

Optical Flow II

Guido Gerig CS 6643, Spring 2017

(credits: Pollefeys Comp 256, UNC, Trucco & Verri, Chapter 8, R. Szelisky, CS 223 Fall 2005)

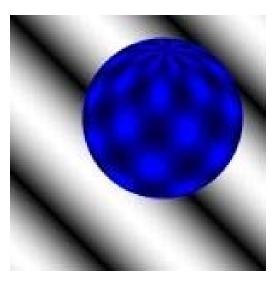


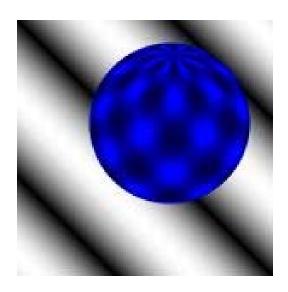
Material

- R. Szelisky Computer Vision: Chapter 7.1-7.2, Chapter 8
- Trucco & Verri Chapter 8 (handout, pdf)
- Hand-written notes G. Gerig (pdf)
- Horn & Schunck Chapter 9
- Pollefeys CV course (ETH/UNC)
- Richard Szeliski, CS223B Fall 2005



Structure from Motion?

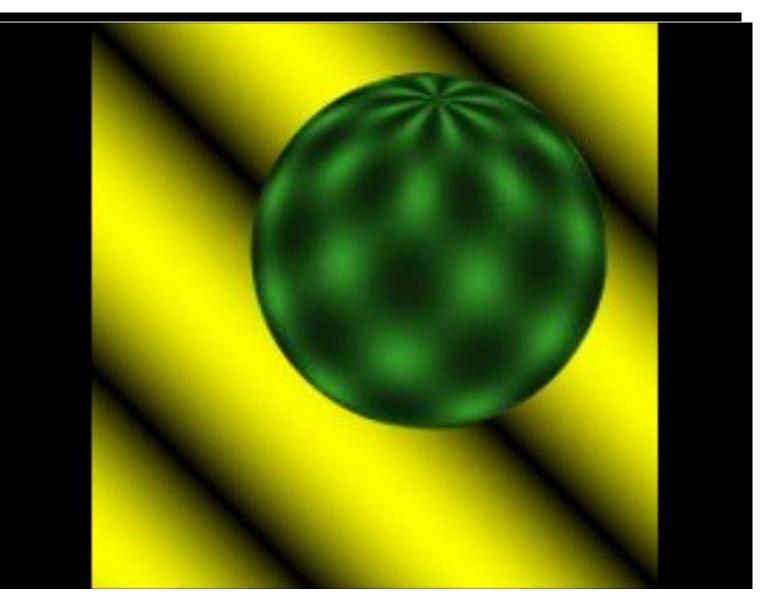




- Known: optical flow (instantaneous velocity)
- Motion of camera / object?



Structure from Motion?





Optical Flow

- Brightness Constancy
- The Aperture problem
- Regularization
- Lucas-Kanade
- Coarse-to-fine
- Parametric motion models
- Direct depth
- SSD tracking
- Robust flow
- Bayesian flow

