$\boxed{\text{Structure from Motion}}$

Object

Nov 3, 20

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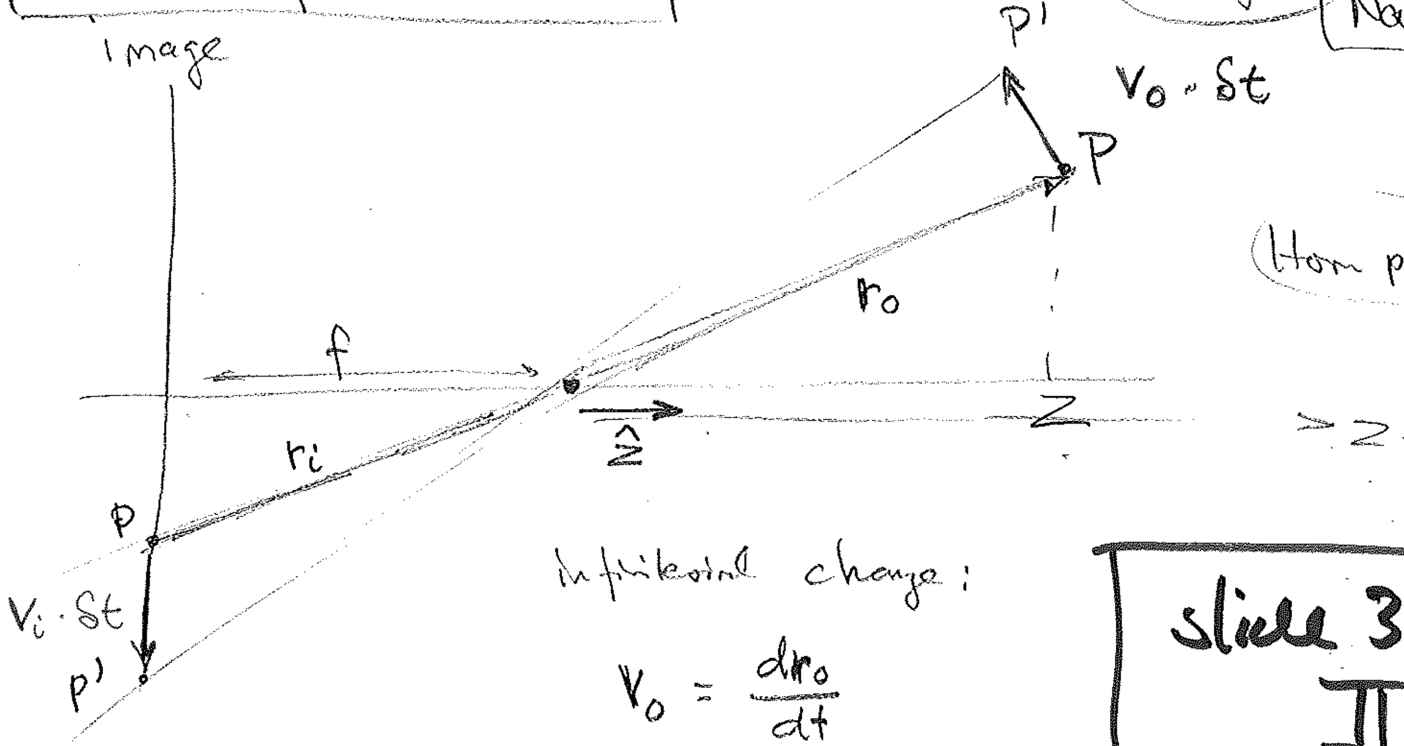
Horn p. 280



infinitesimal change:

$$V_o = \frac{dr_o}{dt}$$

$$V_i = \frac{dr_i}{dt}$$

$\boxed{\text{slide 31 II}}$

Projection:

$$\frac{r_i}{f} = \frac{r_o}{Z} = \frac{r_o}{r_o \cdot \underbrace{\hat{r}_z}_{\hat{Z}}}$$ unit along Z

$\hat{r}_z = \hat{Z}$: normal in Z direction

$$V_i = \frac{dr_i}{dt} = f \cdot \frac{d}{dt}\left(\frac{r_o}{r_o \cdot \underbrace{\hat{r}_z}_{\hat{Z}}}\right)$$

chain rule: $\left(\frac{1}{f(z)}\, g\right)' = \frac{g'}{f(z)} + \left(-\frac{f'(z)}{(f(z))^2} \cdot g\right) \ldots$

$$\Rightarrow\Rightarrow \boxed{V_i = f \cdot \frac{(r_o \times \overset{\left(\frac{dr_o}{dt}\right)}{V_o}) \times \hat{Z}}{(r_o \cdot \hat{Z})^2}}$$

