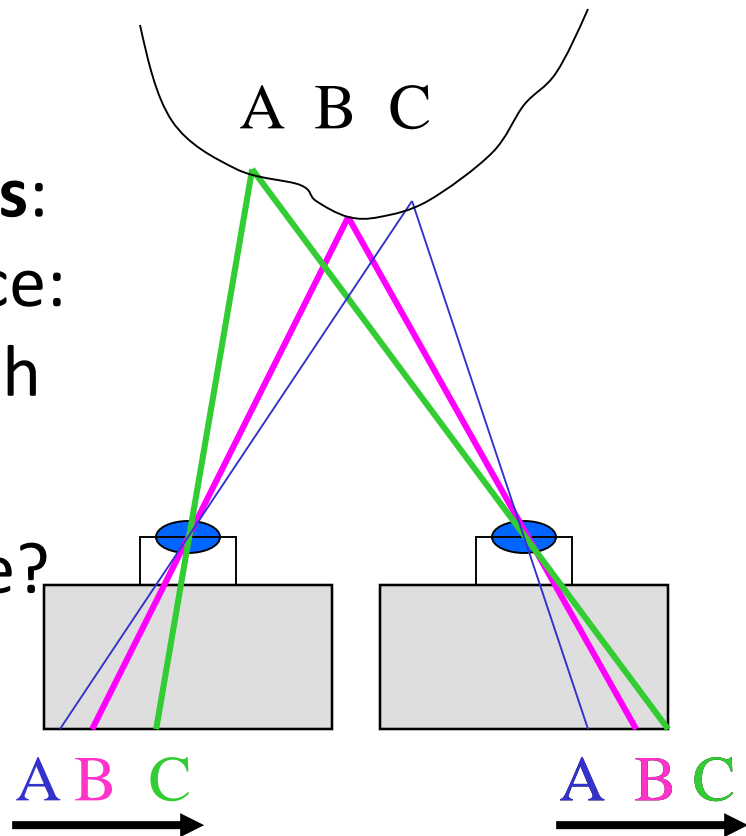




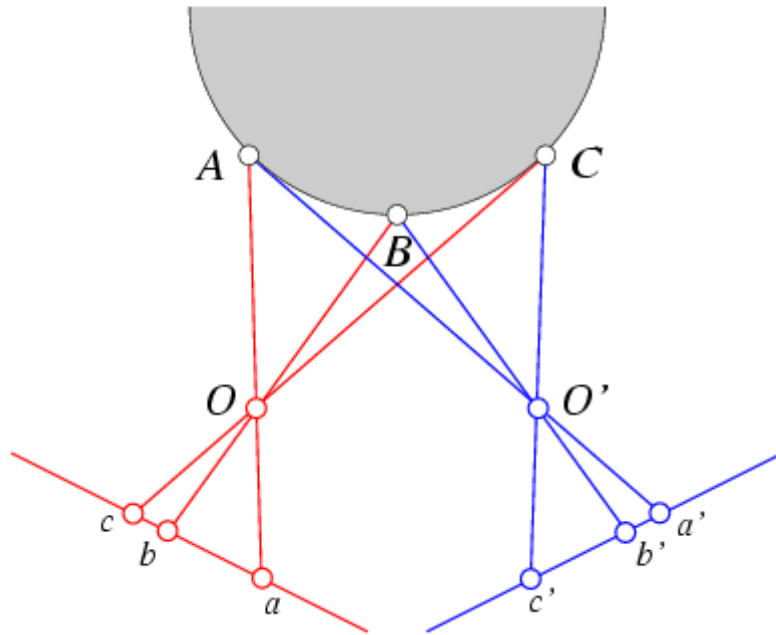
# Additional geometric constraints for correspondence

[Faugeras, pp. 321]

- **Ordering of points:**  
Continuous surface:  
same order in both  
images.
- Is that always true?

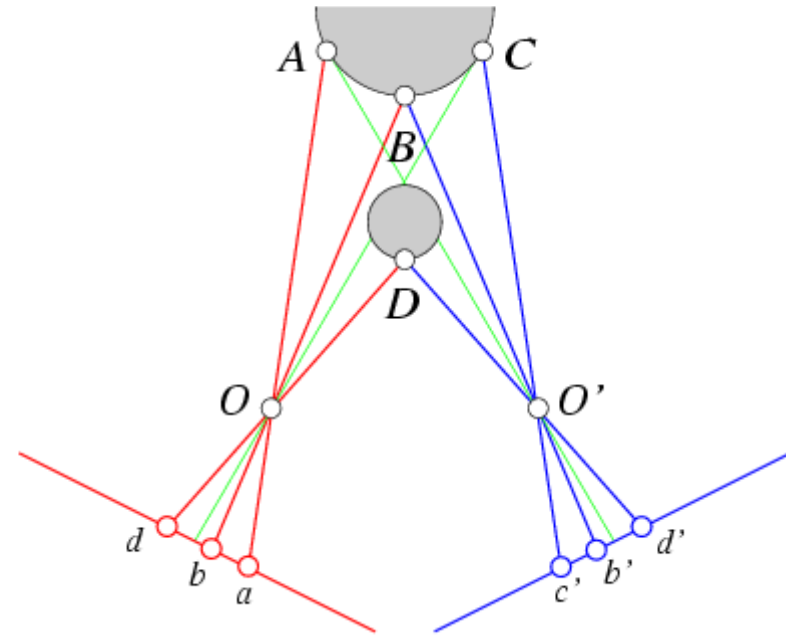
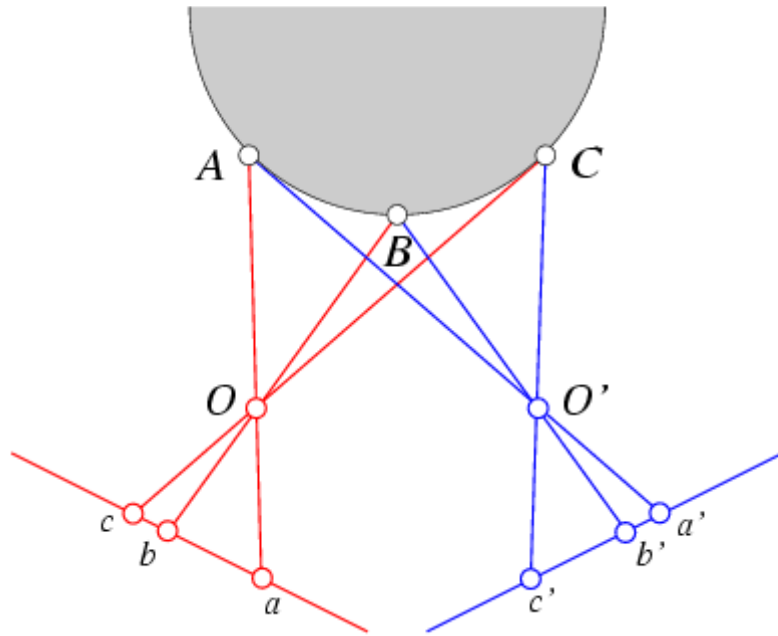


# The Ordering Constraint



In general the points are in the same order on both epipolar lines.

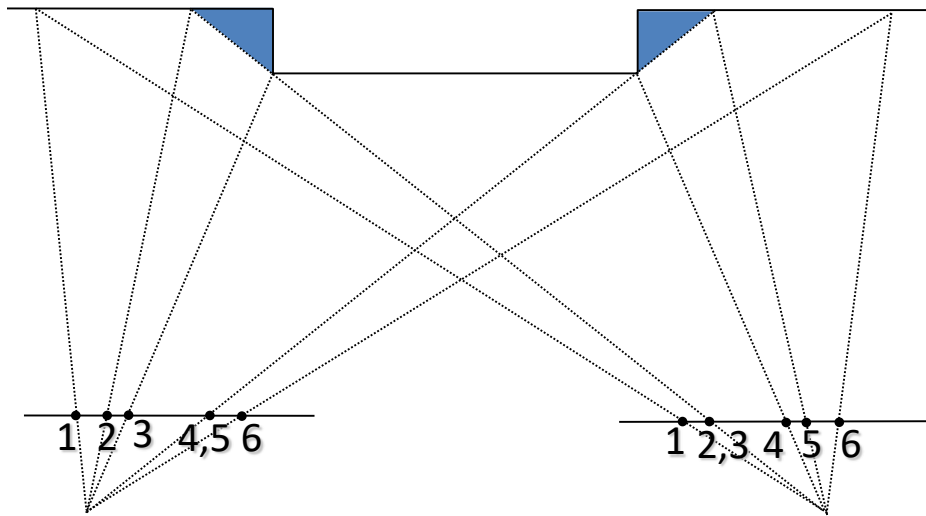
# The Ordering Constraint



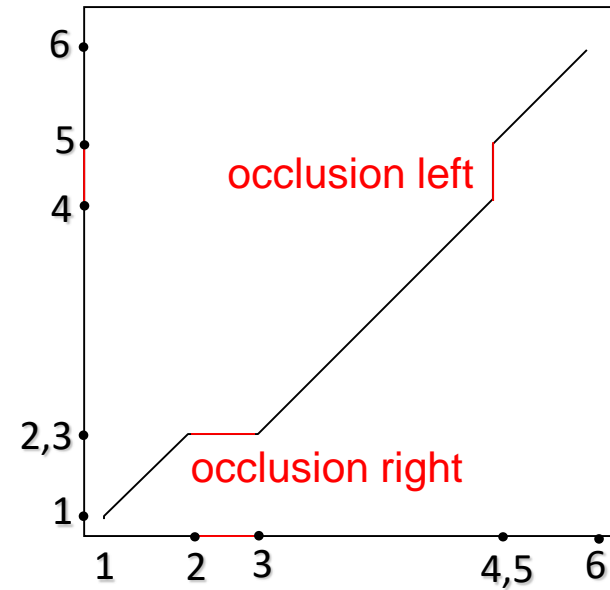
But it is not always the case..