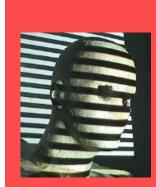


Image velocity of a point moving in the scene

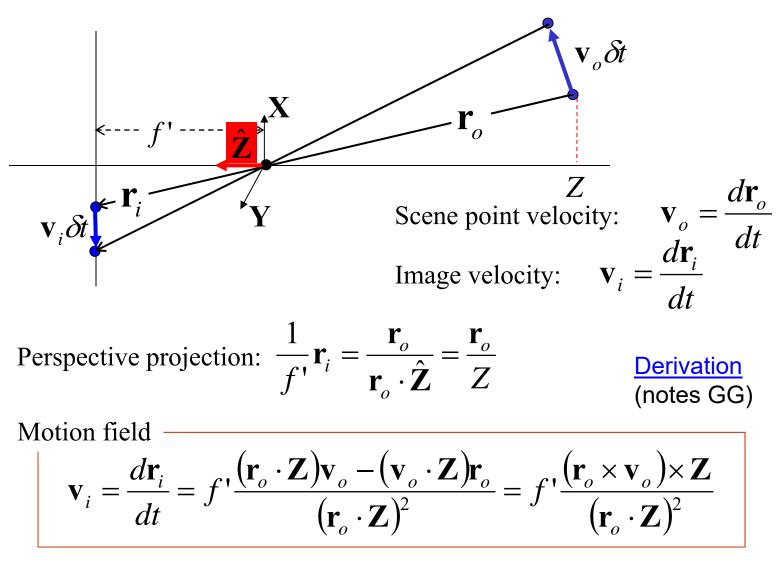
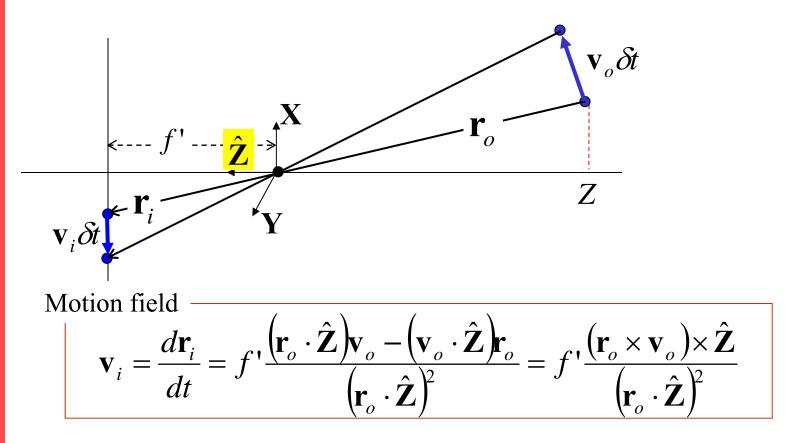
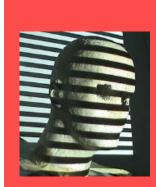




Image velocity of a point moving in the scene



Discussion: \mathbf{v}_i is orthogonal to $(\mathbf{r}_o \times \mathbf{v}_o)$ and $\hat{Z} \rightarrow$ lies in image plane



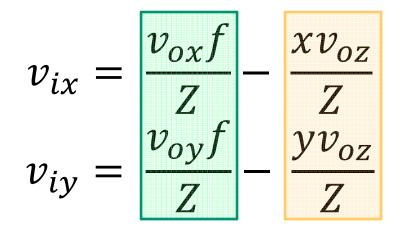
Motion field -

$$\mathbf{v}_{i} = \frac{d\mathbf{r}_{i}}{dt} = f' \frac{\left(\mathbf{r}_{o} \cdot \hat{\mathbf{Z}}\right) \mathbf{v}_{o} - \left(\mathbf{v}_{o} \cdot \hat{\mathbf{Z}}\right) \mathbf{r}_{o}}{\left(\mathbf{r}_{o} \cdot \hat{\mathbf{Z}}\right)^{2}} = f' \frac{\left(\mathbf{r}_{o} \times \mathbf{v}_{o}\right) \times \hat{\mathbf{Z}}}{\left(\mathbf{r}_{o} \cdot \hat{\mathbf{Z}}\right)^{2}}$$

Set $\widehat{Z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ and do the math (see handwritten notes G. Gerig):

$$v_{ix} = \frac{v_{ox}f}{Z} - \frac{xv_{oz}}{Z}$$
$$v_{iy} = \frac{v_{oy}f}{Z} - \frac{yv_{oz}}{Z}$$





Discussion:

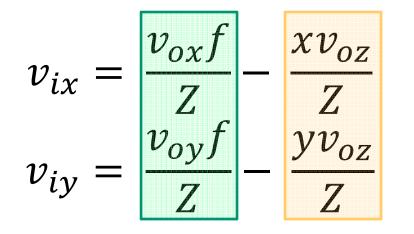


Component of optical flow in image only due to v_x and v_y , object motion parallel to image plane.



Component of optical flow in image only due to v_z , object motion towards/away from camera.





