Depth and Shape Inference (III)

Introduction to Computational and Biological Vision

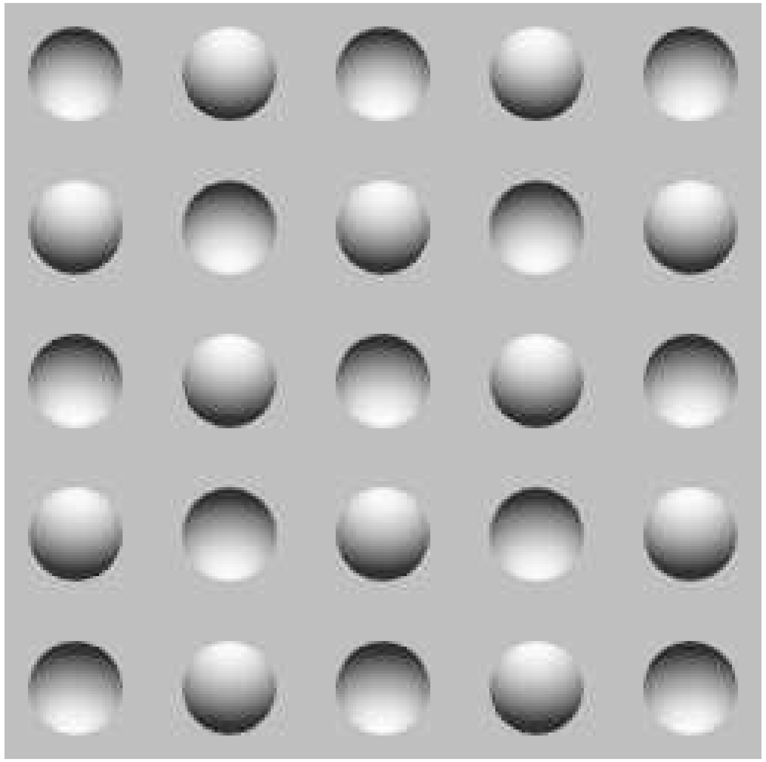
CS 202-1-5261

Computer Science Department, BGU

Ohad Ben-Shahar

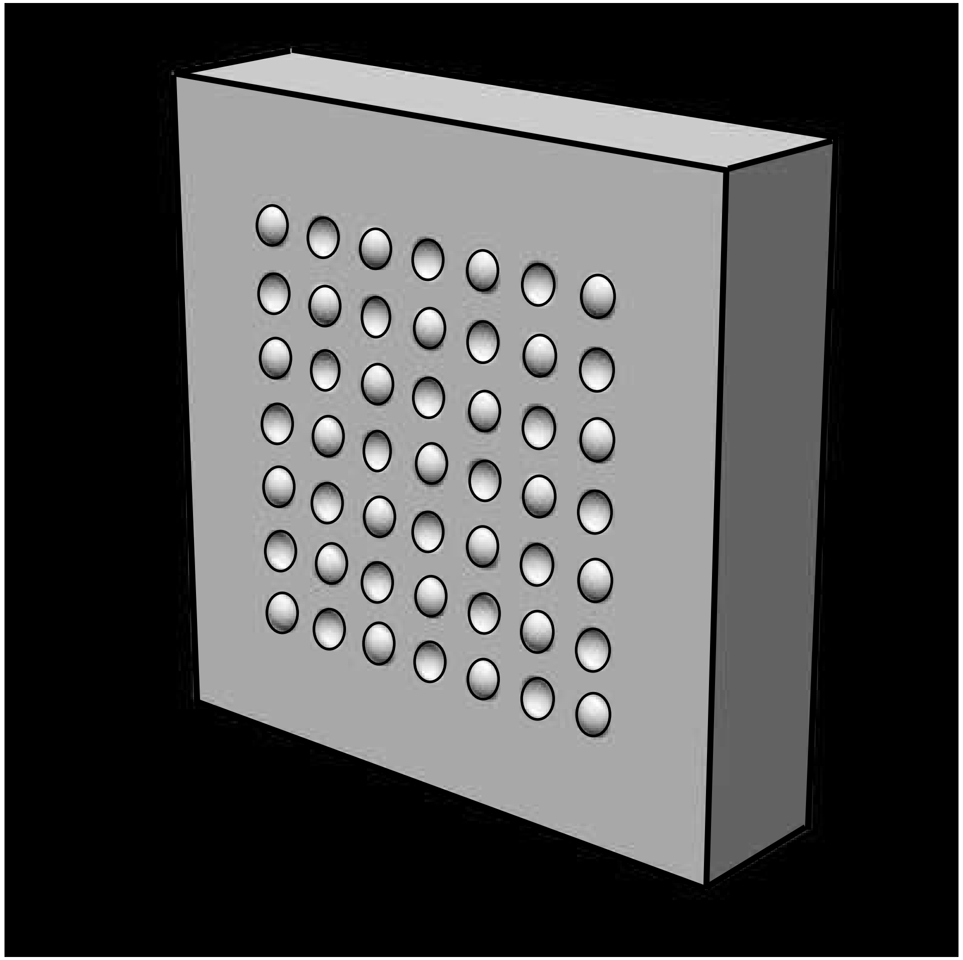
Shape from Shading

Shape from Shading



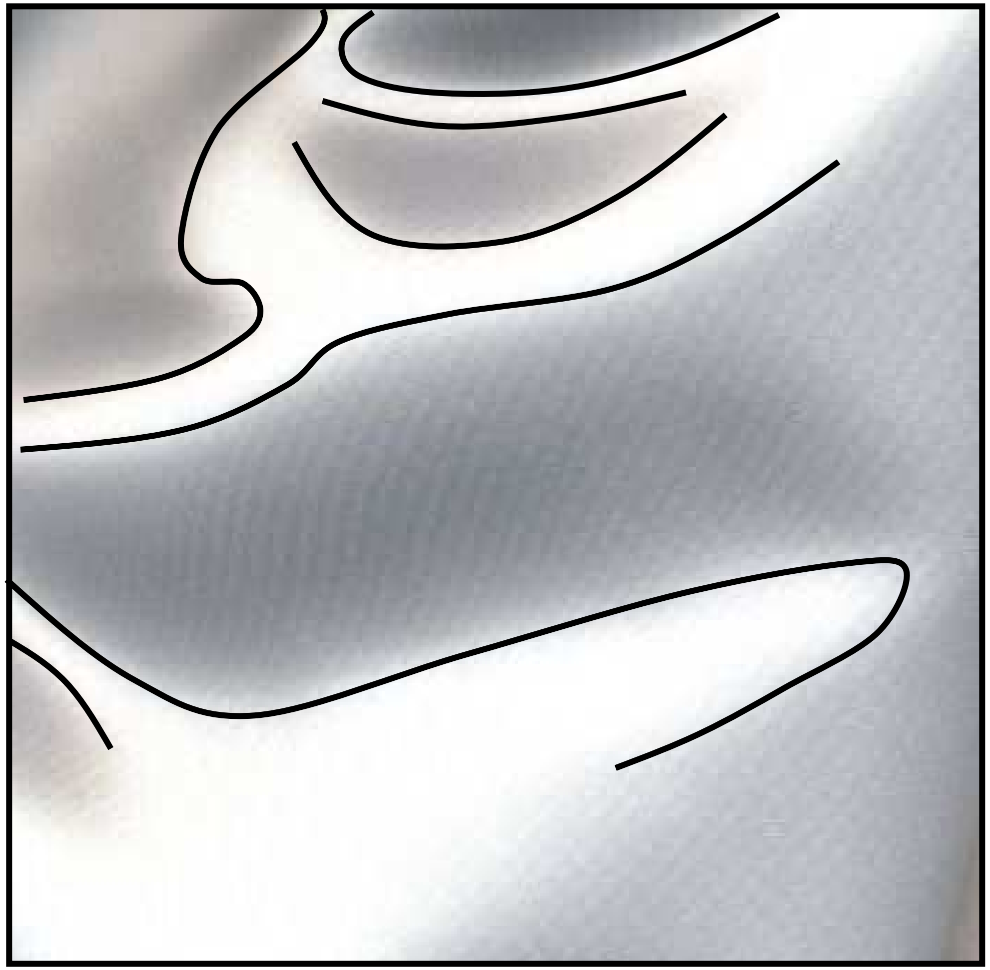
Shape from Shading

Shading is more than contours



Shape from Shading

Shading is more than edge contours



Shape from Shading

Inverting the image formation process

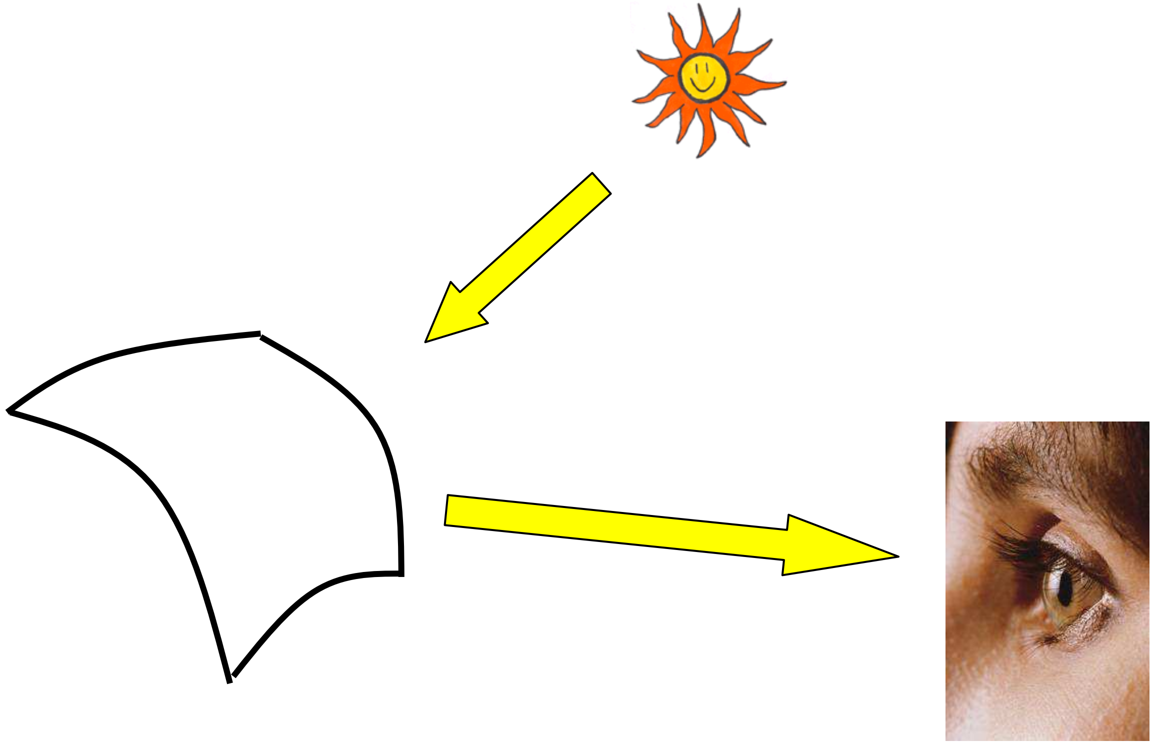
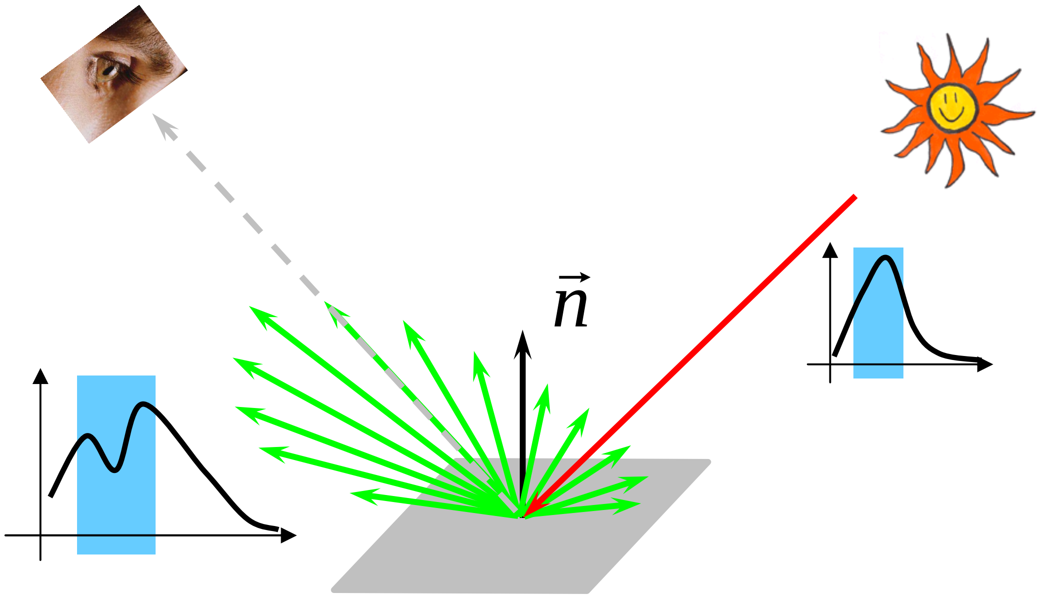