# Ordering constraint

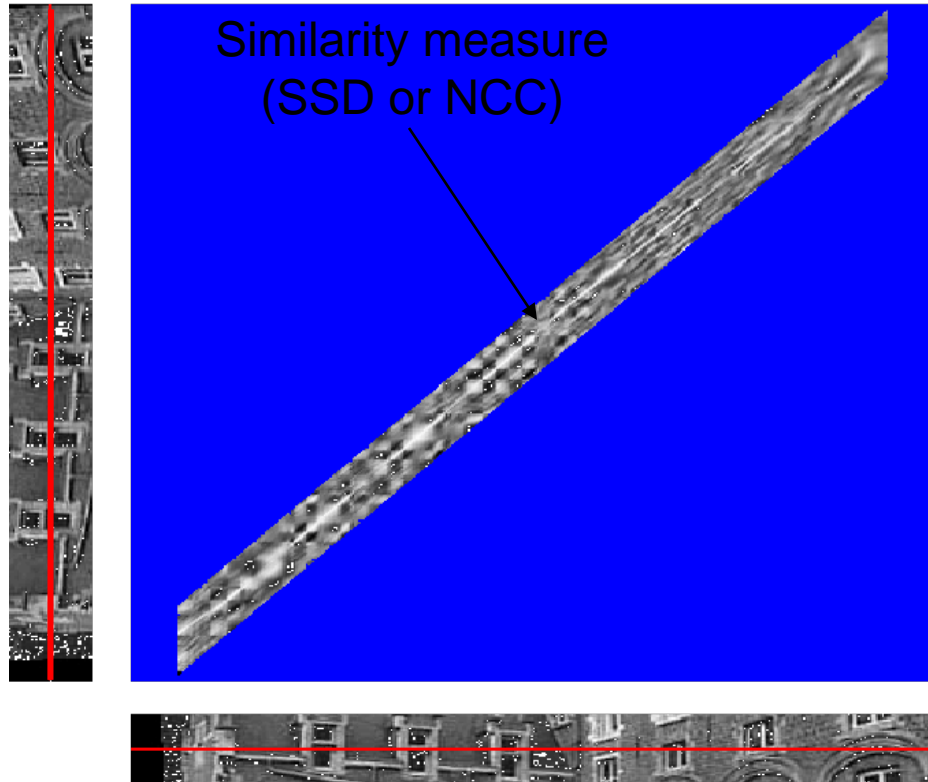
surface slice



surface as a path



# Stereo matching



## Constraints

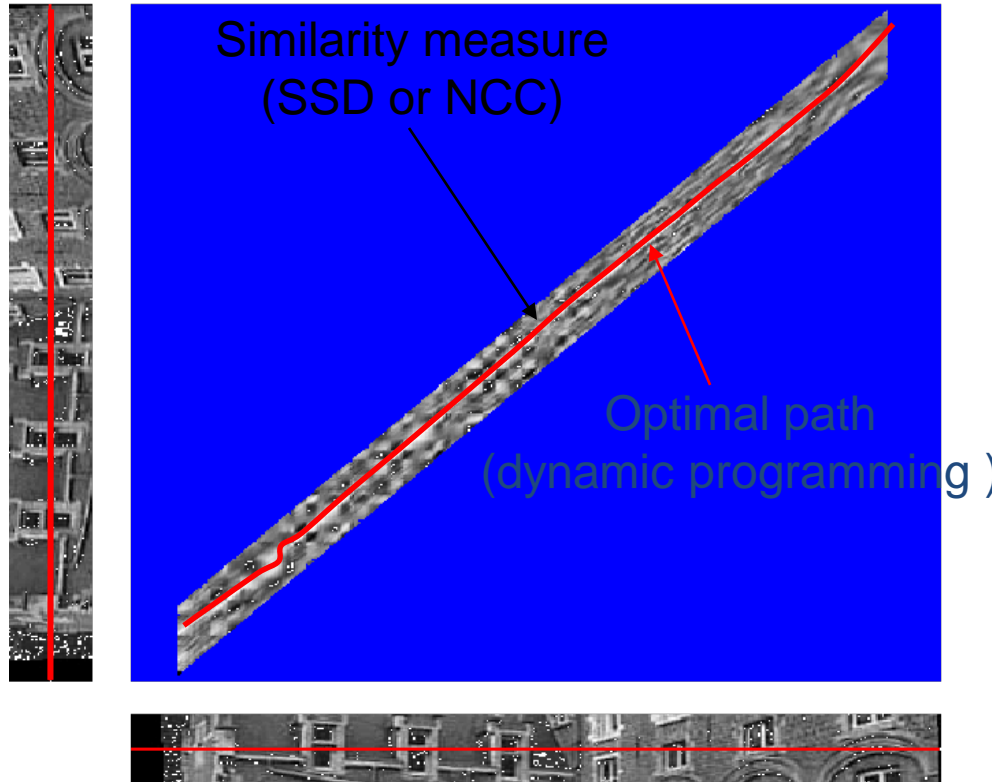
- epipolar
- ordering
- uniqueness
- disparity limit

## Trade-off

- Matching cost (data)
- Discontinuities (prior)

Consider all paths that satisfy the constraints  
pick best using dynamic programming

# Stereo matching



## Constraints

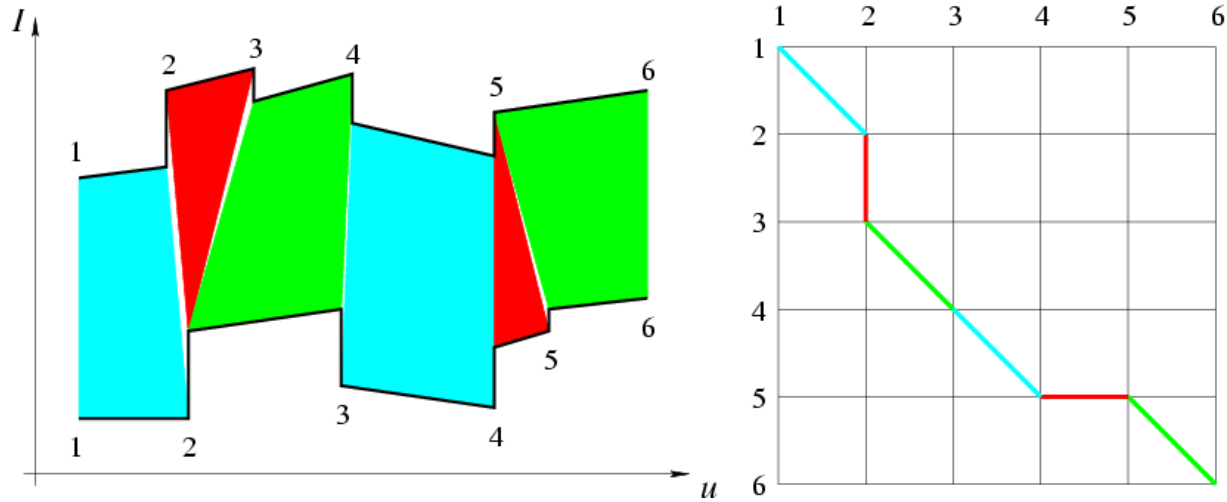
- epipolar
- ordering
- uniqueness
- disparity limit

## Trade-off

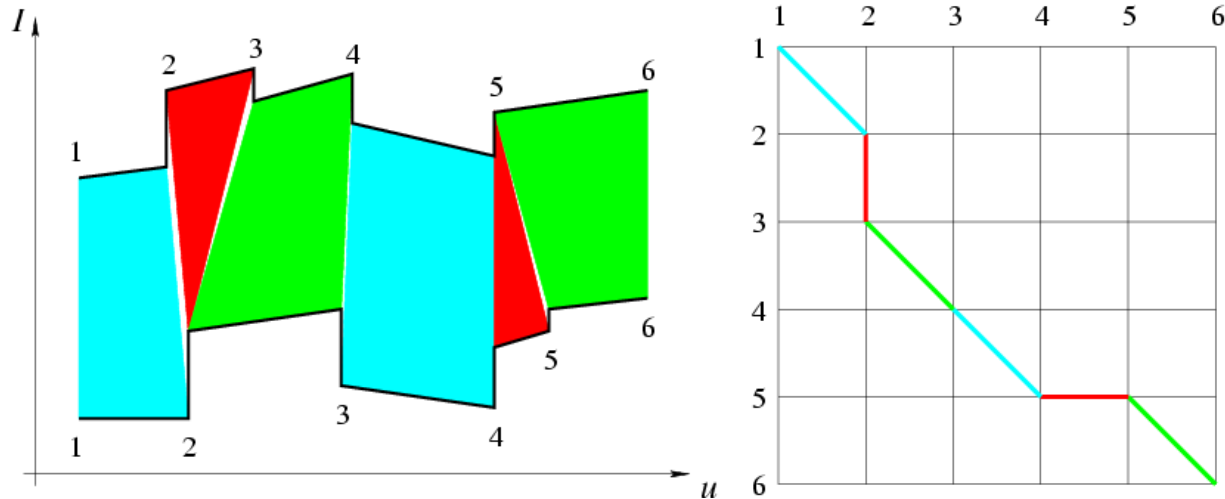
- Matching cost (data)
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Consider all paths that satisfy the constraints  
pick best using dynamic programming