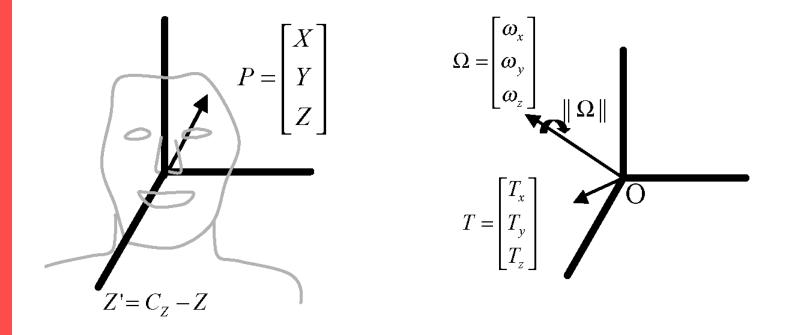
Reformulate: perspective projection of velocity:

$$\begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix} = \begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix} \frac{1}{Z} \begin{bmatrix} v_{ox} \\ v_{oy} \\ v_{oz} \end{bmatrix}$$



Rigid pose estimation

• Head pose model: 6 DOF



Please note notation: T stands for translational motion of object, Ω for rotational component.



• 3-D velocity:

$$V = T + \Omega \times P = T - \hat{\mathbf{P}}\Omega = \begin{bmatrix} \mathbf{I} & -\hat{\mathbf{P}} \begin{bmatrix} T \\ \Omega \end{bmatrix}$$

$$\hat{\mathbf{P}} = [\mathbf{P}_x]$$
(skew-sym.)

$$V = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y \\ 0 & 1 & 0 & -Z & 0 & X \\ 0 & 0 & 1 & Y & -X & 0 \end{bmatrix} \begin{bmatrix} T \\ \Omega \end{bmatrix}$$



• Perspective projection

$$\begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix} = \begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix} \frac{1}{Z} \begin{bmatrix} v_{ox} \\ v_{oy} \\ v_{oz} \end{bmatrix}$$



• Combine

$$V = \begin{bmatrix} V_{x} \\ V_{y} \\ V_{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y \\ 0 & 1 & 0 & -Z & 0 & X \\ 0 & 0 & 1 & Y & -X & 0 \end{bmatrix} \begin{bmatrix} T \\ \Omega \end{bmatrix}$$
$$\begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix} = \begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix} \frac{1}{Z} \begin{bmatrix} v_{ox} \\ v_{oy} \\ v_{oz} \end{bmatrix}$$



• Rigid Motion (for small v): $\begin{vmatrix} v_x \\ v_y \end{vmatrix} = \mathbf{H} \begin{bmatrix} T \\ \Omega \end{bmatrix}$

$$\mathbf{H} = \begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix} \frac{1}{Z'} \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y \\ 0 & 1 & 0 & -Z & 0 & X \\ 0 & 0 & 1 & Y & -X & 0 \end{bmatrix}$$

Perspective projection of 3-D velocity * 3-D velocity at point P (from $\begin{bmatrix} T \\ \Omega \end{bmatrix}$)

Hard to solve with just optic flow vectors! (but see Horn 17.3-17.5).

* Convert from scene to image: $\bar{p} = f \frac{\bar{P}}{Z}$



$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \mathbf{H} \begin{bmatrix} T \\ \Omega \end{bmatrix}$$
$$\mathbf{H} = \begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix} \frac{1}{Z'} \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y \\ 0 & 1 & 0 & -Z & 0 & X \\ 0 & 0 & 1 & Y & -X & 0 \end{bmatrix}$$
$$\mathbf{v}_x = \frac{T_z x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x x y}{f} - \frac{\omega_y x^2}{f}$$
$$v_y = \frac{T_z y - T_y f}{Z} + \omega_x f - \omega_z x - \frac{\omega_y x y}{f} + \frac{\omega_x y^2}{f}.$$