Image formation = "Shading from shape" (and light sources)

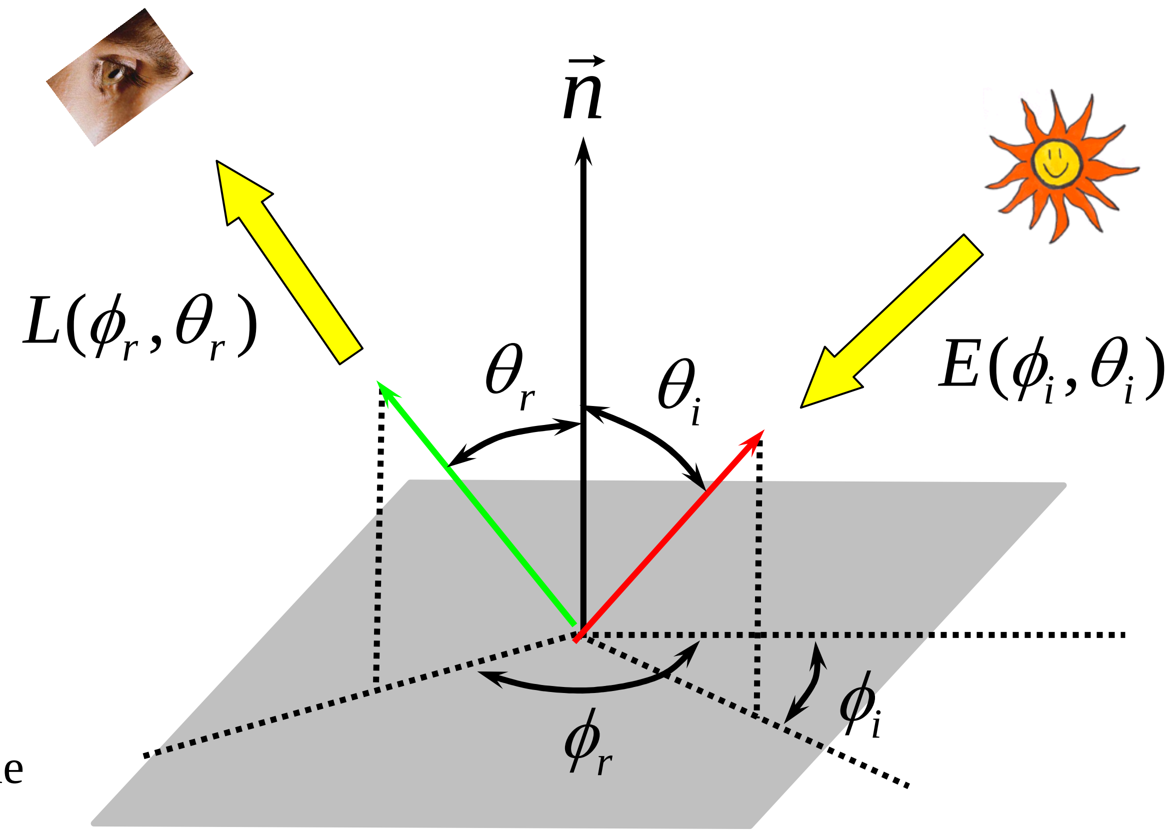
Shape from Shading

Image formation



Shape from Shading

Polar representation of directions



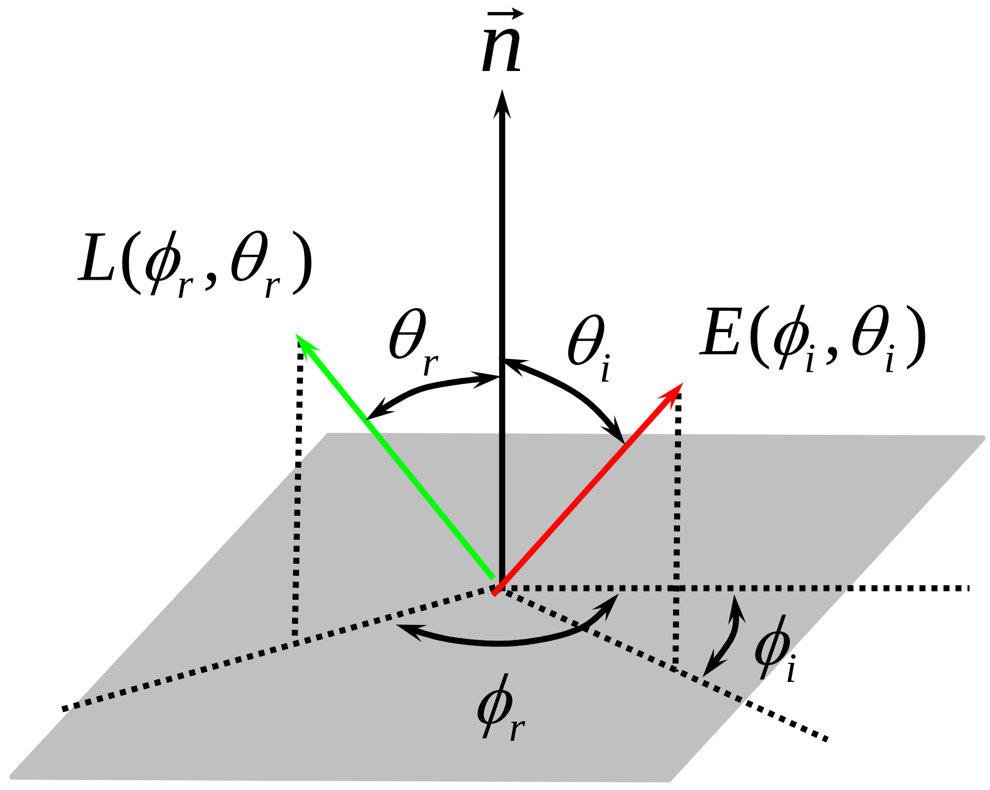
ϕ - Azimuth angle

θ - Zenith angle

Shape from Shading

The Bidirectional Reflectance Distribution Function (BRDF)

$$f_\lambda(\phi_i, \theta_i; \phi_r, \theta_r) = \frac{L_\lambda(\phi_r, \theta_r)}{E_\lambda(\phi_i, \theta_i)}$$



Helmholtz's reciprocity

$$f(\phi_i, \theta_i; \phi_r, \theta_r) = f(\phi_r, \theta_r; \phi_i, \theta_i)$$

Isotropic materials:

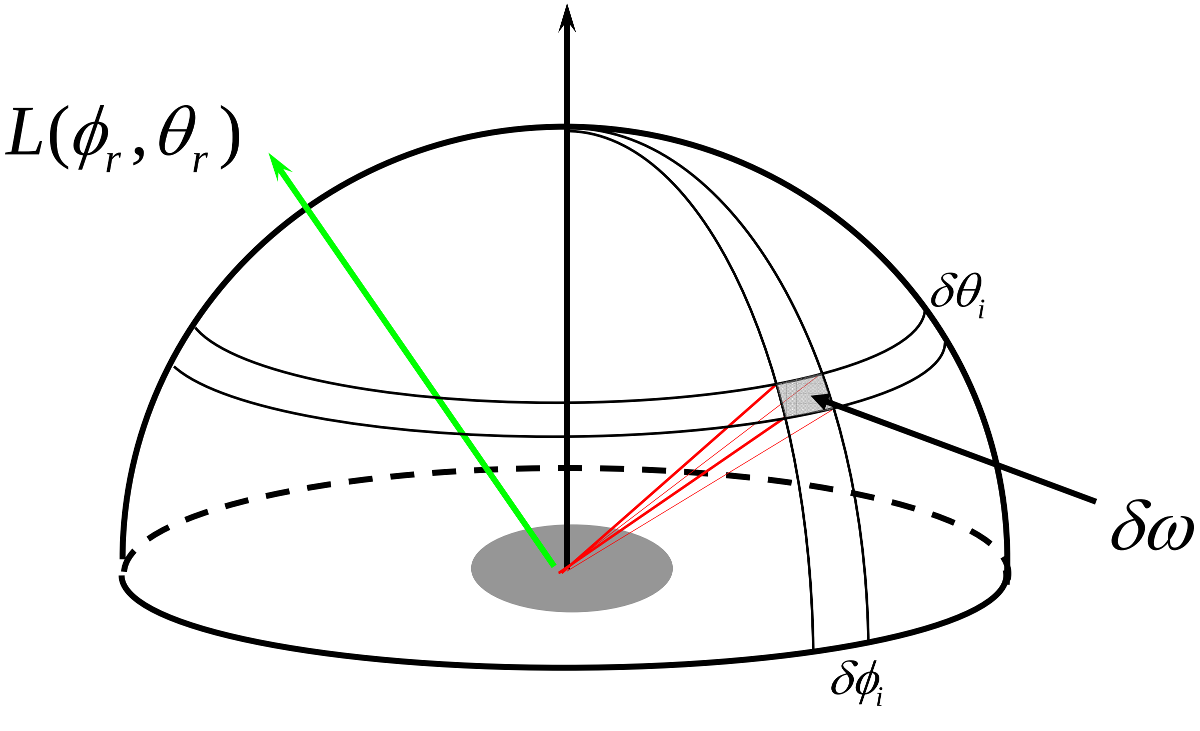
$$f(\phi_i, \theta_i; \phi_r, \theta_r) = f(\phi_i - \phi_r, \theta_i, \theta_r)$$

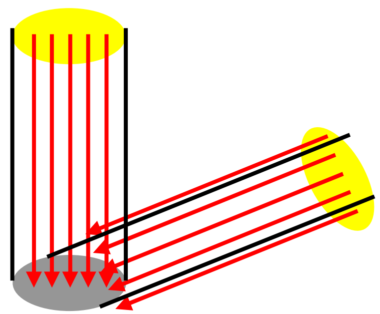
Shape from Shading

Total surface reflection

$$L(\phi_r,\theta_r) = \int_\omega f(\phi_i,\theta_i;\phi_r,\theta_r)\cdot E(\phi_i,\theta_i)\cdot\cos\theta_i d\omega$$

$$\sin\theta_i\,\delta\theta_i\,\delta\phi_i$$

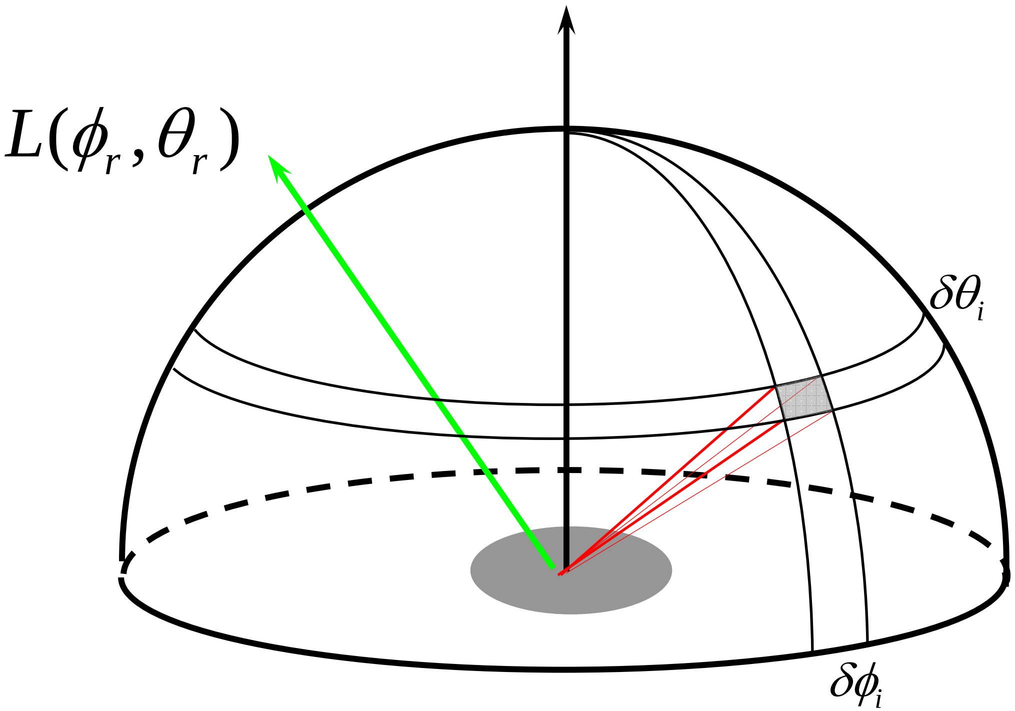




Shape from Shading

Total surface reflection

$$L(\phi_r,\theta_r)=\int_{-\pi}^{\pi}\int_{0}^{\pi/2} f(\phi_i,\theta_i;\phi_r,\theta_r)\cdot E(\phi_i,\theta_i)\cdot\sin\theta_i\cdot\cos\theta_i\cdot\delta\theta_i\delta\phi_i$$

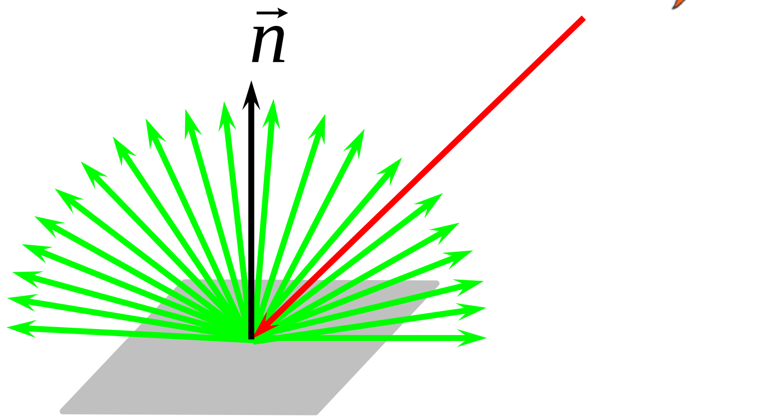


Shape from Shading

Lambertian (perfectly diffused) surfaces

$$f_L(\phi_i,\theta_i;\phi_r,\theta_r) = const = \bar{f} = \rho\frac{1}{\pi}$$

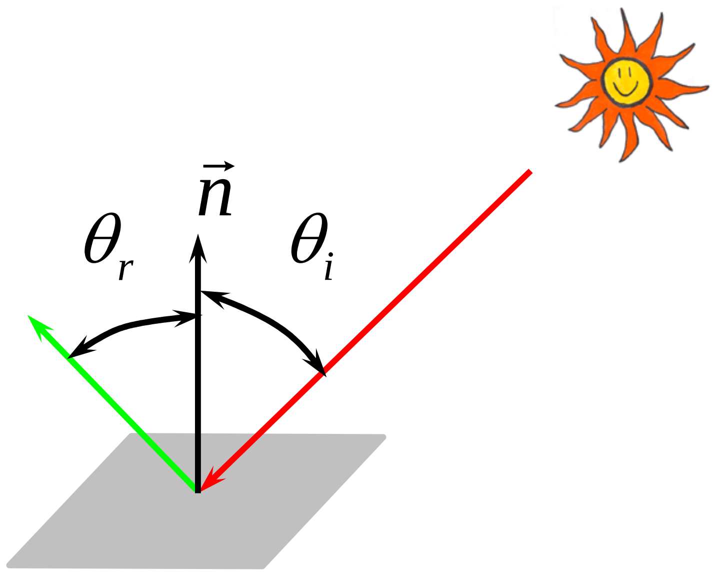
Albedo



Shape from Shading

Mirrored (perfectly secular) surfaces

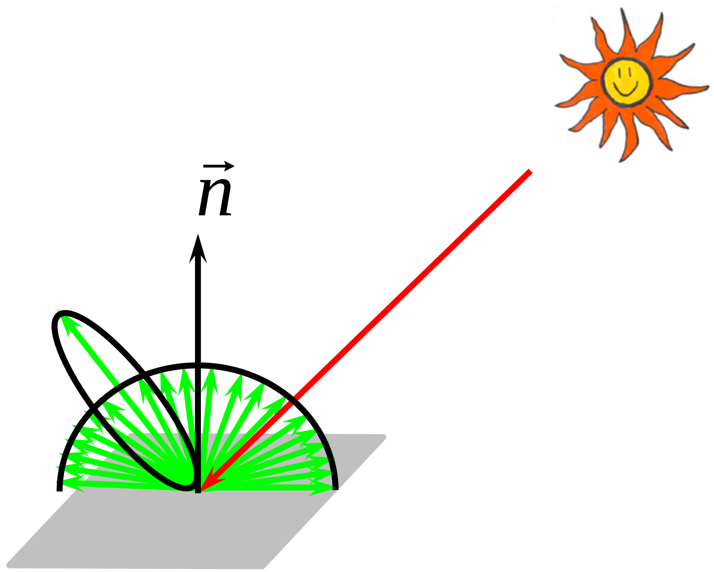
$$f_S(\phi_i, \theta_i; \phi_r, \theta_r) = \frac{\delta(\theta_r - \theta_i)\delta(\phi_r - \phi_i - \pi)}{\sin\theta_i \cos\theta_i}$$