# Dynamic Programming (Baker and Binford, 1981)



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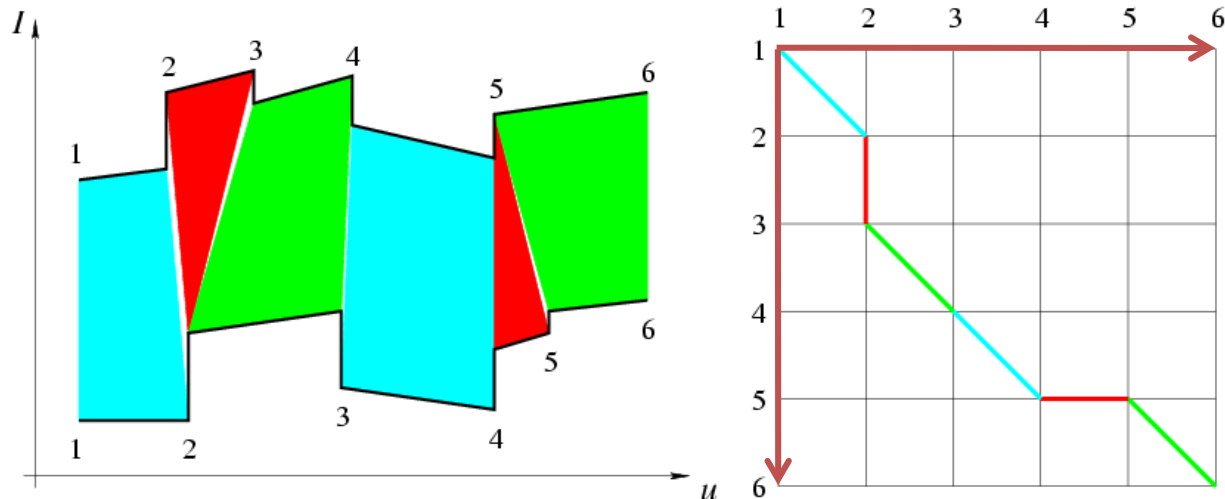


```

% Loop over all nodes (k, l) in ascending order.
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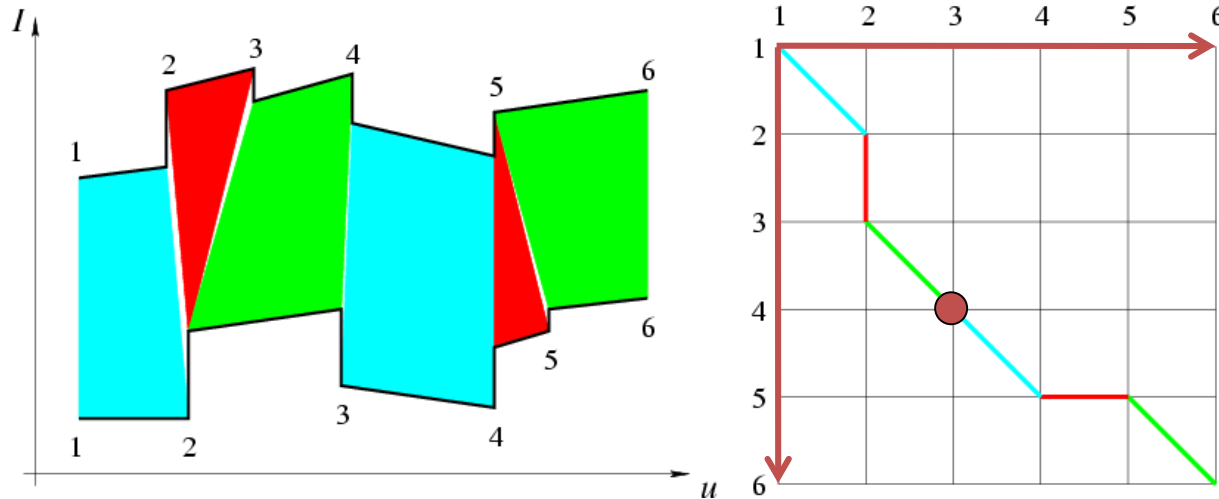
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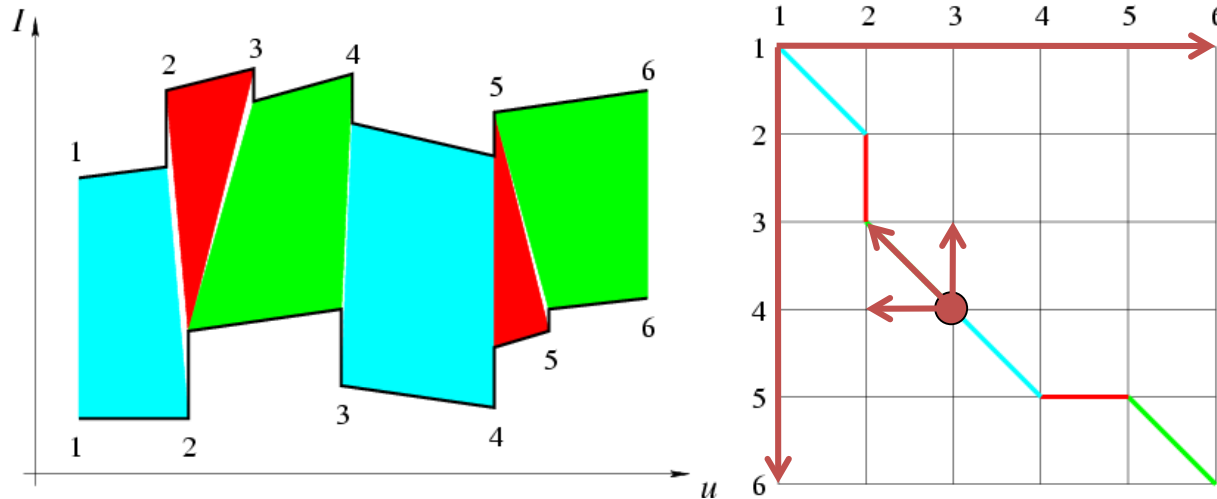
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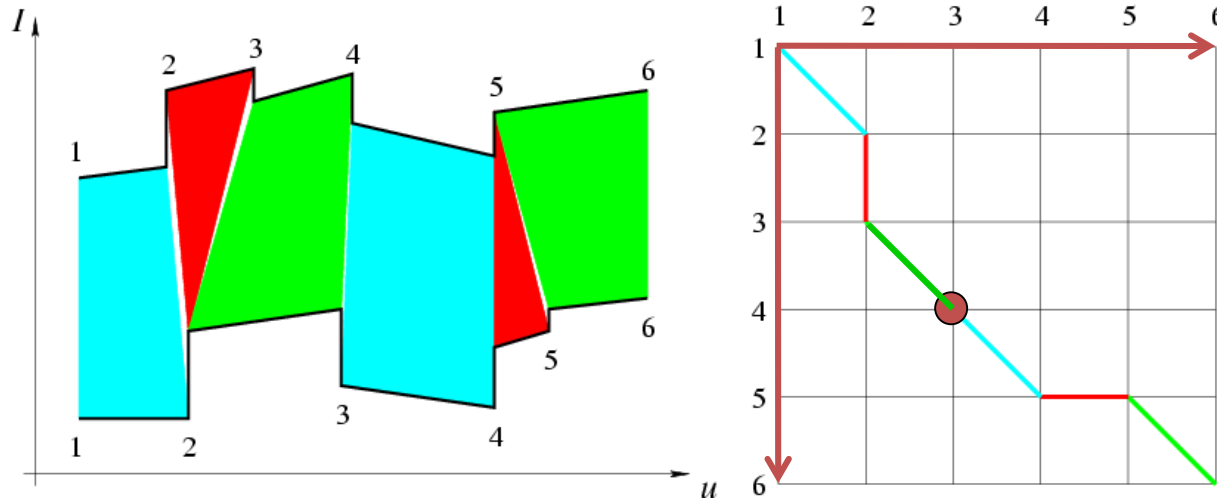
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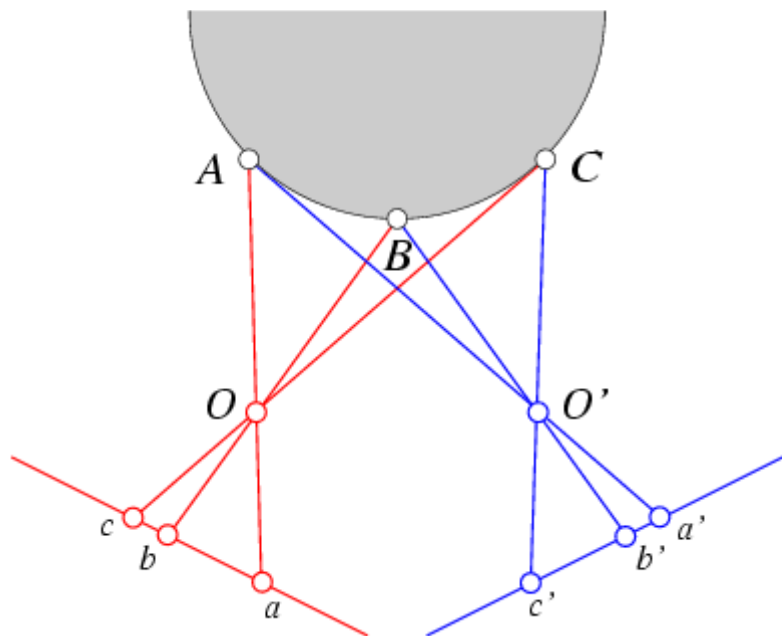
# Dynamic Programming (Baker and Binford, 1981)



```

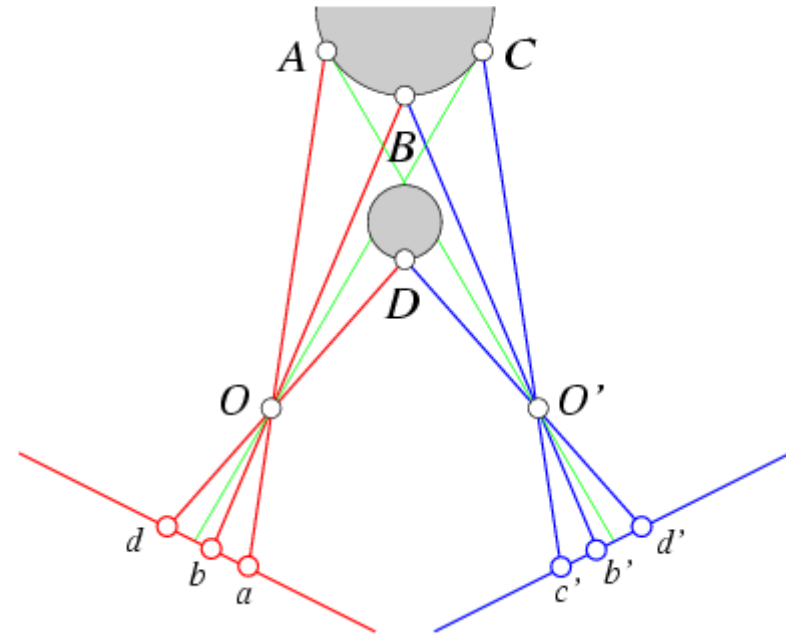
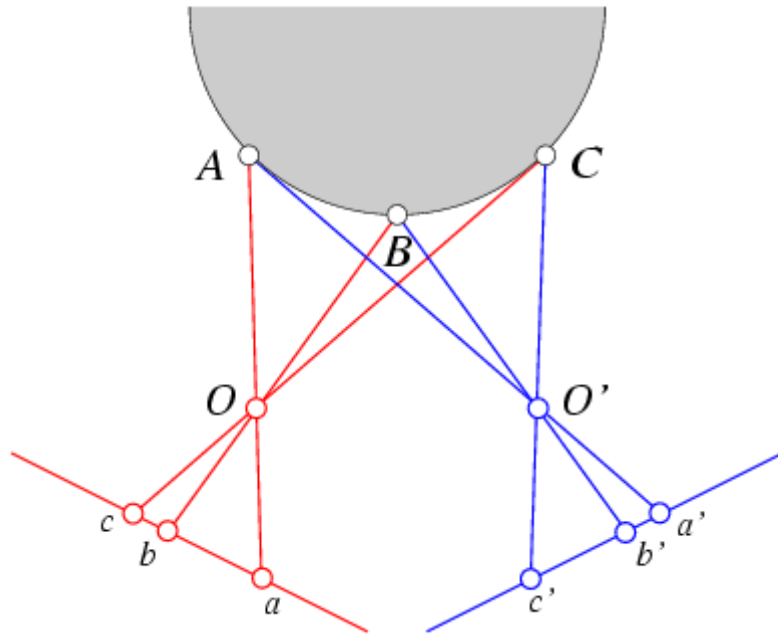
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# The Ordering Constraint



