Image Formation III
Chapter 1 (Forsyth&Ponce)
Cameras *"Lenses"*

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CS-GY 6643, Spring 2016

(slides modified from Marc Pollefeys, UNC Chapel Hill/ ETH Zurich)

Pinhole size / aperture

How does the size of the aperture affect the image we'd get?

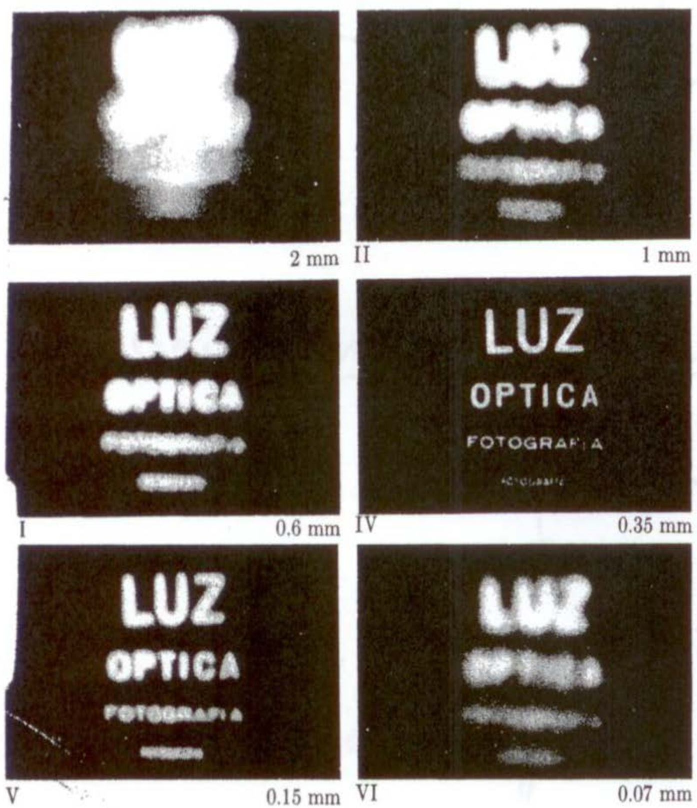
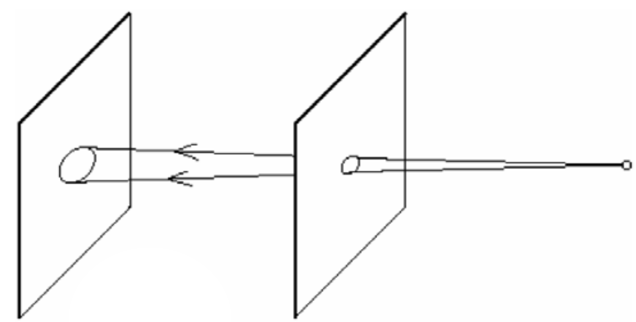


Fig. 5.96 The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]