Shape from Shading

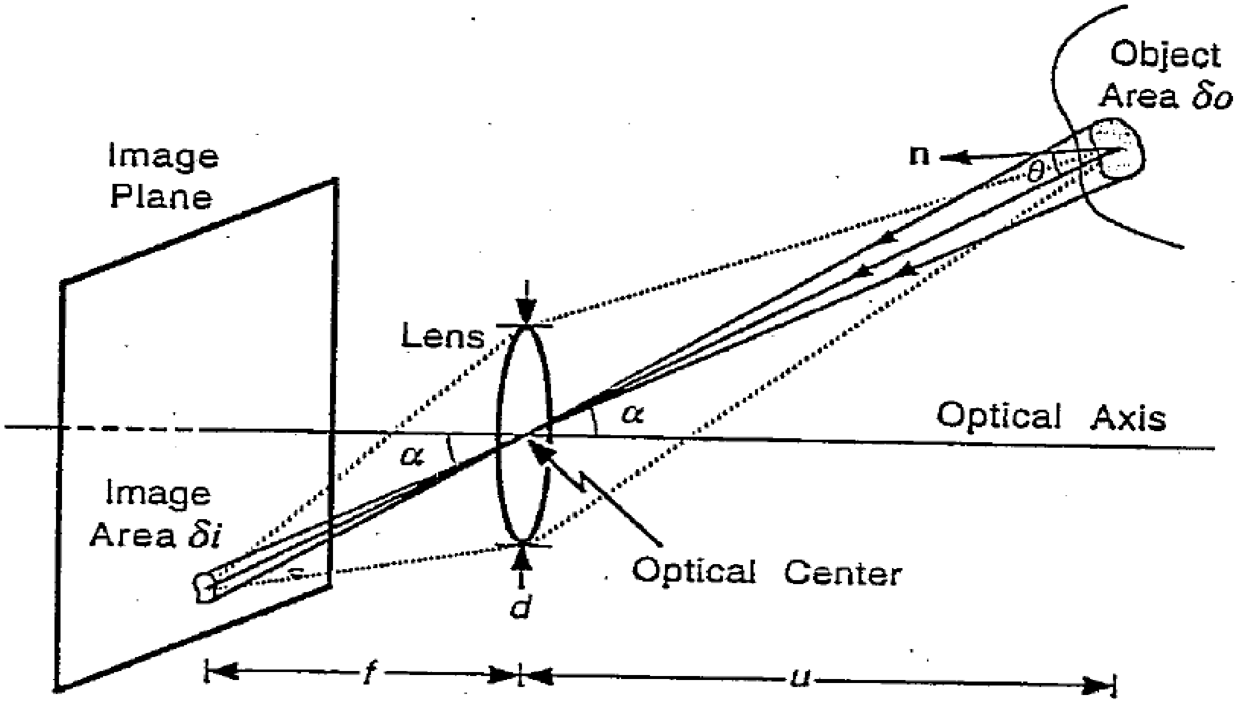
Mixed surfaces

$$f(\phi_i,\theta_i;\phi_r,\theta_r) = \alpha \cdot f_L(\phi_i,\theta_i;\phi_r,\theta_r) + (1-\alpha) f_S(\phi_i,\theta_i;\phi_r,\theta_r)$$



Shape from Shading

The fundamental radiometric relationship



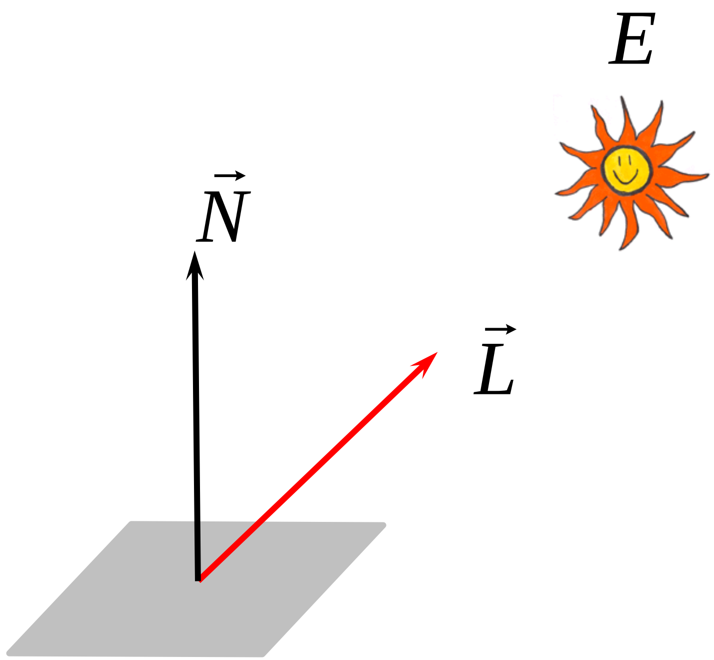
$$I = L \cdot \frac{\pi}{4} \cdot \left(\frac{d}{f}\right)^2 \cdot \cos^4 \alpha \quad \underset{\alpha \to 0}{\Rightarrow} \quad I \propto L$$

Shape from Shading

Point light source from direction (ϕ_L, θ_L)

$$E(\phi_i, \theta_i) = E \cdot \frac{\delta(\theta_L - \theta_i) \cdot \delta(\phi_L - \phi_i)}{\sin \theta_L}$$

$$\int_{-\pi}^{\pi} \int_{0}^{\pi/2} E(\phi_i, \theta_i) \cdot \sin \theta_i \cdot \delta\theta_i \delta\phi_i = E$$



Shape from Shading

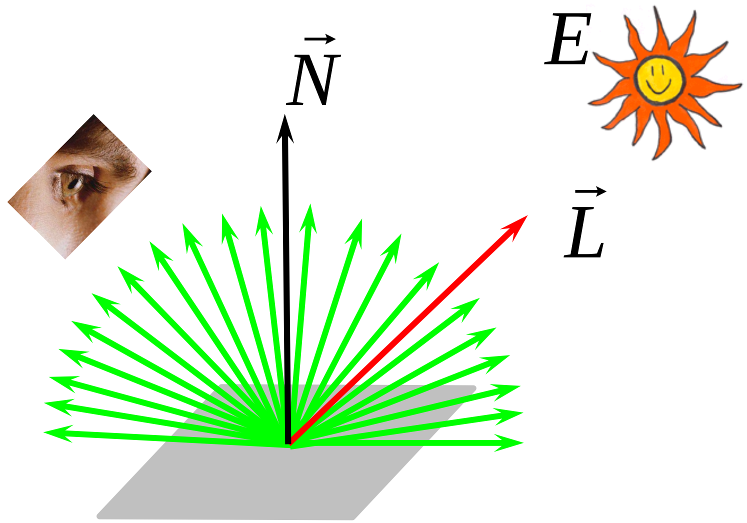
Surface brightness – appearance in the Lambertian case and point light source

$$f_L(\phi_i,\theta_i;\phi_r,\theta_r)=\rho\frac{1}{\pi}$$

$$E(\phi_i,\theta_i)=\frac{\delta(\theta_L-\theta_i)\delta(\phi_L-\phi_i)}{\sin\theta_L}$$

$$I(x,y)\propto L(\phi_r,\theta_r)=\int_{-\pi}^{\pi}\int_{0}^{\pi/2} f(\phi_i,\theta_i;\phi_r,\theta_r)\cdot E(\phi_i,\theta_i)\cdot\sin\theta_i\cdot\cos\theta_i\cdot\delta\theta_i\delta\phi_i$$