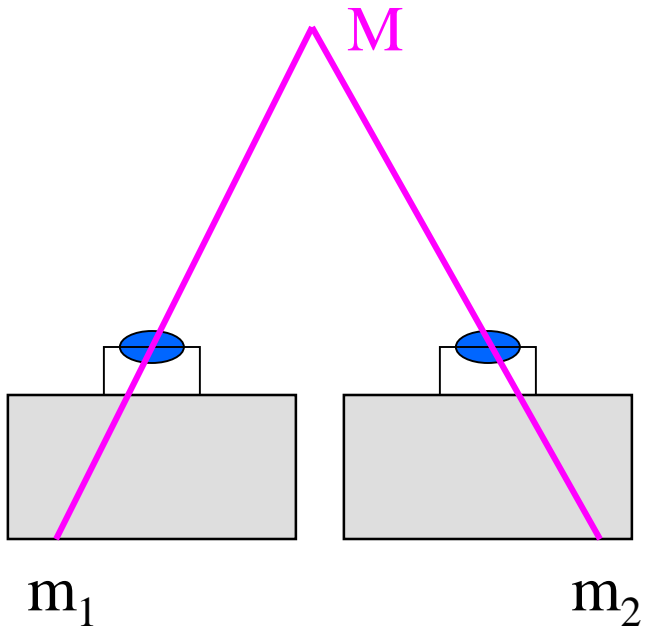
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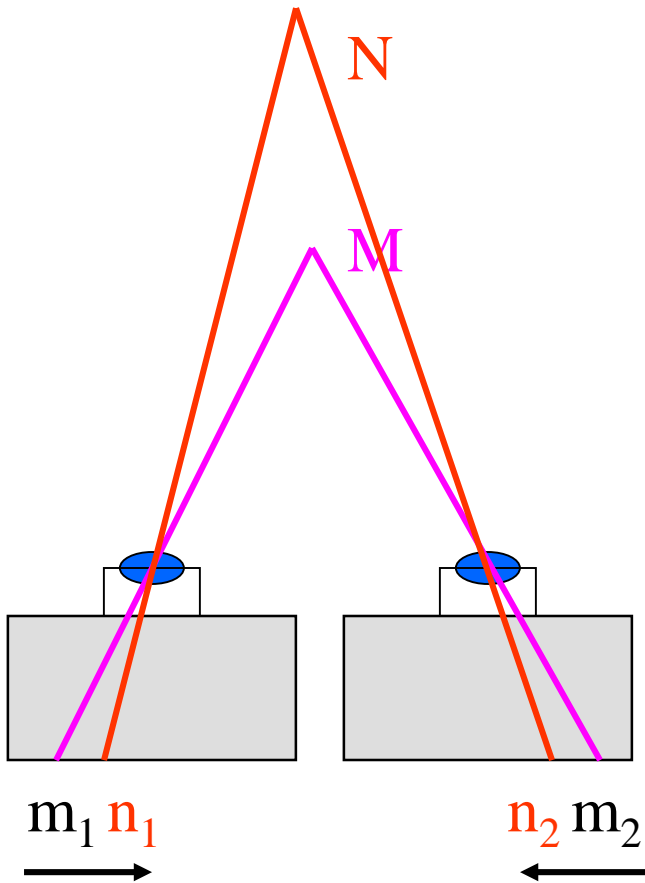


But it is not always the case..

# Forbidden Zone

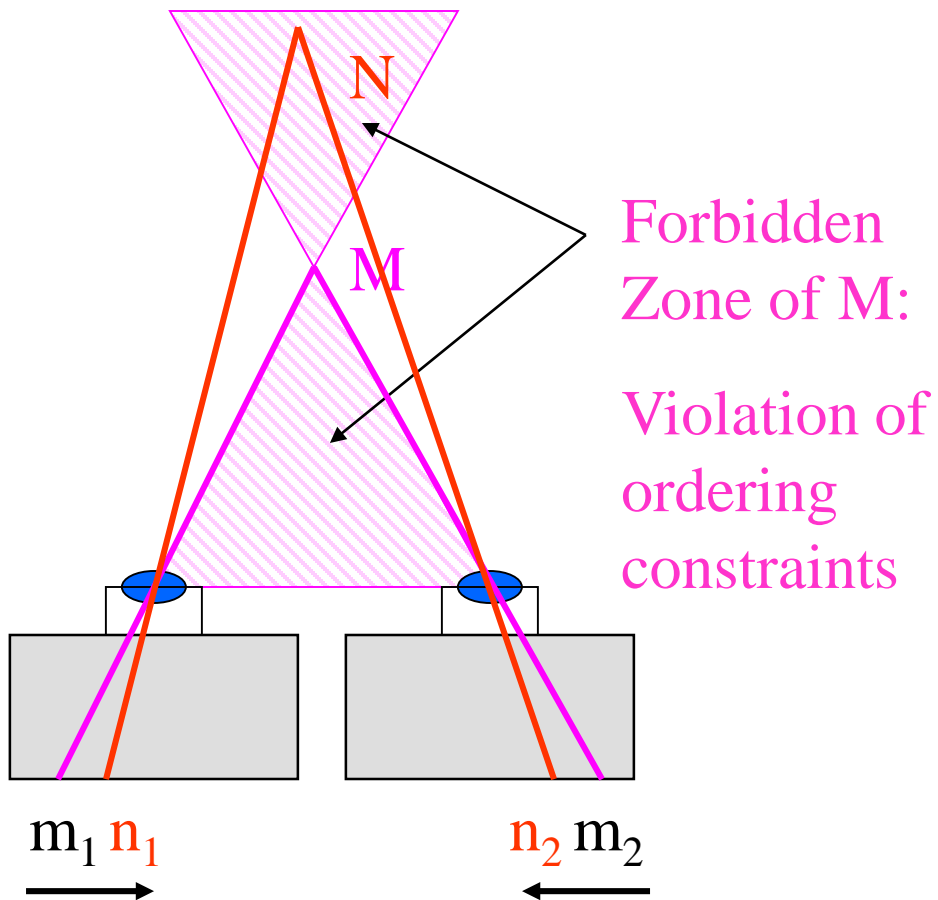


# Forbidden Zone

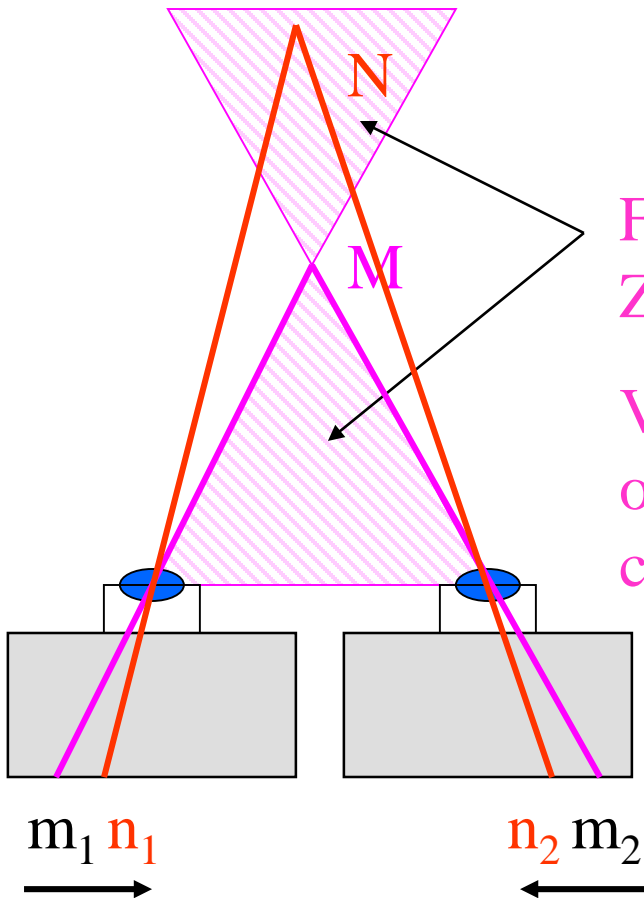




# Forbidden Zone



# Forbidden Zone



## Practical applications:

- Object bulges out: ok
- In general: ordering across whole image is not reliable feature
- Use ordering constraints for neighbors of M within small neighborhood only

# Disparity map

image  $I(x,y)$



Disparity map  $D(x,y)$

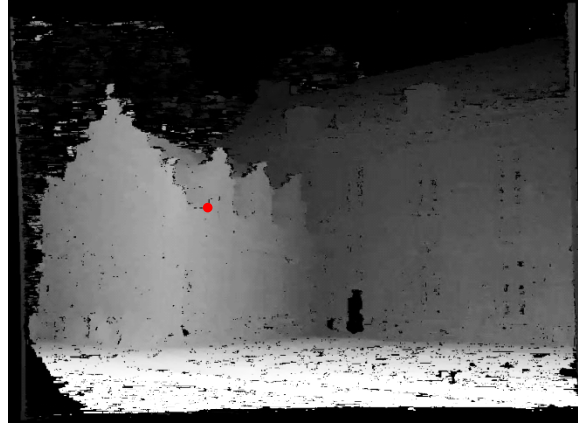


image  $I'(x',y')$



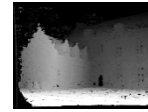
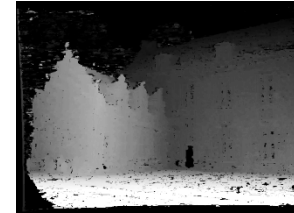
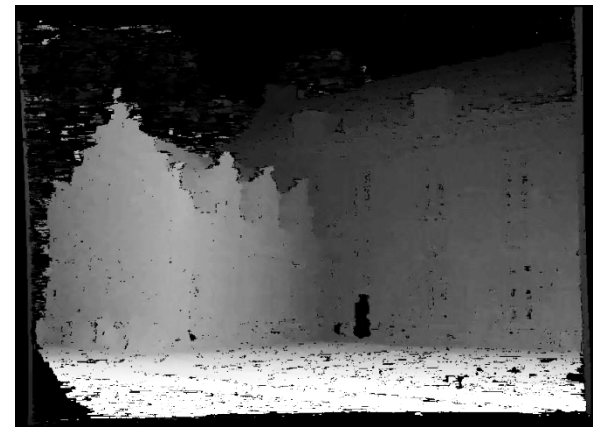
$$(x',y')=(x+D(x,y),y)$$

# Hierarchical stereo matching

Allows faster computation

Deals with large disparity ranges

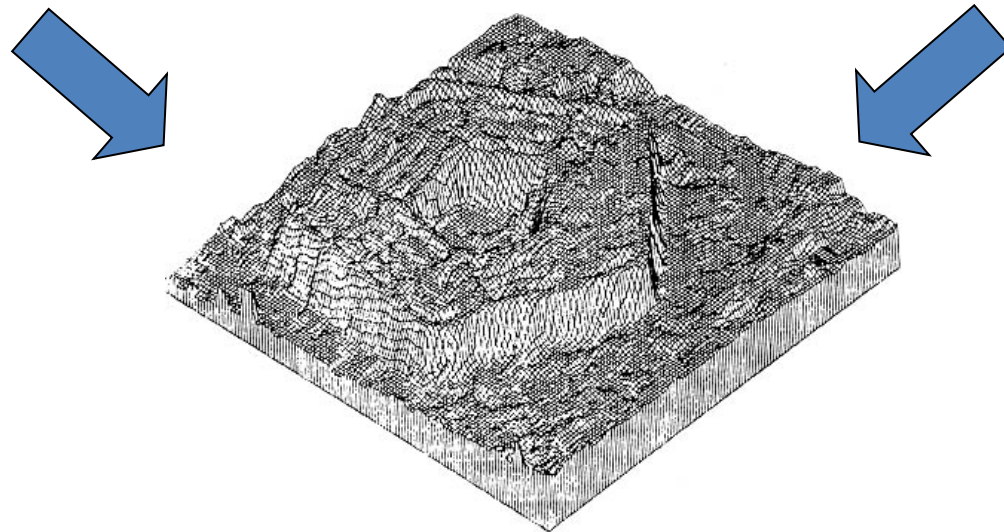
Downsampling  
(Gaussian pyramid)



Disparity propagation



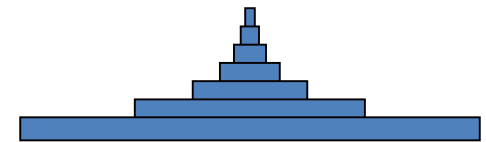
# Dynamic Programming (Ohta and Kanade, 1985)