Flow field of rigid motion

In components, and using (8.5), (8.6) read

$$v_x = \frac{T_z x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x x y}{f} - \frac{\omega_y x^2}{f}$$
$$v_y = \frac{T_z y - T_y f}{Z} + \omega_x f - \omega_z x - \frac{\omega_y x y}{f} + \frac{\omega_x y^2}{f}.$$

Notice that the motion field is the sum of two components, one of which depends translation only, the other on rotation only. In particular, the translational compone of the motion field are

$$v_x^T = \frac{T_z x - T_x f}{Z}$$
$$v_y^T = \frac{T_z y - T_y f}{Z},$$

and the rotational components are

$$v_x^{\omega} = -\omega_y f + \omega_z y + \frac{\omega_x x y}{f} - \frac{\omega_y x^2}{f}$$
$$v_y^{\omega} = \omega_x f - \omega_z x - \frac{\omega_y x y}{f} + \frac{\omega_x y^2}{f}.$$

Trucco & Verri p. 184



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Discussion:

- Motion field of translational component depends on T and depth Z. For increasing Z, velocity becomes smaller.
- Motion field that depends on angular velocity does NOT carry information on depth Z!



Special Case: Pure Translation

$$v_x = \frac{T_z x - T_x f}{Z}$$
$$v_y = \frac{T_z y - T_y f}{Z}$$

Choose x₀ and y₀ so that v becomes 0

$$x_0 = f T_x / T_z$$
$$y_0 = f T_y / T_z,$$

$$v_x = (x - x_0) \frac{T_z}{Z}$$

$$v_y = (y - y_0) \frac{T_z}{Z}.$$

Says that motion field of a pure translation is radial, it consists of vectors radiating from a common origin $p_0=(x_0,y_0)$, which is the vanishing point.

Trucco & Verri p. 184/185 See also F&P Chapter 10.1.3 p. 218



Special Case: Pure Translation

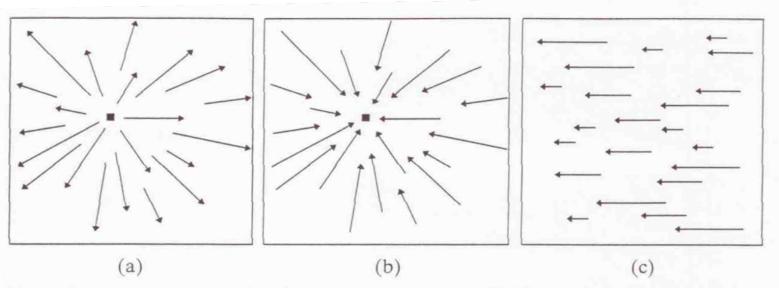


Figure 8.4 The three types of motion field generated by translational motion. The filled square marks the instantaneous epipole.

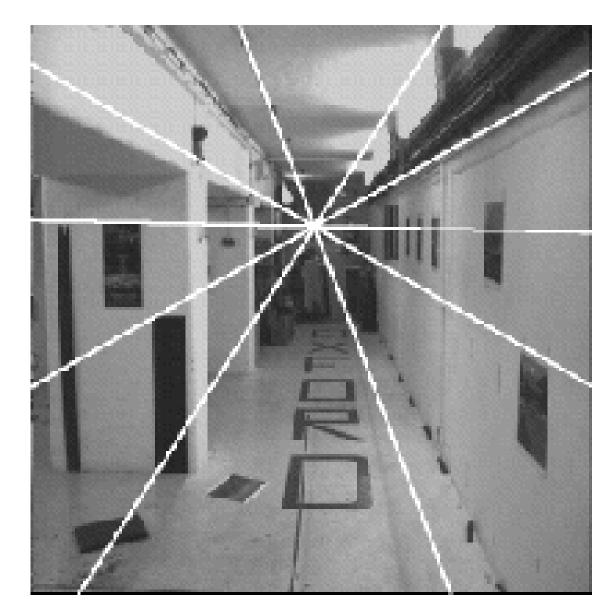
Focus of expansion/contraction:

$$x_0 = f T_x / T_z$$
$$y_0 = f T_y / T_z,$$

Trucco & Verri p. 184/185 See also F&P Chapter 10.1.3 p. 218



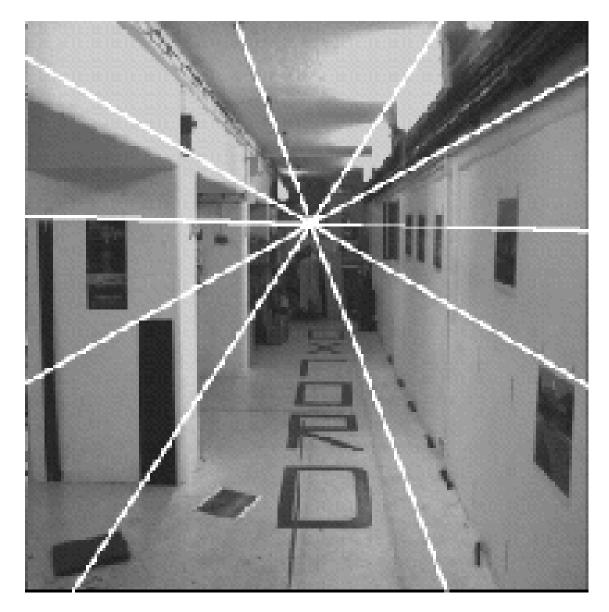
Example: forward motion



courtesy of Andrew Zisserman



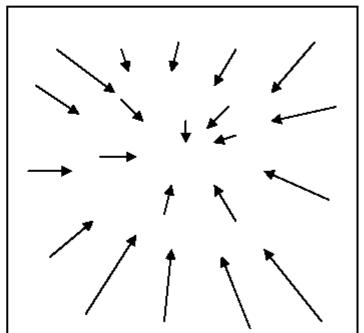
Example: forward motion



courtesy of Andrew Zisserman

FOE for Translating Camera



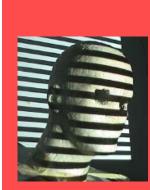




Moving Plane (Trucco&Verri p.187) $v_{x} = \frac{1}{fd}(a_{1}x^{2} + a_{2}xy + a_{3}fx + a_{4}fy + a_{5}f^{2})$ $v_{y} = \frac{1}{fd}(a_{1}xy + a_{2}y^{2} + a_{6}fy + a_{7}fx + a_{8}f^{2})$ $a_{1} = -d\omega_{y} + T_{z}n_{x}, \quad a_{2} = d\omega_{x} + T_{z}n_{y},$ $a_{3} = T_{z}n_{z} - T_{x}n_{x}, \quad a_{4} = d\omega_{z} - T_{x}n_{y},$ $a_{5} = -d\omega_{y} - T_{x}n_{z}, \quad a_{6} = T_{z}n_{z} - T_{y}n_{y},$

 $a_7 = -d\omega_z - T_y n_x, \qquad a_8 = d\omega_x - T_y n_z.$

- Motion field of planar surface is quadratic polynomial in (f,x,y)
- Same motion field produced by two different planes w. two different 3D motions
- Not unique: co-planar set of points (remember 8 point algorithm for calibration)



Application (Szeklisky): Motion representations

• How can we describe this scene?





Optical Flow Field



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Layered motion

• Break image sequence up into "layers":



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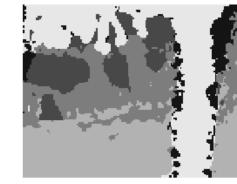
Describe each layer's motion

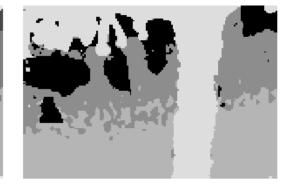


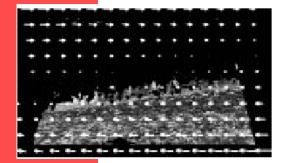
Results

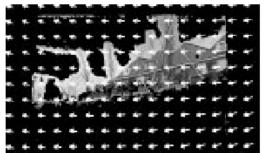


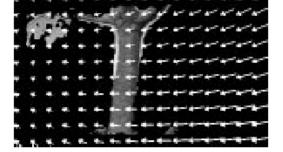
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Additional Slides, not discussed in class.