$$L=\rho\frac{1}{\pi}E\cos\theta_L\propto\rho(\hat{N}\cdot\hat{L})$$



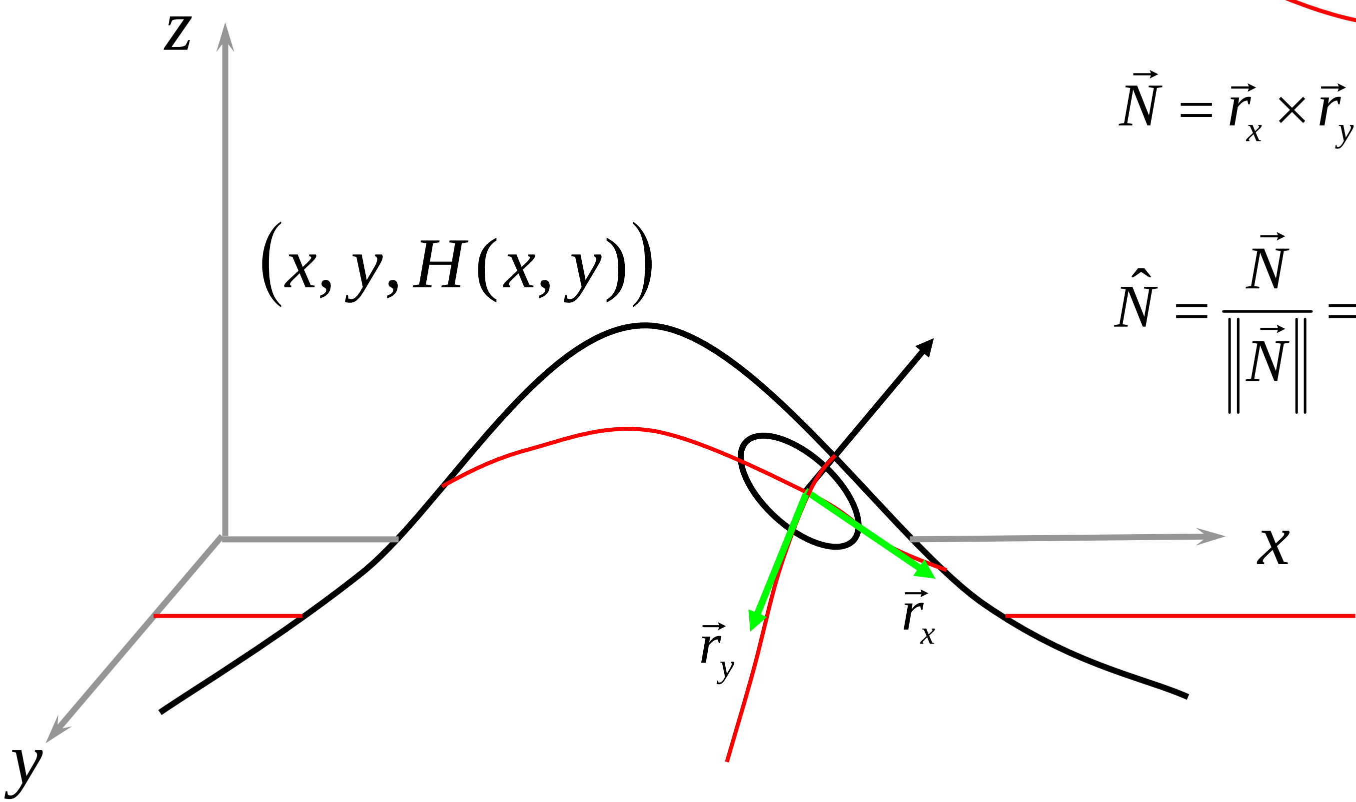
Shape from Shading

Shape description – Tangent plane and normal vectors

$$\vec{r}_x = \left(1, 0, \frac{\partial H}{\partial x}\right) \qquad \vec{r}_y = \left(0, 1, \frac{\partial H}{\partial y}\right)$$

$$\vec{N} = \vec{r}_x \times \vec{r}_y = (-p, -q, 1)$$

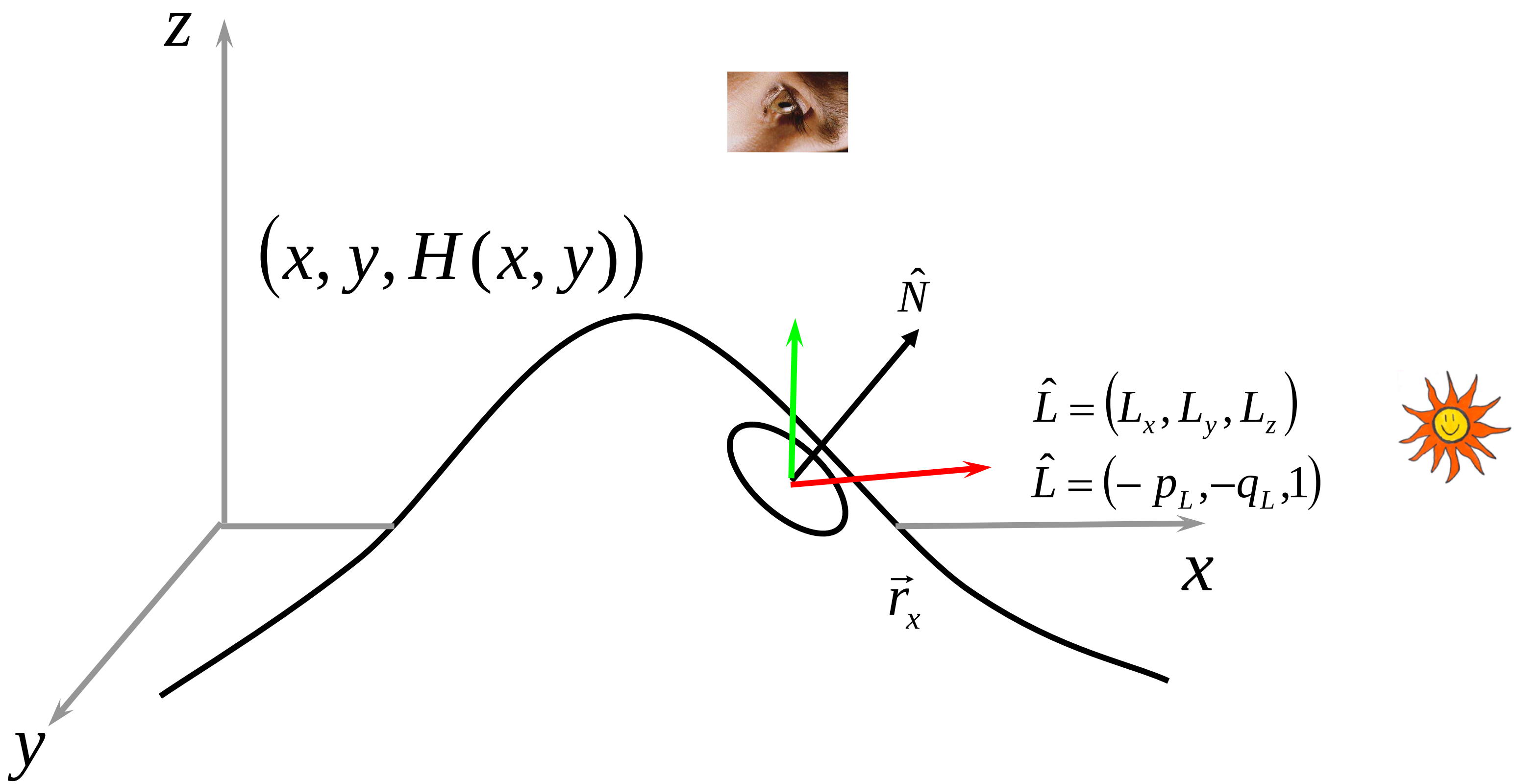
$$\hat{N} = \frac{\vec{N}}{\left\|\vec{N}\right\|} = \frac{(-p, -q, 1)}{\sqrt{p^2 + q^2 + 1}}$$



Shape from Shading

Shading on Lambertian surface – General point source

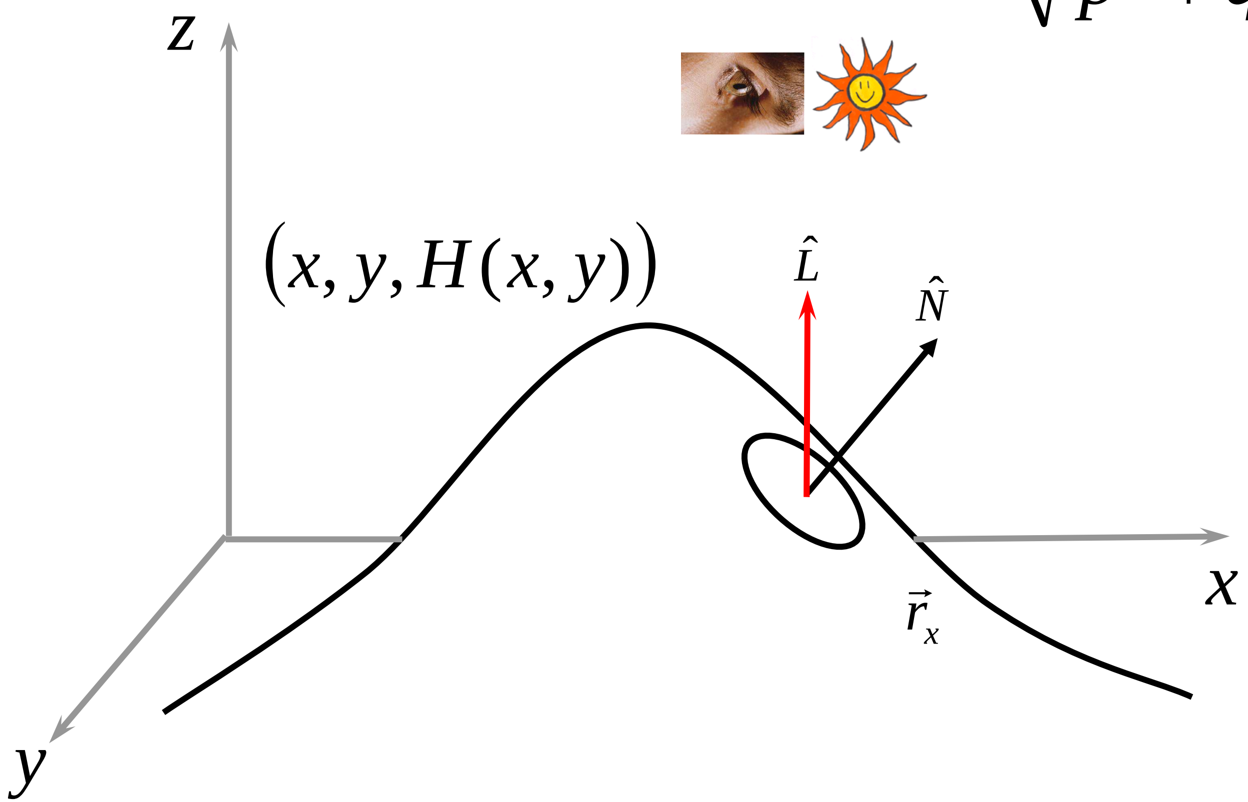
$$I = \rho(\hat{N} \cdot \hat{L}) = \rho \frac{-p \cdot L_x - q \cdot L_y + L_z}{\sqrt{p^2 + q^2 + 1}\sqrt{L_x^2 + L_y^2 + L_z^2}} = \rho \frac{p \cdot p_L + q \cdot q_L + 1}{\sqrt{p^2 + q^2 + 1}\sqrt{p_L^2 + q_L^2 + 1}}$$