# Real-time stereo on graphics hardware

Ruigang Yang and Marc Pollefeys, UNC

- Computes Sum-of-Square-Differences
- Hardware mip-map generation used to aggregate results over support region
- Trade-off between small and large support window



Shape of a kernel  
for summing up 6 levels

140M disparity hypothesis/sec on Radeon 9700pro  
e.g. 512x512x20disparities at 30Hz



# Stereo results

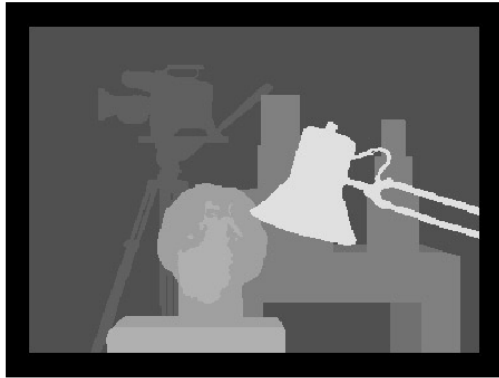
- Data from University of Tsukuba
- Similar results on other images without ground truth



Scene



Ground truth



True disparities



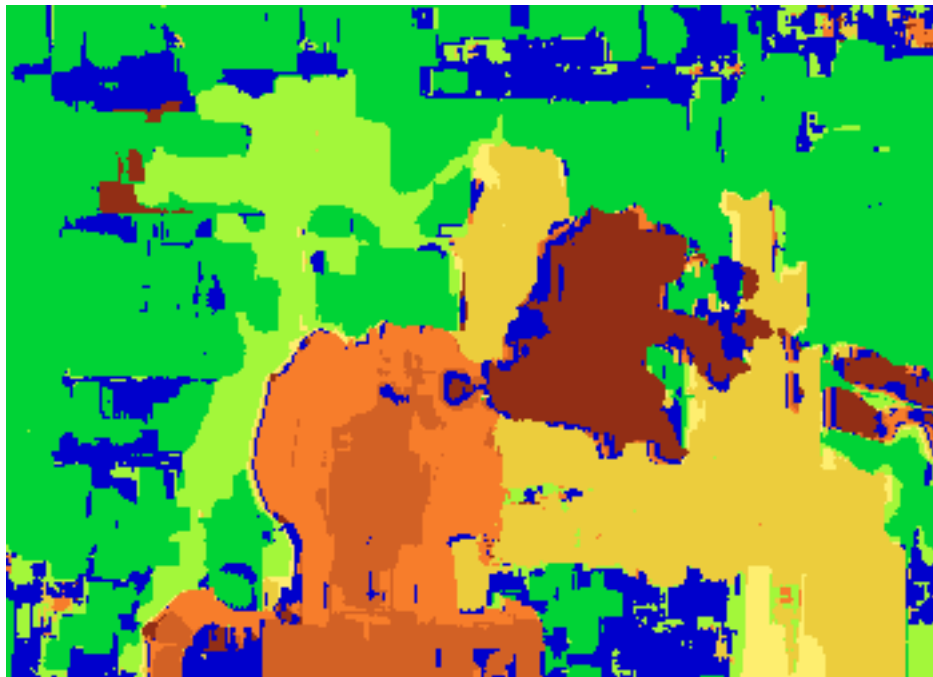
16 – Fast Correlation



\*1 – SSD+MF



# Results with window correlation



Window-based matching  
(best window size)



Ground truth

# Results with better method



State of the art method

Boykov et al., [Fast Approximate Energy Minimization via Graph Cuts](#),  
International Conference on Computer Vision, September 1999.



Ground truth

# Material I

- <http://vision.middlebury.edu/stereo/>
- (online stereo pairs and truth (depth maps))
- Stereo correspondence software: e.g.  
<http://vision.middlebury.edu/stereo/data/scenes2001/data/imagehtml/tsukuba.html>
- CVonline compendium:  
<http://homepages.inf.ed.ac.uk/rbf/CVonline/>

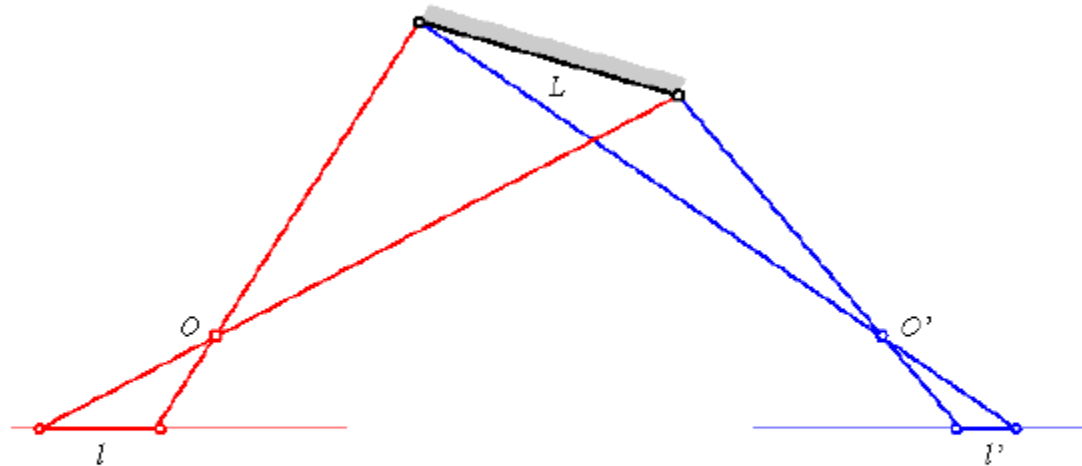
# Material II

- Epipolar Geometry, Rectification:
  - [http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL\\_COPIES/FUSIELLO2/rectif\\_cvol.html](http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/FUSIELLO2/rectif_cvol.html)
  - and:  
[http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL\\_COPIES/OWENS/LECT11/node11.html](http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/OWENS/LECT11/node11.html)
- Stereo:
  - [http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL\\_COPIES/OWENS/LECT11/lect11.html](http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/OWENS/LECT11/lect11.html)
- 3D Reconstruction:
  - [http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL\\_COPIES/OWENS/LECT11/node8.html](http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/OWENS/LECT11/node8.html)

# Additional Materials

# Problem: Foreshortening

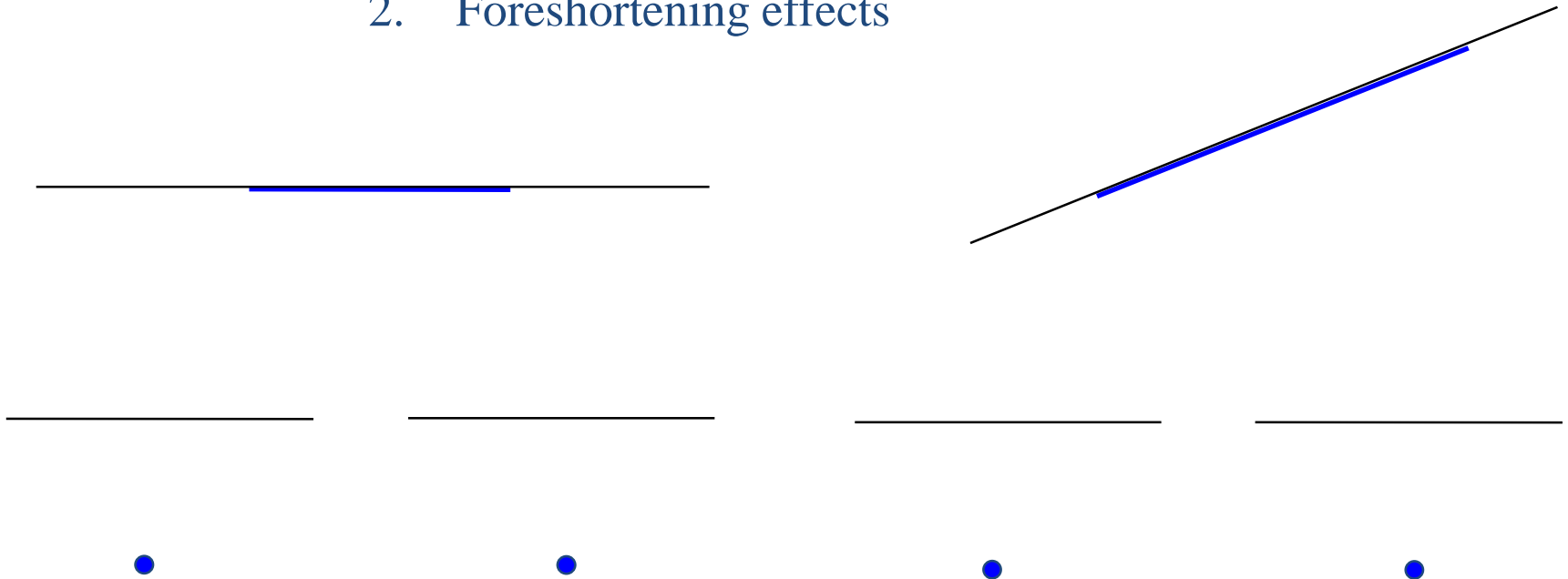
Window methods assume fronto-parallel surface at 3-D point.



Initial estimates of the disparity can be used to warp the correlation windows to compensate for unequal amounts of foreshortening in the two pictures [Kass, 1987; Devernay and Faugeras, 1994].

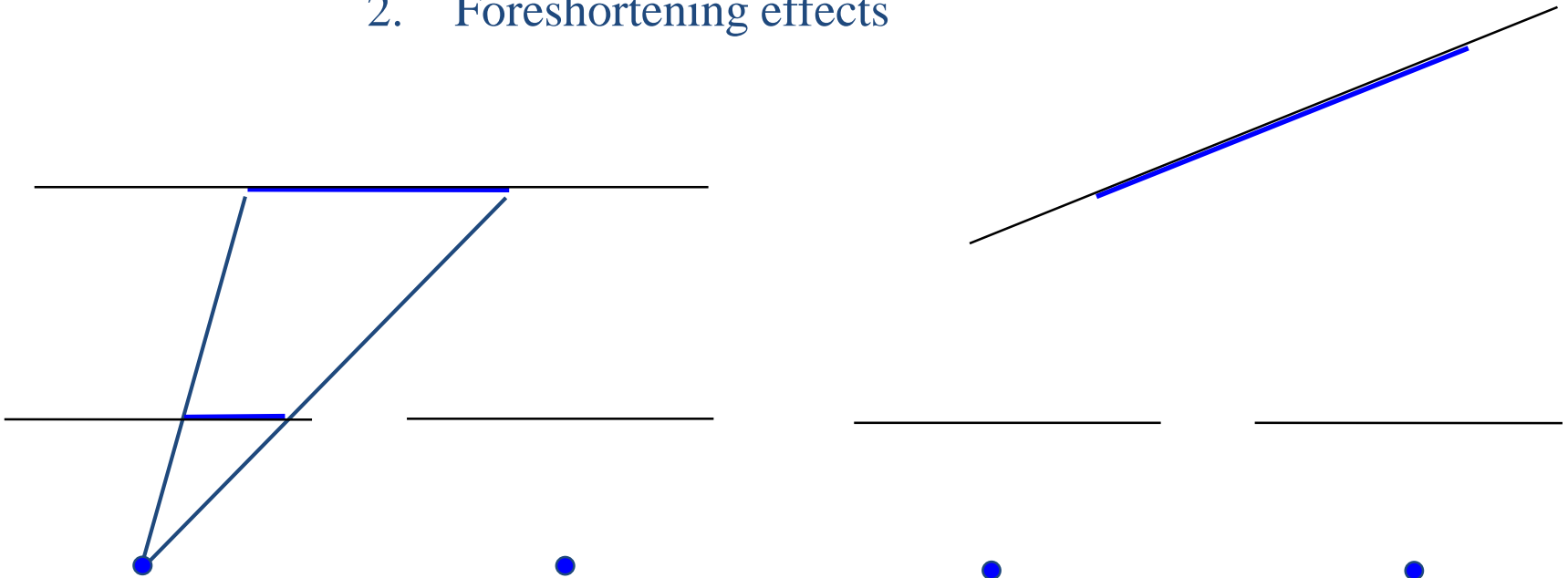
Why is cross-correlation such a poor measure in the second case?