perpendicular to r_o and V_o ⇒ ⊙ comes out

then perp. to $\hat{Z}$ ⊙→ →

projection to img plane

18. Nov. 2008

Horn

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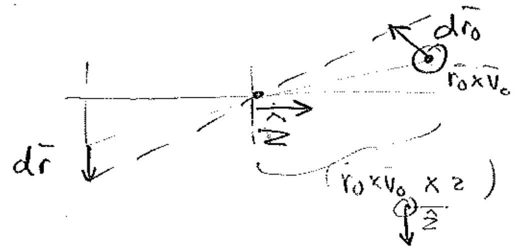
$$\frac{1}{f'}\,\bar r_i = \left(\frac{1}{\bar r_0\cdot\hat z}\right)\cdot \bar r_0$$

$$\bar v_i\cdot\frac{1}{f'} = \frac{1}{f'}\,\frac{d\bar r_i}{dt} = \left(\left(\frac{1}{\bar r_0\cdot\hat z}\right)\cdot\bar r_0\right)'$$

$$\left(\frac{1}{f\cdot z}\,g\right)' = \frac{g'}{f\cdot z} - \frac{f'\cdot z}{(f\cdot z)^2}\cdot g$$

$$= \frac{\frac{d\bar r_0}{dt}}{\bar r_0\cdot\hat z} - \frac{\frac{d\bar r_0}{dt}\cdot z\cdot\bar r_0}{(\bar r_0\cdot\hat z)^2}$$



$$= \frac{\bar v_0}{\bar r_0\cdot\hat z} - \frac{\bar v_0\cdot\hat z\cdot\bar r_0}{(\bar r_0\cdot\hat z)^2}$$

$$= \frac{(\bar r_0\cdot\hat z)\cdot\bar v_0 - (\bar v_0\cdot\hat z)\,\bar r_0}{(\bar r_0\cdot\hat z)^2}$$ ✓

Horn p.280

$$\overset{\frac{d\bar r_i}{dt}}{\bar v_i}\cdot\frac{1}{f'} = \boxed{\frac{(\bar r_0\times\overset{?\,\frac{d\bar r_0}{dt}}{\bar v_0})\times\hat z}{(\bar r_0\cdot\hat z)^2}}$$ ✓

perpendicular to $\bar r_0$ and $d\bar r_0$ ⇒ with $\hat z$! in image plane!

$$\bar v_0 = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \qquad \bar v_i = \begin{pmatrix} v_{ix} \\ v_{iy} \end{pmatrix}$$

sei: $\hat z = (0,0,z_n) = (0,0,1)$

$$\Rightarrow \bar v_i\cdot\frac{1}{f'} = \left(\frac{r_z v_x - r_x v_z}{z^2},\ \frac{r_z v_y - r_y v_z}{z^2},\ 0\right)$$

$\frac{r_z}{z} = 1$ $\frac{r_x\cdot f}{z} = x$

$$\Rightarrow \frac{v_{ix}}{f} = \frac{r_z v_{0x}}{z^2} - \frac{r_x v_{0z}}{z^2}$$

$$\frac{v_{iy}}{f} = \frac{r_z v_{0y}}{z^2} - \frac{r_y v_{0z}}{z^2}$$

$$v_{ix} = \frac{r_z v_{0x}\cdot f}{z^2} - \frac{r_x v_{0z}}{z^2}\cdot f = \frac{v_x\cdot f}{z} - \frac{x\cdot v_z}{z}$$

$$v_{iy} = \frac{v_y\cdot f}{z} - \frac{y\cdot v_z}{z}$$

→ slide 25

Nov. 3, 200

image object

$$\boxed{V_i = f\,\frac{(r_o \times V_o) \times \hat{Z}}{(r_o \cdot \hat{Z})^2}}$$

$$V_o = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$V_i = \begin{pmatrix} V_{ix} \\ V_{iy} \end{pmatrix}$$

do the math:

$$V_i = f\,\frac{\left(\begin{pmatrix} r_{ox} \\ r_{oy} \\ r_{oz} \end{pmatrix} \times \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}\right) \times \begin{pmatrix} Z_x \\ Z_y \\ Z_z \end{pmatrix}}{(r_o \cdot \hat{Z})^2}$$

set $\hat{Z} = \begin{pmatrix} 0 \\ 0 \\ Z_n \end{pmatrix}$ $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$= f\,\frac{\begin{pmatrix} r_{oy} \cdot V_z - r_{oz} V_y \\ -r_{ox} \cdot V_z + r_{oz} V_x \\ r_{ox} V_y - r_{oy} V_x \end{pmatrix} \times \begin{pmatrix} Z_x \\ Z_y \\ Z_z \end{pmatrix}}{(r_o \hat{Z})^2} \quad \begin{pmatrix} 0 \\ 0 \\ Z_n \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$P(X, Y, Z)$