Shape from Shading

Shading on Lambertian surface – Overhead point source

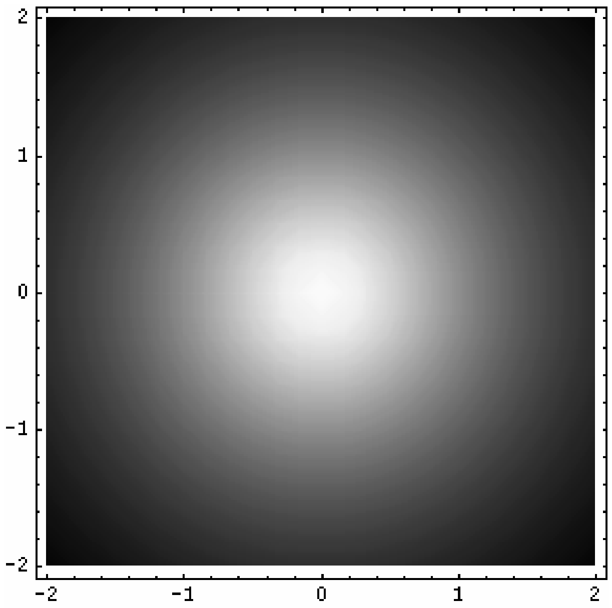
$$I(x,y) = \rho(\hat{N} \cdot [0,0,1]) = \rho \frac{1}{\sqrt{p^2+q^2+1}} = R(p,q)$$

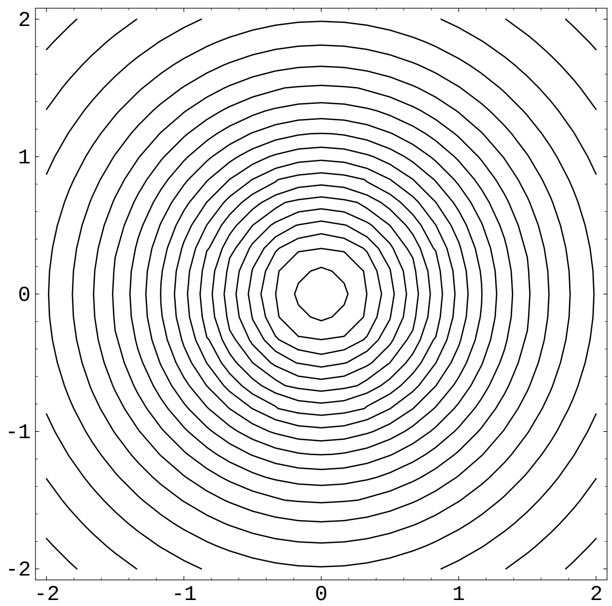


Shape from Shading

The Reflectance Map – Lambertian surface from overhead source position

$$R(p,q) = \frac{1}{\sqrt{p^2 + q^2 + 1}}$$

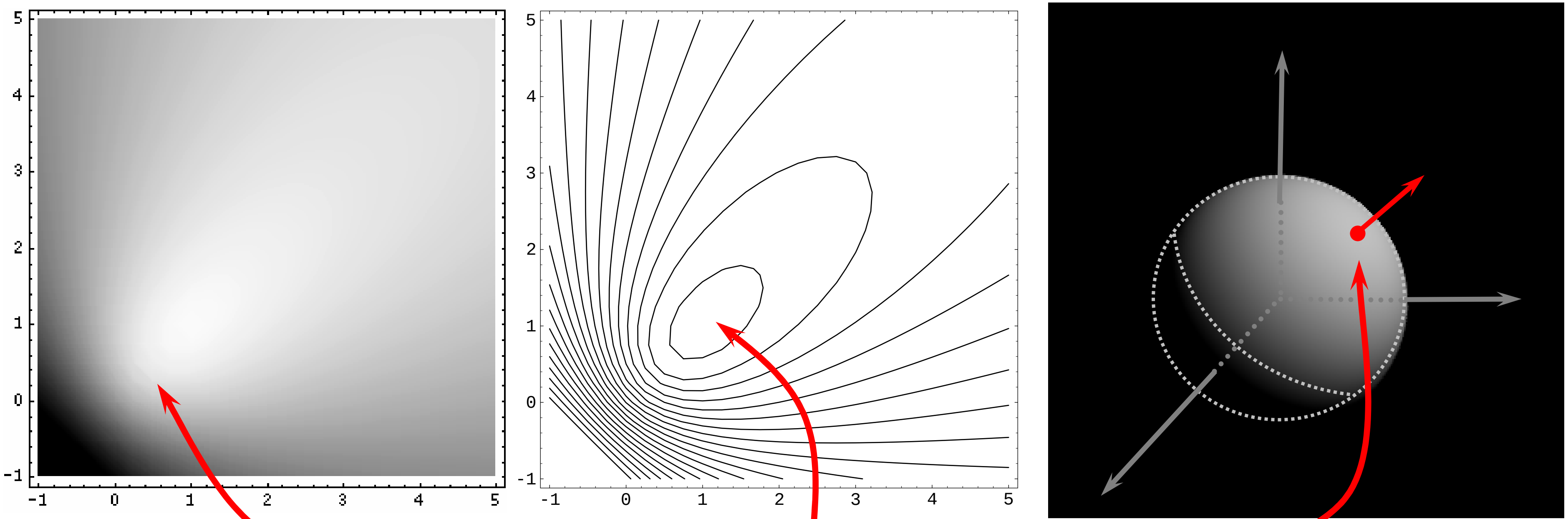




Shape from Shading

The Reflectance Map – Lambertian surface from general source position

$$R(p,q) = \frac{p \cdot p_L + q \cdot q_L + 1}{\sqrt{p^2 + q^2 + 1}\sqrt{p_L{}^2 + q_L{}^2 + 1}}$$