1. The neighbourhood region does not have a “distinctive” spatial intensity distribution
2. Foreshortening effects



Why is cross-correlation such a poor measure in the second case?

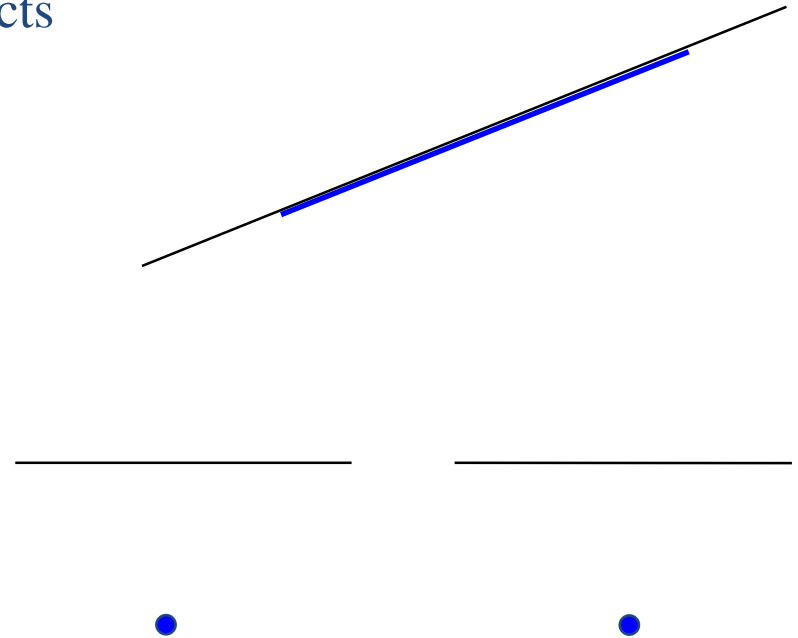
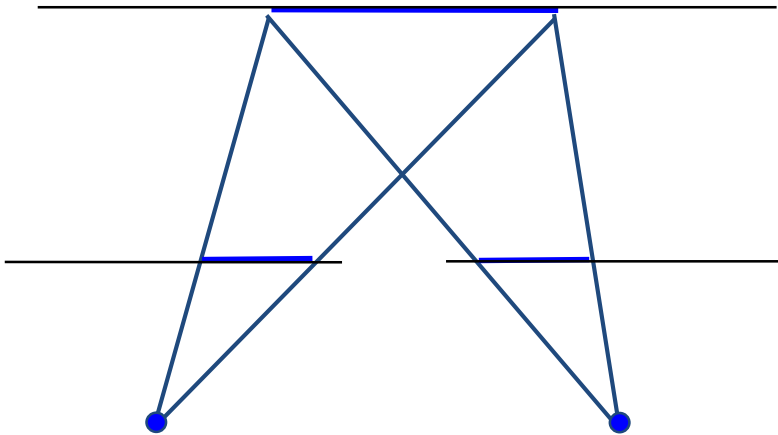
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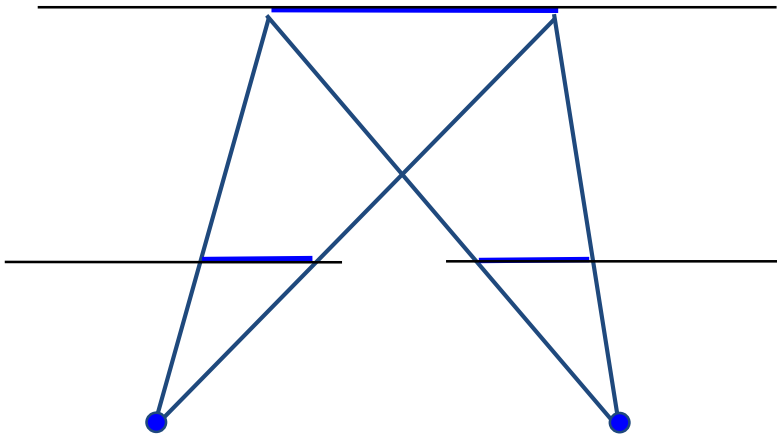
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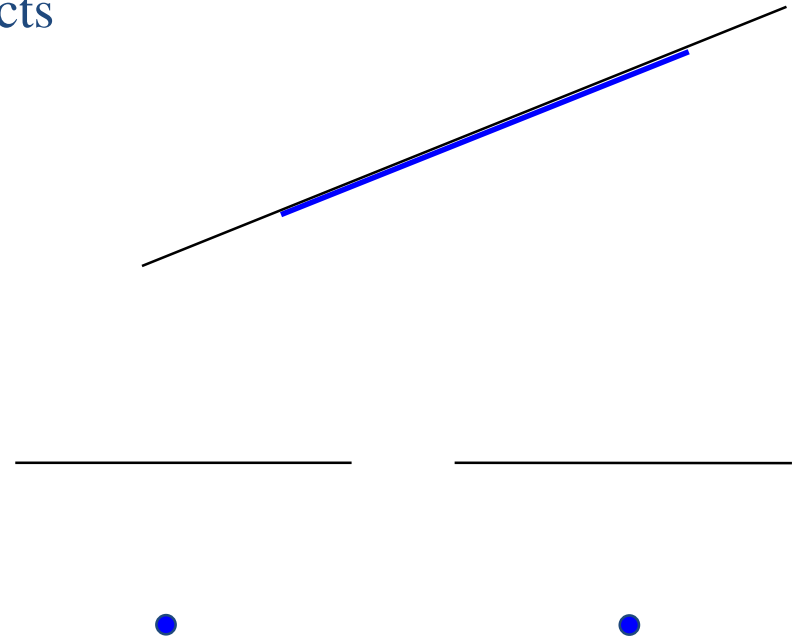
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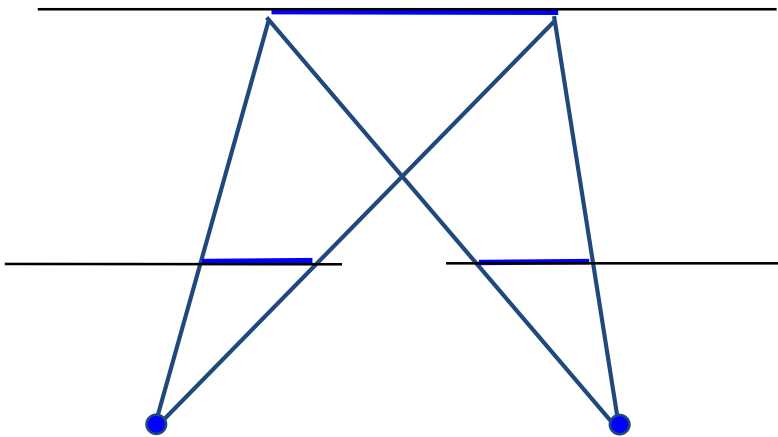
fronto-parallel surface

imaged length the same



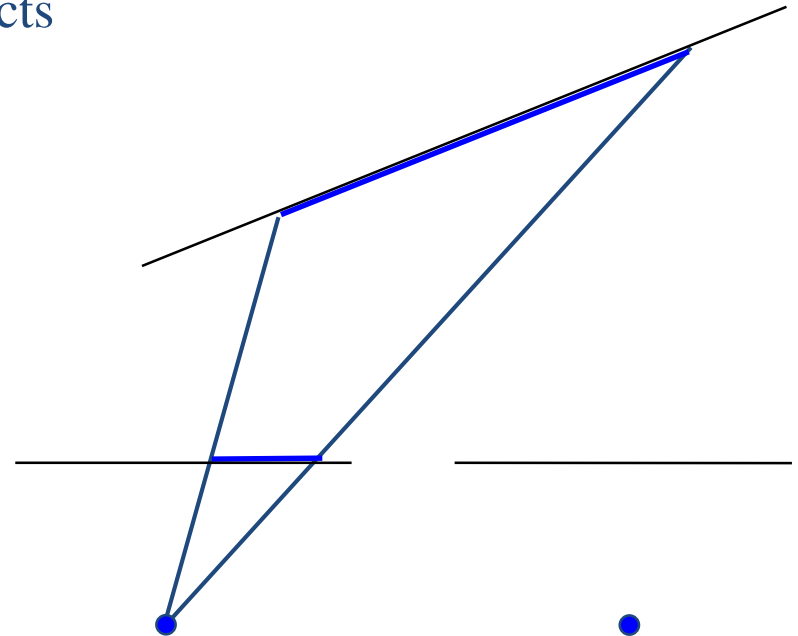
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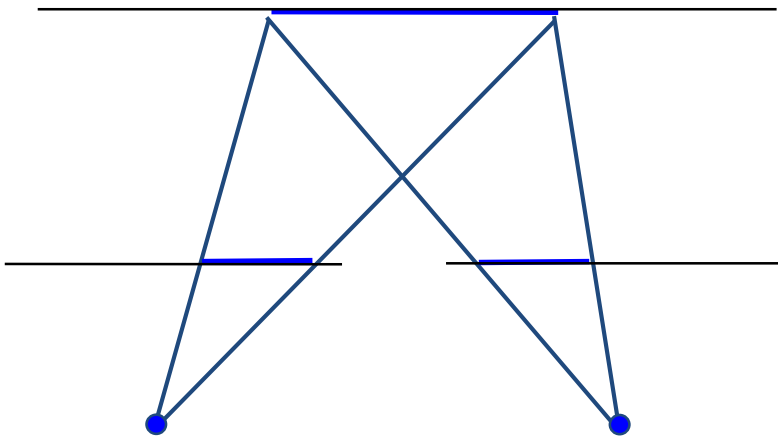
fronto-parallel surface