$p(x, y)$

$$= f\,\frac{\begin{pmatrix} (-r_{ox} V_z + r_{oz} V_x)\, Z_n \\ (-r_{oy} V_z + r_{oz} V_y)\, Z_n \\ 0 \end{pmatrix}}{\underbrace{(r_o \hat{Z})^2}_{\approx Z^2}}$$

r_x $\frac{X}{Z} = \frac{x}{f}$

$\frac{Y}{Z} = \frac{y}{f}$

r_o in direction $\hat{Z}$ $\Rightarrow Z$

$$\Rightarrow \frac{V_{ix}}{f} = \frac{r_z V_x}{Z^2} - \frac{r_x V_z}{Z^2}$$

$$\frac{V_{iy}}{f} = \frac{r_z V_y}{Z^2} - \frac{r_y V_z}{Z^2}$$

$\frac{r_z}{Z} = 1$

$\frac{r_x \cdot f}{Z} = x$

$\frac{r_y \cdot f}{Z} = y$

$$\Rightarrow V_{ix} = \frac{V_x \cdot f}{Z} - \frac{x \cdot V_z}{Z}$$

$$V_{iy} = \frac{V_y \cdot f}{Z} - \frac{y \cdot V_z}{Z}$$

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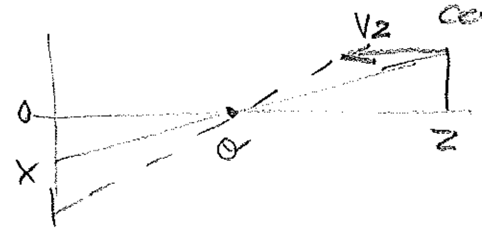
(11)

$$\begin{bmatrix} V_{ix} \\ V_{iy} \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix}}_{\text{perspective projection of 3D velocity}} \frac{1}{Z} \begin{bmatrix} V_{ox} \\ V_{oy} \\ V_{oz} \end{bmatrix}$$

Intuition: look at $V_{ix} = \frac{V_X \cdot f}{Z} - \frac{x V_Z}{Z}$

$\frac{V_{ix}}{f} = \frac{V_X}{Z}$ component due to V_X

$\frac{V_{ix}}{x} = -\frac{V_Z}{Z}$ component due to V_Z (however, camera)



8.2 Trucco

now: $\Omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$ angular velocity of 3D motion

(slide 6 III) $T = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}$ translational component of 3D motion

$$V = T + \Omega \times P$$

| P: 3D point in camera reference frame

$$\Rightarrow \begin{aligned} V_X &= T_X + \omega_Y Z - \omega_Z Y \\ V_Y &= T_Y + \omega_Z X - \omega_X Z \\ V_Z &= T_Z + \omega_X Y - \omega_Y X \end{aligned} \Rightarrow [V] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & Z & -Y \\ 0 & 1 & 0 & -Z & 0 & X \\ 0 & 0 & 1 & Y & -X & 0 \end{array}\right] \begin{bmatrix} T \\ \Omega \end{bmatrix}$$

(slide 7)

Nov. 5, 2005

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continue;

slide 9 $\overline{\text{III}}$

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = \underbrace{\overset{2\times 3}{\begin{bmatrix} & \end{bmatrix}} \frac{1}{Z} \overset{3\times 6}{\begin{bmatrix} & & \end{bmatrix}}}_{H\ [2\times 6]} \begin{bmatrix} T \\ \Omega \end{bmatrix}$$

persp. proj. 3D velocity

(slide 11)

$$\frac{1}{Z}\begin{bmatrix} \underbrace{\begin{matrix} f & 0 & -x \\ 0 & f & -y \end{matrix}}_{\text{Transl.}} & \underbrace{\begin{matrix} -xY & (Zf+xX) & -fY \\ (-fZ-yY) & -yY & fY \end{matrix}}_{\text{rotation}} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \underbrace{\begin{bmatrix} V_x \\ V_y \end{bmatrix}_{transl.}}_{\frac{1}{Z}\begin{bmatrix} ft_x - xt_z \\ ft_y - yt_z \end{bmatrix} = f(T, Z)} + \underbrace{\begin{bmatrix} V_x \\ V_y \end{bmatrix}_{angular}}_{\begin{matrix} -\frac{xy}{f}\omega_x + (f+\frac{x^2}{f})\omega_y - y\omega_z \\ -f\omega_x - \frac{y^2}{f}\omega_x - \frac{yx}{f}\omega_y + y\omega_z \end{matrix}}$$

(slide 11 (II))
(Trucco p. 184)

$f(\Omega, x, y)$

$\neq f(Z)$!

motion field that depends on angular velocity does not carry information on depth !