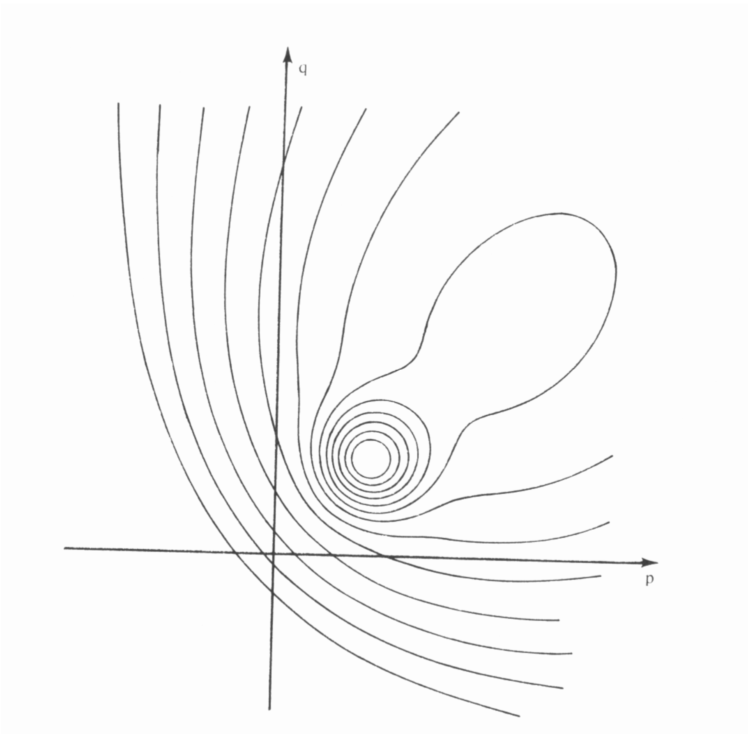


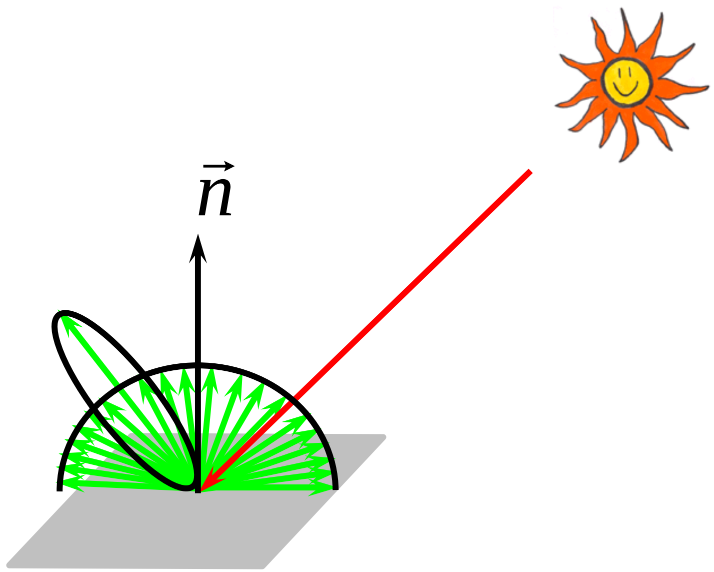
Gradient point of maximum brightness

Shape from Shading

The Reflectance Map – typical real surfaces

$$R(p, q)$$

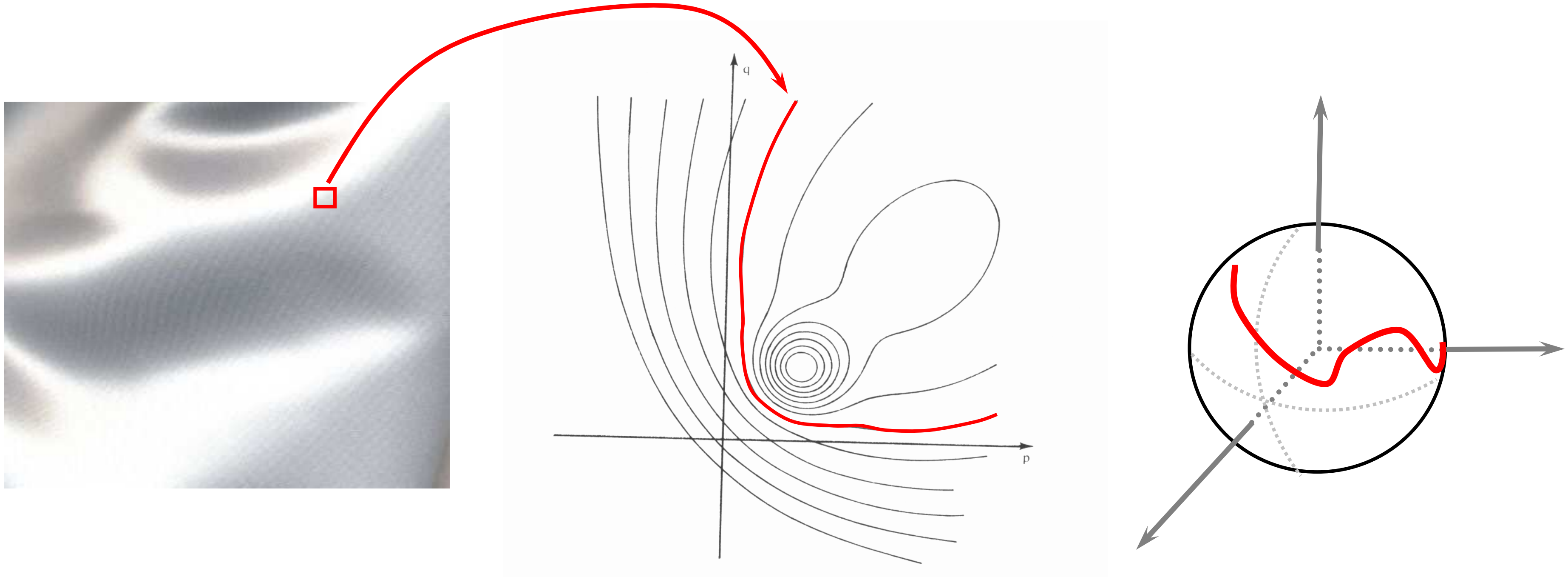




Shape from Shading

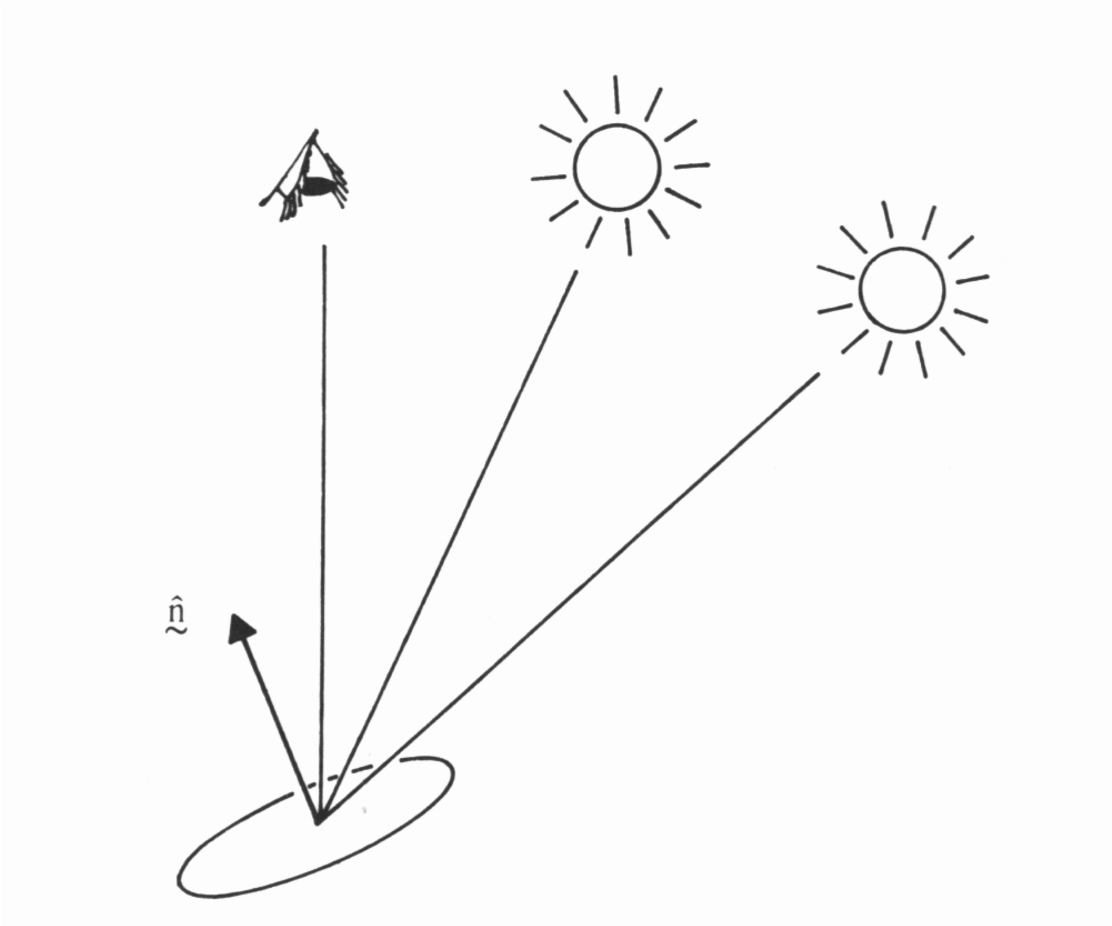
Surface orientation from shading

$$I(x,y) = R(p,q)$$



Shape from Shading

Photometric stereo

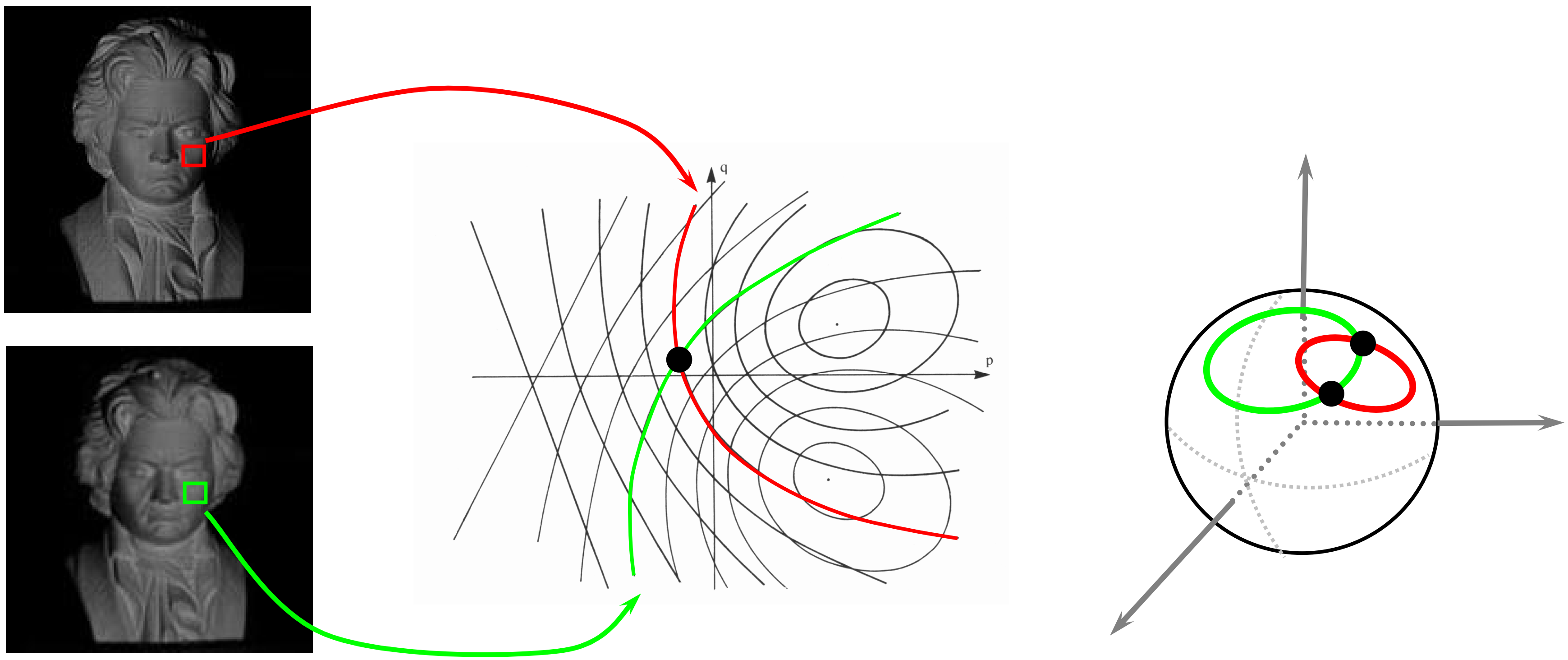


Shape from Shading

Photometric stereo

$$I_1(x,y) = R_1(p,q)$$

$$I_2(x,y) = R_2(p,q)$$



Shape from Shading

The SFS problem (special case)

Given *I(x,y)* of an (orthographic) projection of *H(x,y)*, and the reflectance map *R(p,q)*, find *H(x,y)* everywhere.

$$I(x,y) = R(p,q) = R\left(\frac{\partial}{\partial x} H(x,y), \frac{\partial}{\partial y} H(x,y)\right)$$