Direct Motion Estimation

• One equation per pixel:

$$\begin{bmatrix} -\frac{dI}{dt} \end{bmatrix} = \begin{bmatrix} \frac{dI}{dx} & \frac{dI}{dy} \end{bmatrix} \begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix} \frac{1}{Z'} \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -yZ'/f \\ 0 & 1 & 0 & -Z & 0 & xZ'/f \\ 0 & 0 & 1 & yZ'/f & -xZ'/f & 0 \end{bmatrix} \begin{bmatrix} T \\ \Omega \end{bmatrix}$$

• Still hard!

. . .

- Z unknown; assume surface shape...
 - Negahdaripour & Horn Planar
 - Black and Yacoob Affine
 - Basu and Pentland; Bregler and Malik Ellipsoidal
 - Essa et al. Polygonal approximation



Frame 0



Frame 80



Background scene (players removed)

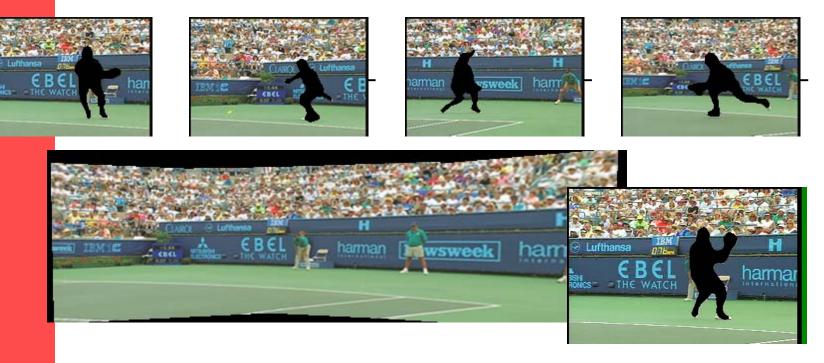


Complete synopsis of the video



Background modeling (MPEG-4)

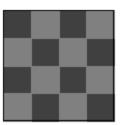
 Convert masked images into a background sprite for layered video coding



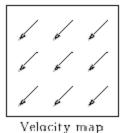


What are layers?

- [Wang & Adelson, 1994]
- intensities \bullet
- alphas
- velocities \bullet

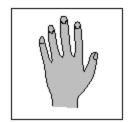






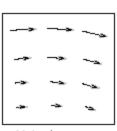
Intensity map

Alpha map

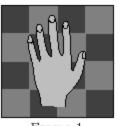


Intensity map

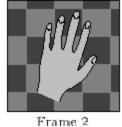


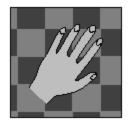


Velocity map



Frame 1





Frame 3

Alpha map





How do we form them?

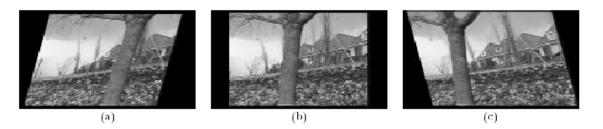


Figure 7: (a) Frame 1 warped with an affine transformation to align the flowerbed region with that of frame 15. (b) Original frame 15 used as reference. (c) Frame 30 warped with an affine transformation to align the flowerbed region with that of frame 15.

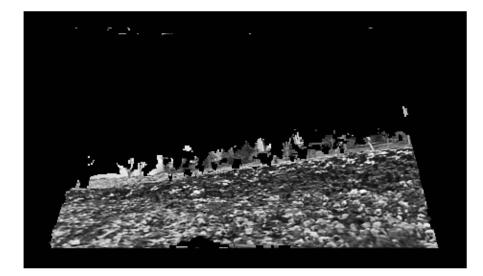


Figure 8: Accumulation of the flowerbed. Image intensities are obtained from a temporal median operation on the motion compensated images. Only the regions belonging to the flowerbed layer is accumulated in this image. Note also occluded regions are correctly recovered by accumulating data over many frames.