imaged length the same



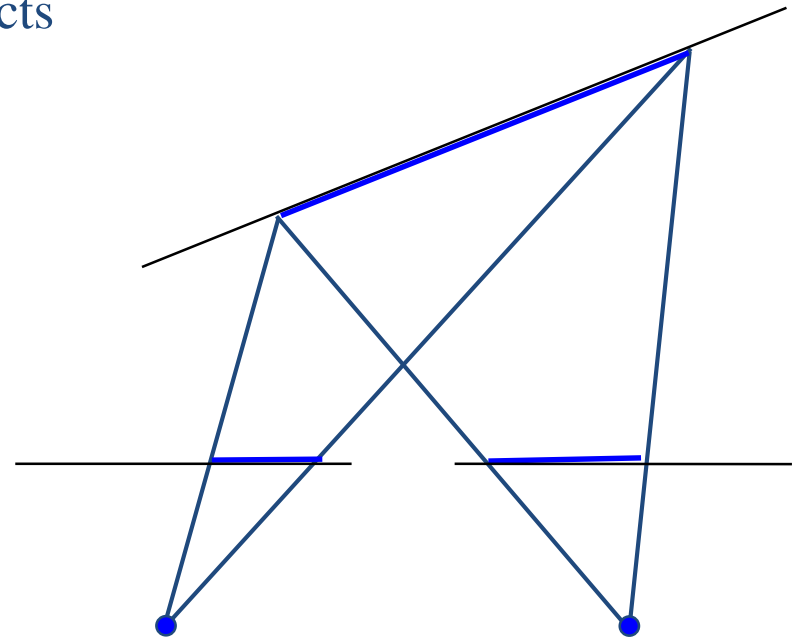
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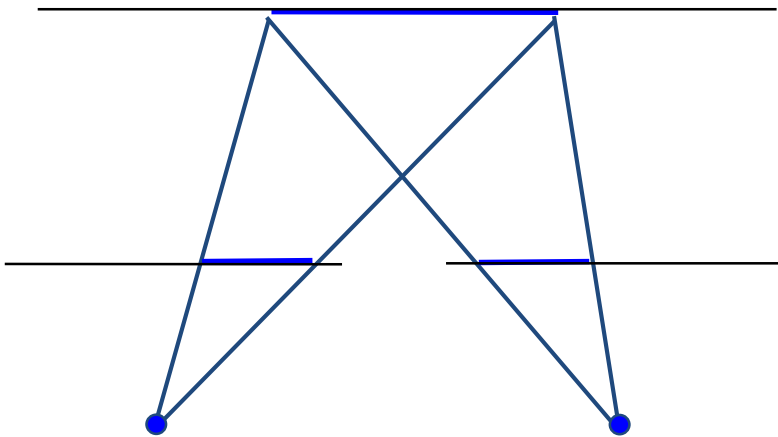
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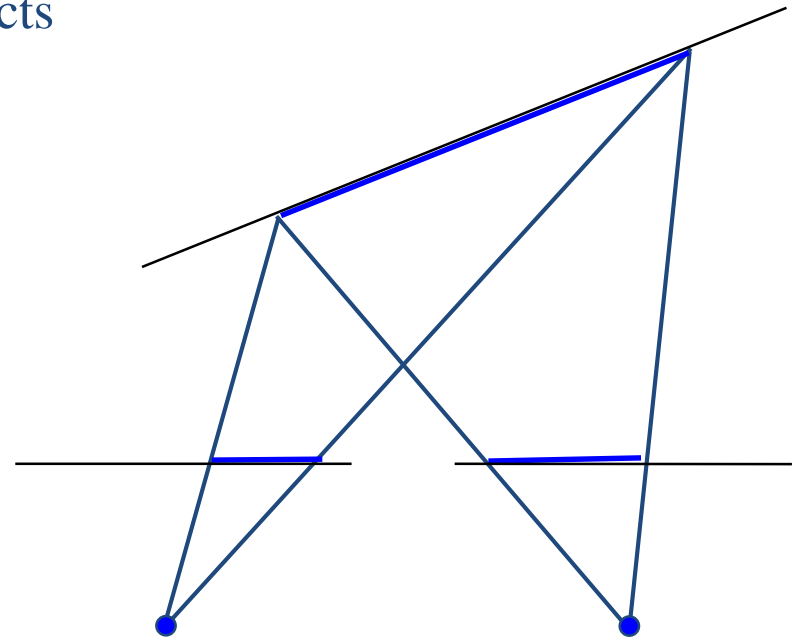
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fronto-parallel surface

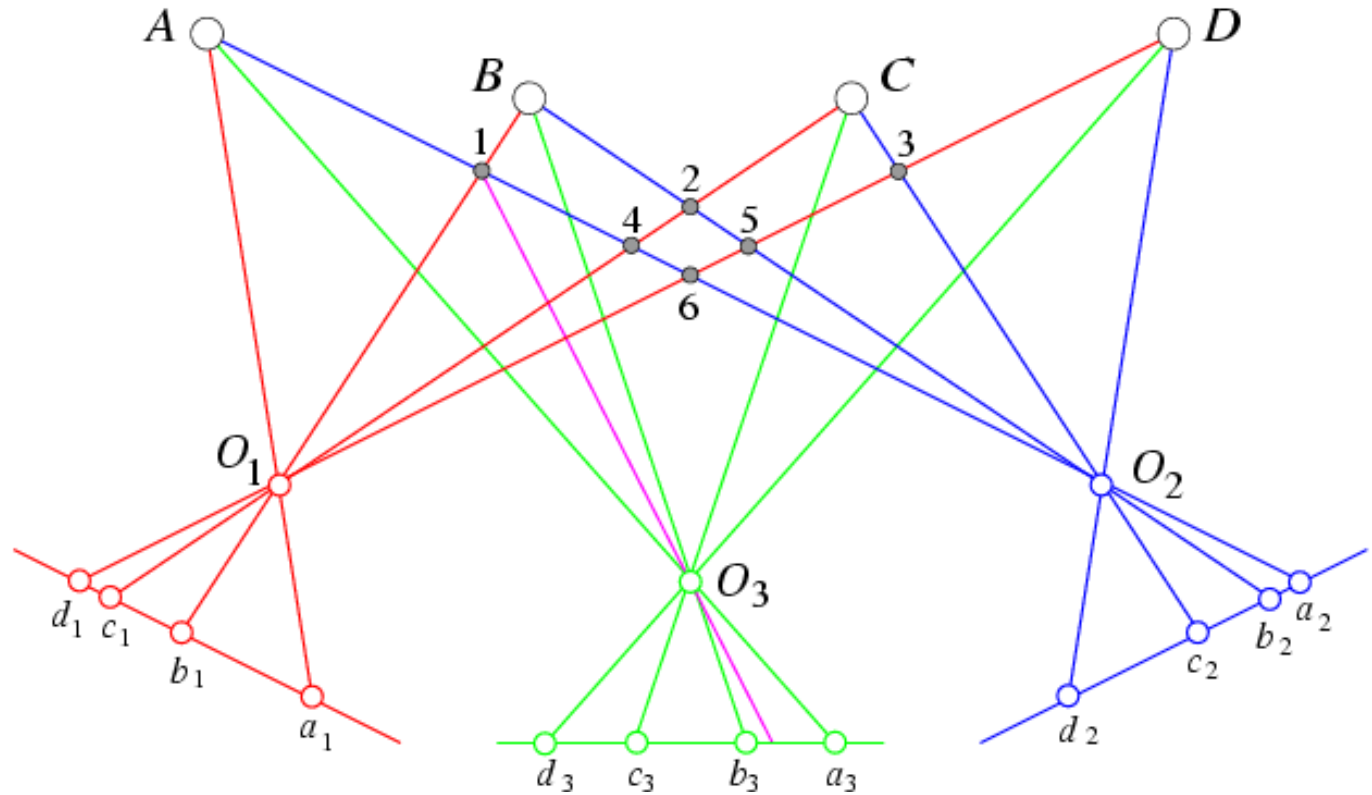
imaged length the same



slanting surface

imaged lengths differ

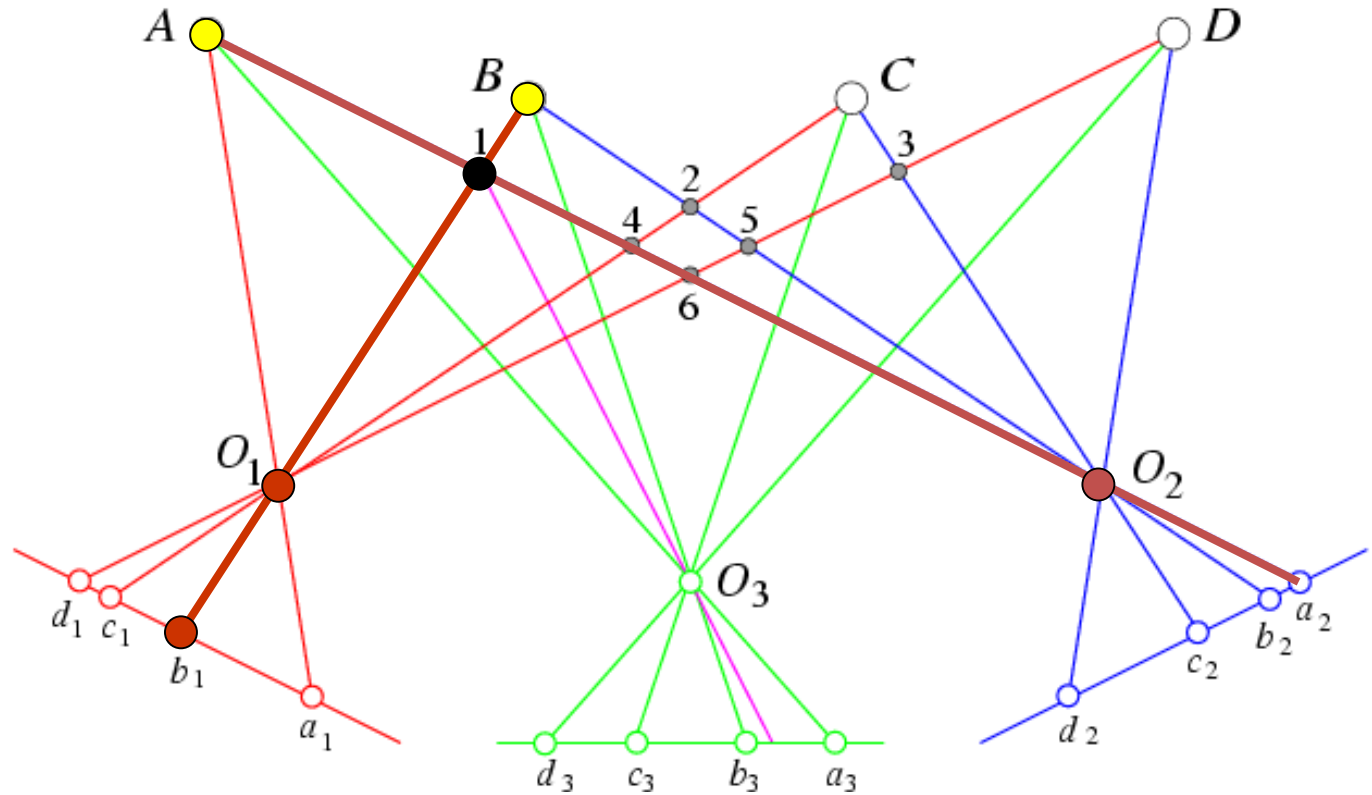
# Three Views



The third eye can be used for verification..