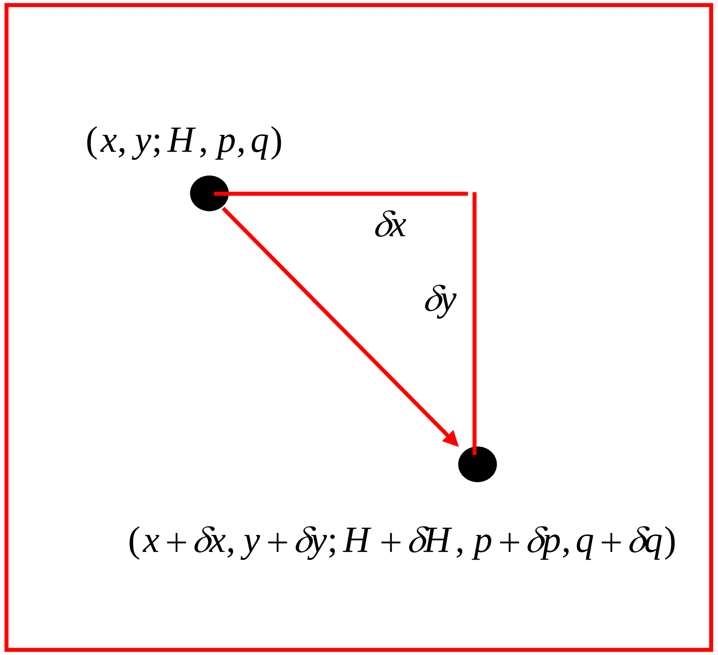
Shape from Shading

Shape recovery via characteristic strips

$$I(x,y) = R(p(x,y), q(x,y))$$

$$p(x,y) = \frac{\partial}{\partial x} H(x,y)$$

$$q(x,y) = \frac{\partial}{\partial y} H(x,y)$$



$$H(x+\delta x, y+\delta y) \cong H(x,y) + p\,\delta x + q\,\delta y$$

$$p(x+\delta x, y+\delta y) \cong p(x,y) + \frac{\partial p}{\partial x}\delta x + \frac{\partial p}{\partial y}\delta y$$

$$q(x+\delta x, y+\delta y) \cong q(x,y) + \frac{\partial q}{\partial x}\delta x + \frac{\partial q}{\partial y}\delta y$$

Shape from Shading

Shape recovery via characteristic strips

$$I(x,y) = R(p(x,y), q(x,y))$$

? ?

$$\frac{\partial}{\partial x} I(x,y) = \frac{\partial R(p,q)}{\partial p} \frac{\partial p(x,y)}{\partial x} + \frac{\partial R(p,q)}{\partial q} \frac{\partial q(x,y)}{\partial x}$$

$$\frac{\partial}{\partial y} I(x,y) = \frac{\partial R(p,q)}{\partial p} \frac{\partial p(x,y)}{\partial y} + \frac{\partial R(p,q)}{\partial q} \frac{\partial q(x,y)}{\partial y}$$

Shape from Shading

Shape recovery via characteristic strips

$$I(x, y) = R(p(x, y), q(x, y))$$

$$\frac{\partial}{\partial x} I(x, y) = \frac{\partial R(p, q)}{\partial p} \frac{\partial p(x, y)}{\partial x} + \frac{\partial R(p, q)}{\partial q} \frac{\partial p(x, y)}{\partial y}$$

$$\frac{\partial}{\partial y} I(x, y) = \frac{\partial R(p, q)}{\partial p} \frac{\partial q(x, y)}{\partial x} + \frac{\partial R(p, q)}{\partial q} \frac{\partial q(x, y)}{\partial y}$$

Shape from Shading

Shape recovery via characteristic strips

$$I(x,y) = R(p(x,y), q(x,y))$$

$$\delta H \cong p\,\delta x + q\,\delta y$$

$$\frac{\partial}{\partial x} I(x,y) = \nabla R \cdot \nabla p$$

$$\delta p \cong \frac{\partial p}{\partial x}\delta x + \frac{\partial p}{\partial y}\delta y \quad = \nabla p \cdot (\delta x, \delta y)$$

$$\frac{\partial}{\partial y} I(x,y) = \nabla R \cdot \nabla q$$

$$\delta q \cong \frac{\partial q}{\partial x}\delta x + \frac{\partial q}{\partial y}\delta y \quad = \nabla q \cdot (\delta x, \delta y)$$

A smart choice

$$\delta x = \frac{\partial R(p,q)}{\partial p}\delta s$$

$$\delta y = \frac{\partial R(p,q)}{\partial q}\delta s$$

$$\Rightarrow$$

$$\delta p \cong \frac{\partial}{\partial x} I(x,y) \cdot \delta s$$

$$\delta q \cong \frac{\partial}{\partial y} I(x,y) \cdot \delta s$$

Shape from Shading

Shape recovery via characteristic strips

$$\delta x = R_p \delta s \qquad \dot{x} = R_p$$

$$\delta y = R_q \delta s \qquad \dot{y} = R_q$$

$$\delta H = \left(pR_p + qR_q\right)\delta s \quad \Rightarrow \quad \dot{H} = pR_p + qR_q$$

$$\delta p = I_x \delta s \qquad \dot{p} = I_x$$

$$\delta q = I_y \delta s \qquad \dot{q} = I_y$$

Shape from Shading

Shape recovery via characteristic strips

Shape from Shading via Characteristic Curves

Given

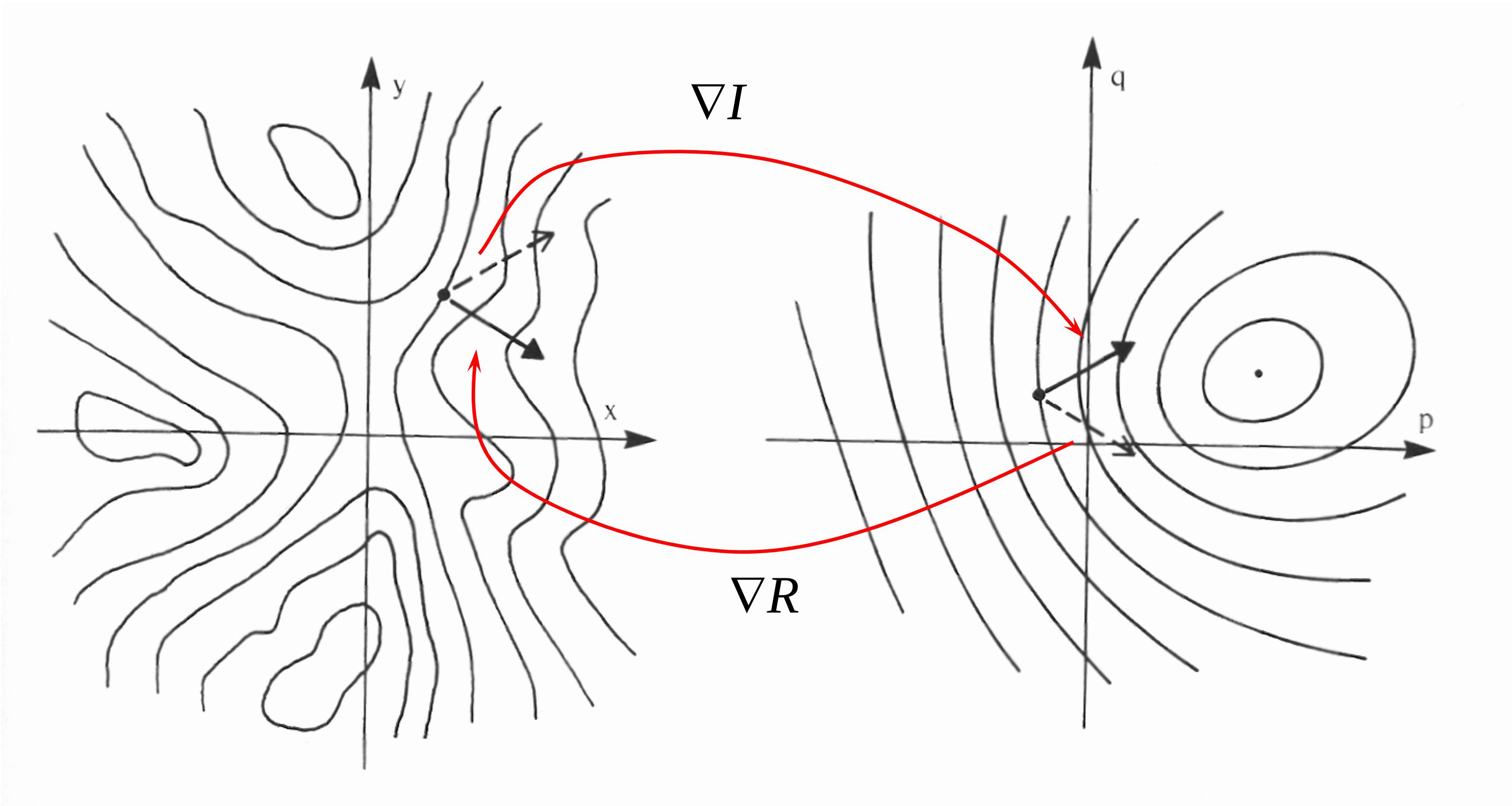
- $I(x,y)$ of an (orthographic) projection of unknown $H(x,y)$
- The reflectance map $R(p,q)$
- Initial data x_0, y_0, $H(x_0,y_0)$, $p(x_0,y_0)$, $q(x_0,y_0)$

Develop a curve solution on $H(x,y)$ by taking small steps of size δs via the system

$$\delta x = R_p \delta s$$
$$\delta y = R_q \delta s$$
$$\delta H = \left(pR_p + qR_q\right)\delta s$$
$$\delta p = I_x \delta s$$
$$\delta q = I_y \delta s$$

Shape from Shading

Shape recovery via characteristic strips



Shape from Shading

Shape recovery via characteristic strips