Larger

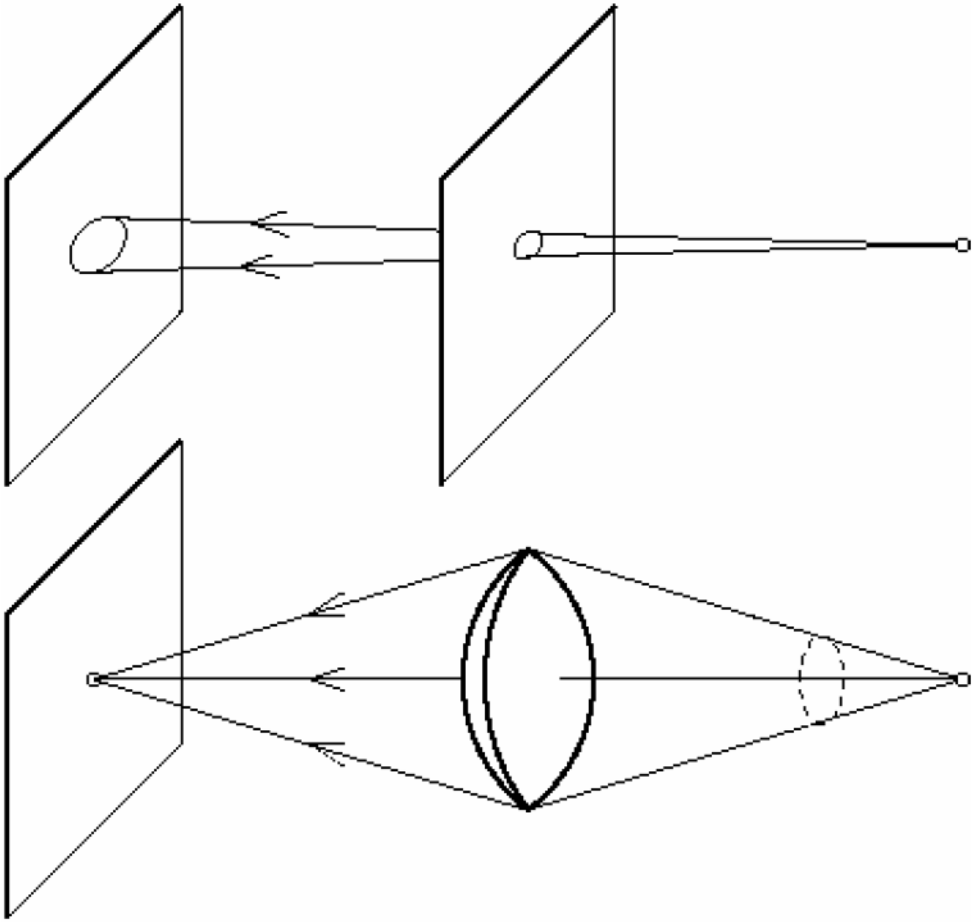
↓

Smaller



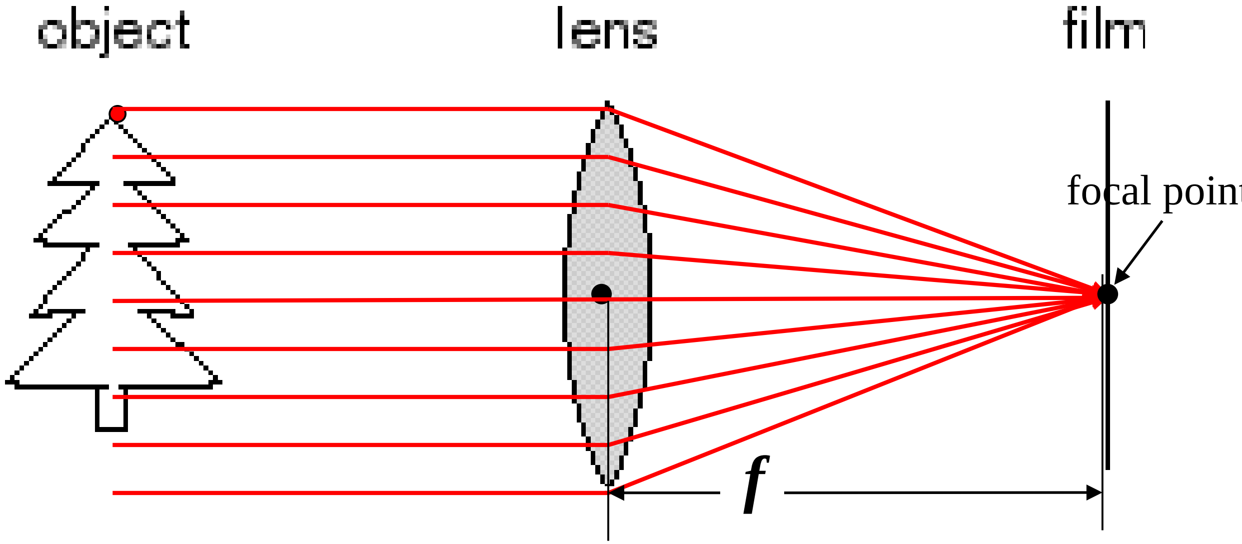
K. Grauman

Pinhole vs. lens



K. Grauman

Adding a lens

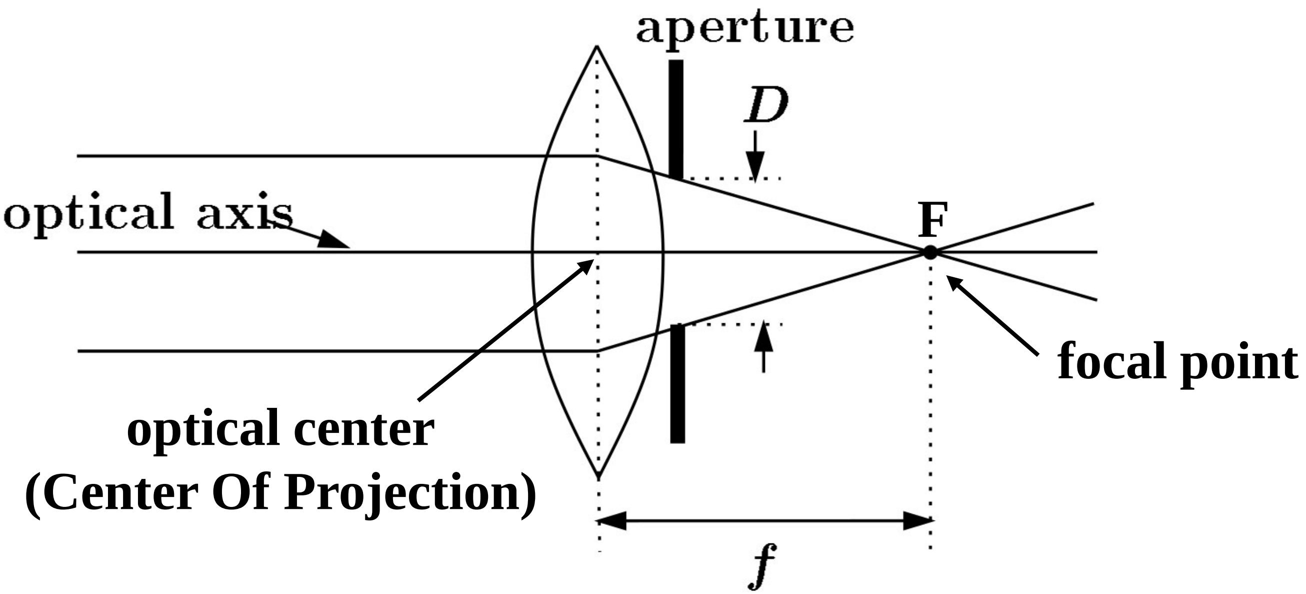


A lens focuses light onto the film

- Rays passing through the center are not deviated
- All parallel rays converge to one point on a plane located at the *focal length f*