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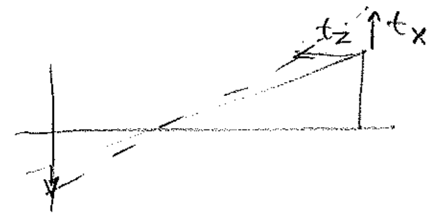
Pure translation



$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \frac{1}{Z} \begin{bmatrix} f T_x - x T_z \\ f T_y - y T_z \end{bmatrix}$$

introduce: $p_0 = (x_0, y_0)^T$

Choose p0 so that vx and vy get 0: point which does not move.

let: $x_0 T_z = f T_x \Rightarrow x_0 = \frac{f T_x}{T_z}$

$y_0 T_z = f T_y \Rightarrow y_0 = \frac{f T_y}{T_z}$

plug in

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \frac{1}{Z} \begin{bmatrix} x_0 T_z - x T_z \\ y_0 T_z - y T_z \end{bmatrix} = \frac{T_z}{Z} \begin{bmatrix} x_0 - x \\ y_0 - y \end{bmatrix}$$

motion field of pure translation is radial!

(slide 12 III)

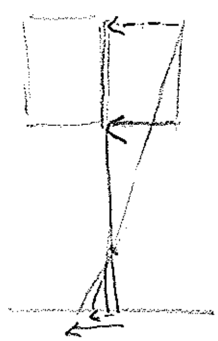
$p_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ focus of expansion

vanishing point

cases: $T_z = 0 \Rightarrow \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \frac{f}{Z} \begin{bmatrix} T_x \\ T_y \end{bmatrix}$

a)

- all motion vectors parallel
- amount V inverse proportional to depth



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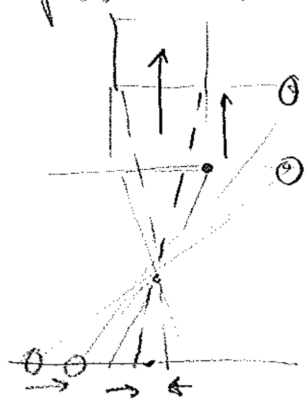
b) pure T_Z : $T_x, T_y = 0$

$\Rightarrow x_0, y_0 = (0,0)^T$



vanishing point

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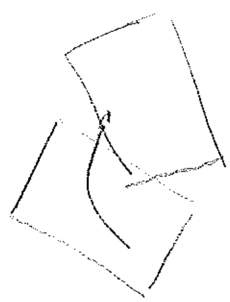
c) moving plane (Trucco p. 187)

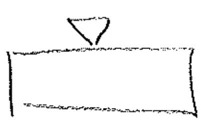
slide 13

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = f(x, y, xy, x^2, y^2)$$

quadratic polynomial



pls. look up!



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final idea: epipolar constraint

Moving plane

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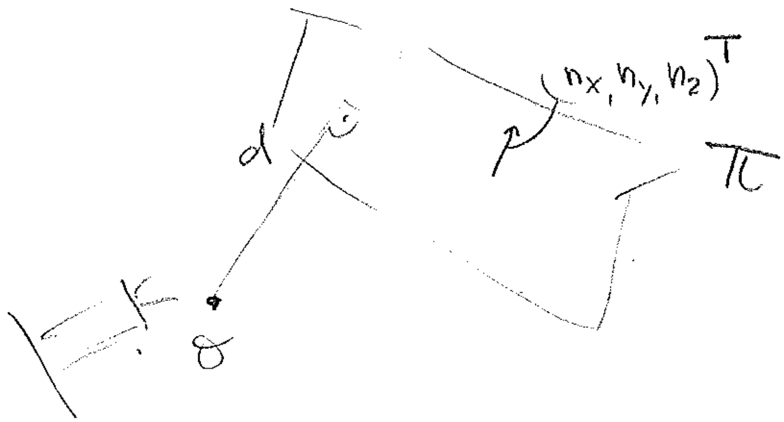

$$\bar{n}^t \cdot P = d$$

moving with $T \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$ and $\omega = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$

$\Rightarrow \bar{n}(t), \; T(t)$

$\bar{n}^t \cdot \underbrace{P}_{\frac{Z \cdot p}{f}} = d \qquad \Big| \qquad p = f \cdot \frac{P}{Z}$

$$\Rightarrow \frac{(n_x \cdot x + n_y \cdot y + n_z \cdot f)}{f} \cdot Z = d$$

$\Rightarrow$ solve for Z

$\Rightarrow$ plug into $\begin{bmatrix} v_x \\ v_y \end{bmatrix} = H \begin{bmatrix} T \\ \Omega \end{bmatrix}$

p.187 Trucco

$\Rightarrow v_x = \frac{1}{fd}(x^2, xy, fx, fy, f^2)$

$v_y = \frac{1}{fd}(xy, y^2, fy, fx, f^2)$

$\Rightarrow$ motion field of a moving planar surface is quadratic polynomial in (x, y, f)

$\Rightarrow$ p. 187/188: motion field not unique?