[Demo epipolar geometry](#)

# Three Views



The third eye can be used for verification..

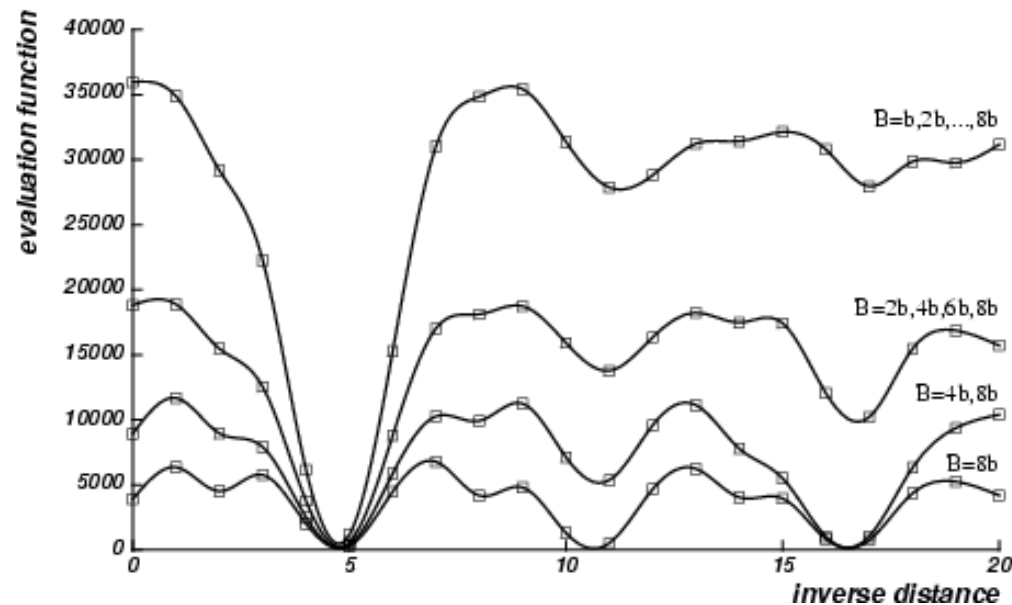
[Demo epipolar geometry](#)





## More Views (Okutami and Kanade, 1993)


New book: Ch7.6 p. 215: Pick a reference image, and slide the corresponding window along the corresponding epipolar lines of all other images, using **inverse depth ( $Z^{-1}$ )** relative to the first image as the search parameter.



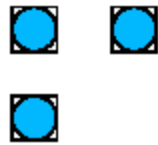
Reprinted from "A Multiple-Baseline Stereo System," by M. Okutami and T. Kanade, IEEE Trans. on Pattern Analysis and Machine Intelligence, 15(4):353-363 (1993). \copyright 1993 IEEE.

Use the sum of correlation scores to rank matches: SSD used as global evaluation function: Find  $Z^{-1}$  that minimizes SSD.

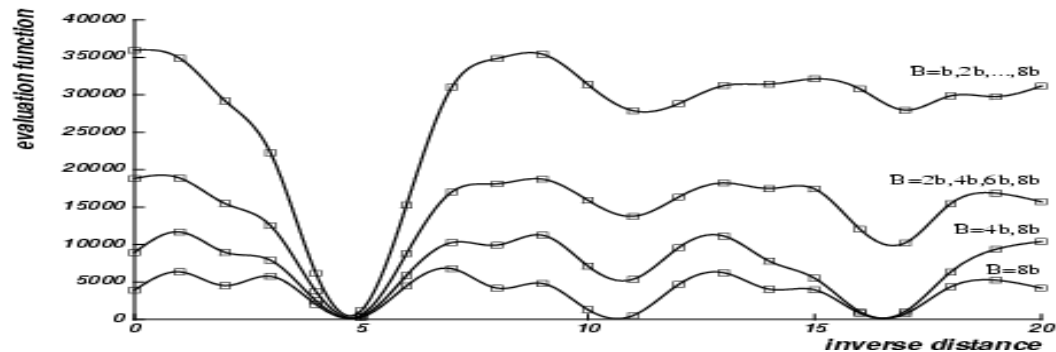
# Multi-camera configurations

 3 cameras give both robustness and precision

 4 cameras give additional redundancy

 3 cameras in a T arrangement allow the system to see vertical lines.

(illustration from Pascal Fua)



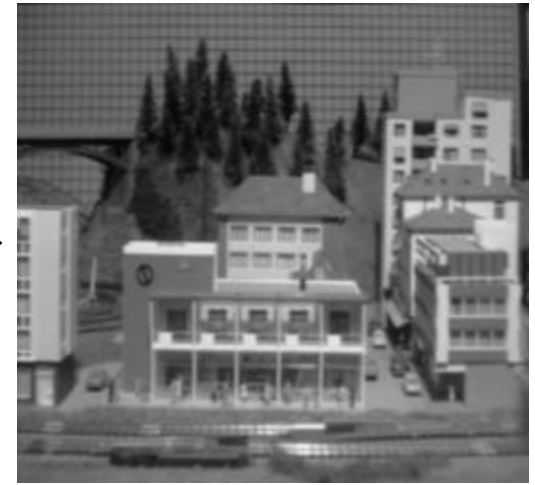
Okutami and Kanade



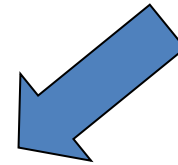
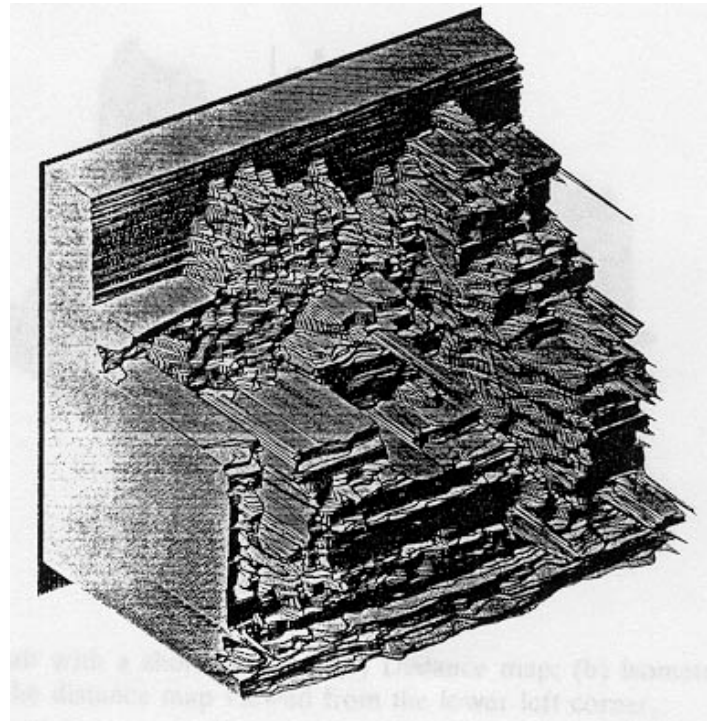
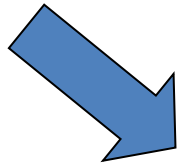
I1



I2



I10





# Normalized cross correlation

subtract mean:  $A \leftarrow A - \langle A \rangle, B \leftarrow B - \langle B \rangle$

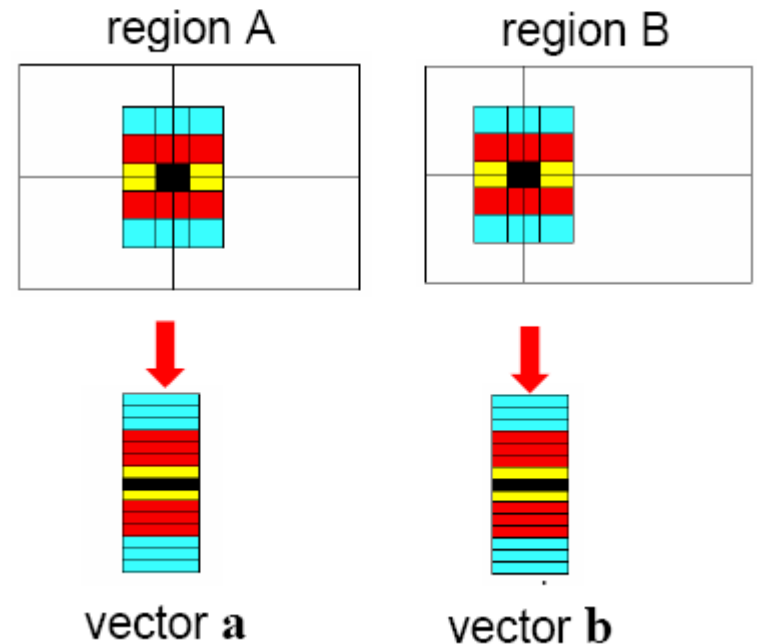
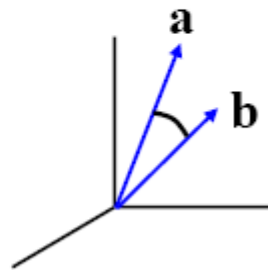
$$\text{NCC} = \frac{\sum_i \sum_j A(i, j) B(i, j)}{\sqrt{\sum_i \sum_j A(i, j)^2} \sqrt{\sum_i \sum_j B(i, j)^2}}$$

Write regions as vectors

$A \rightarrow \mathbf{a}, B \rightarrow \mathbf{b}$

$$\text{NCC} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$

$$-1 \leq \text{NCC} \leq 1$$



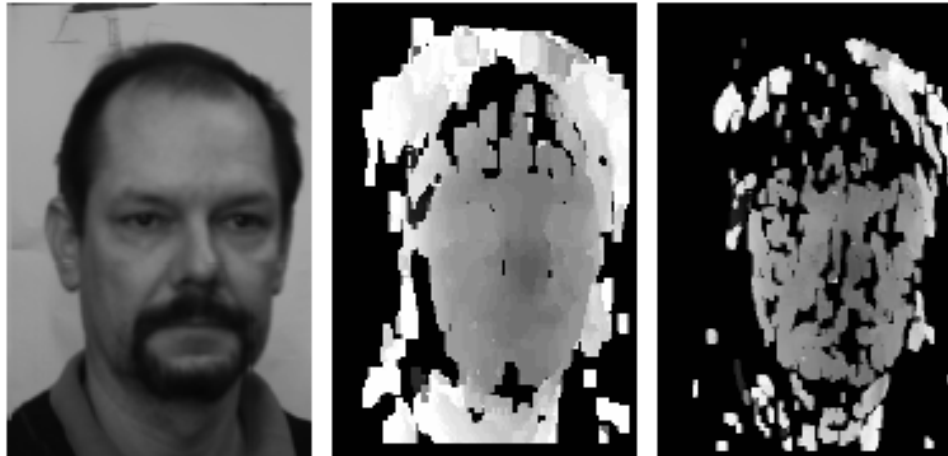
# Aggregation window sizes

## Small windows

- disparities similar
- more ambiguities
- accurate when correct

## Large windows

- larger disp. variation
- more discriminant
- often more robust
- use shiftable windows to deal with discontinuities



14x14

7x7

(Illustration from Pascal Fua)