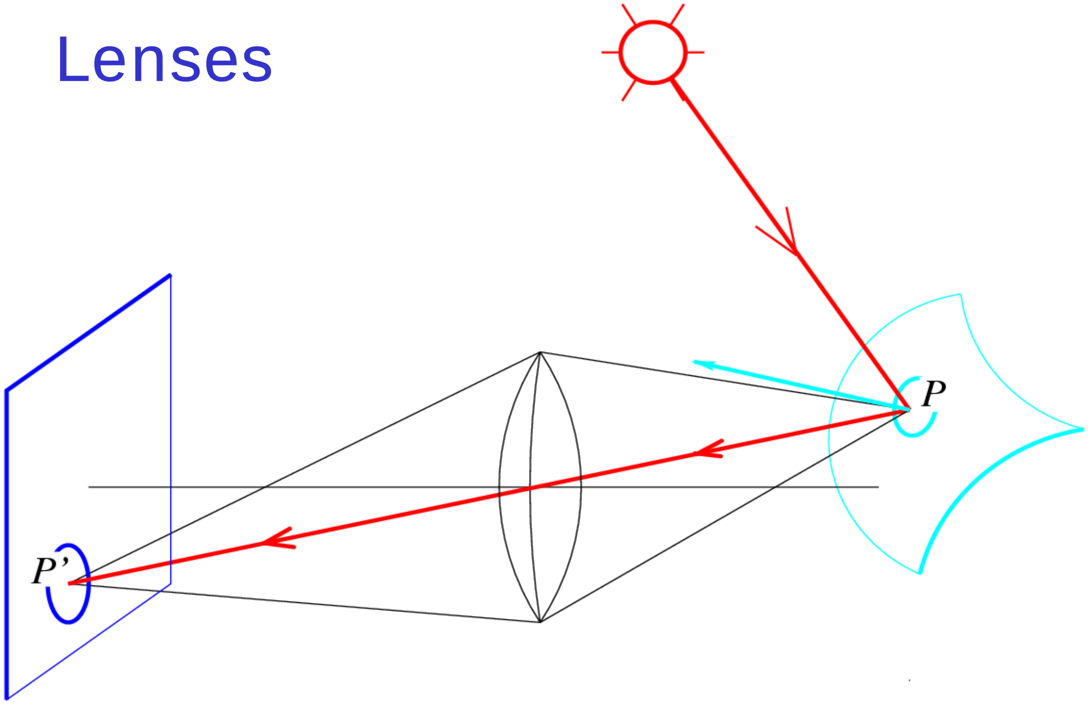
Slide by Steve Seitz

Cameras with lenses



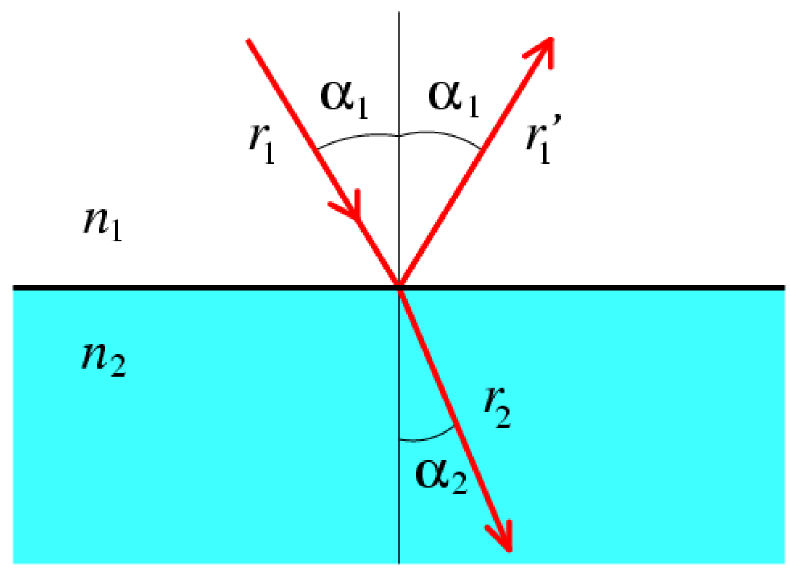
- A lens focuses parallel rays onto a single focal point
- Gather more light, while keeping focus; make pinhole perspective projection practical

Lenses



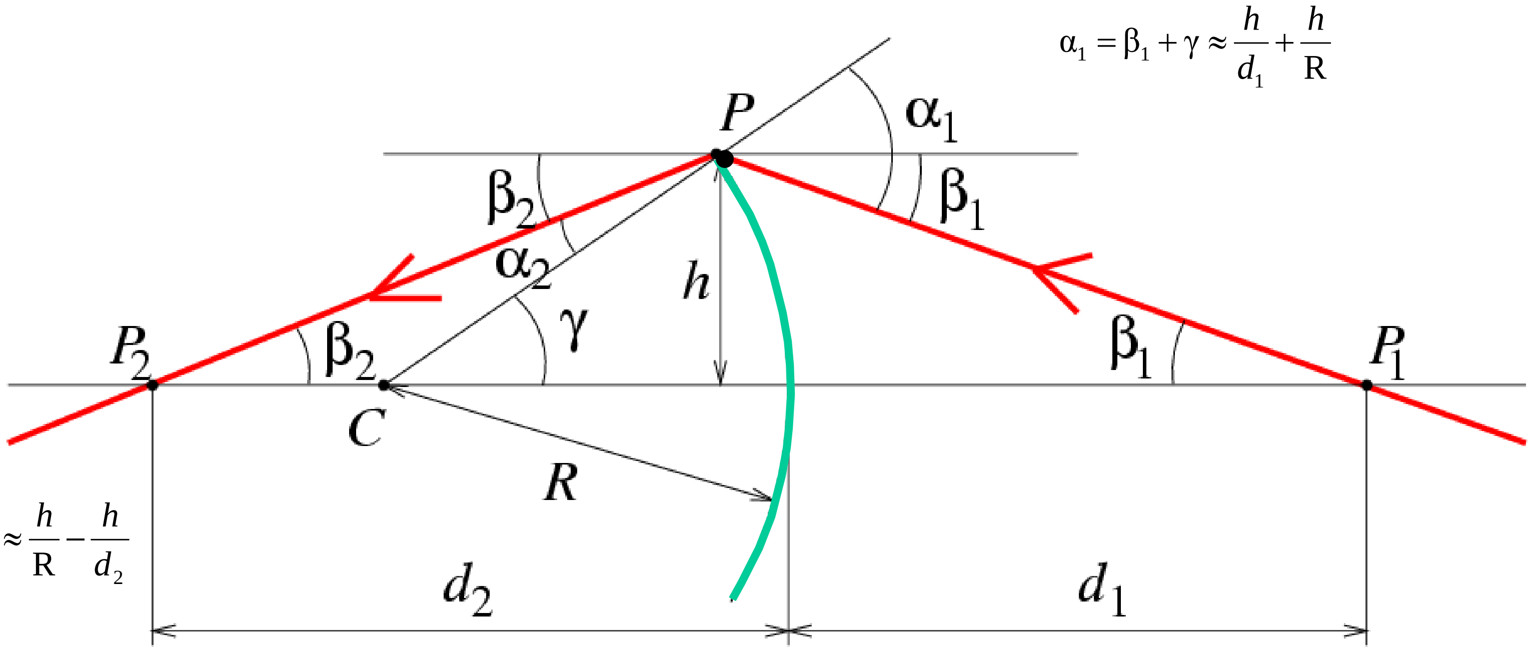
Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$



Paraxial (or first-order) optics

$$\alpha_1 = \beta_1 + \gamma \approx \frac{h}{d_1} + \frac{h}{R}$$



$$\alpha_2 = \gamma - \beta_2 \approx \frac{h}{R} - \frac{h}{d_2}$$

Snell's law:

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

Small angles:

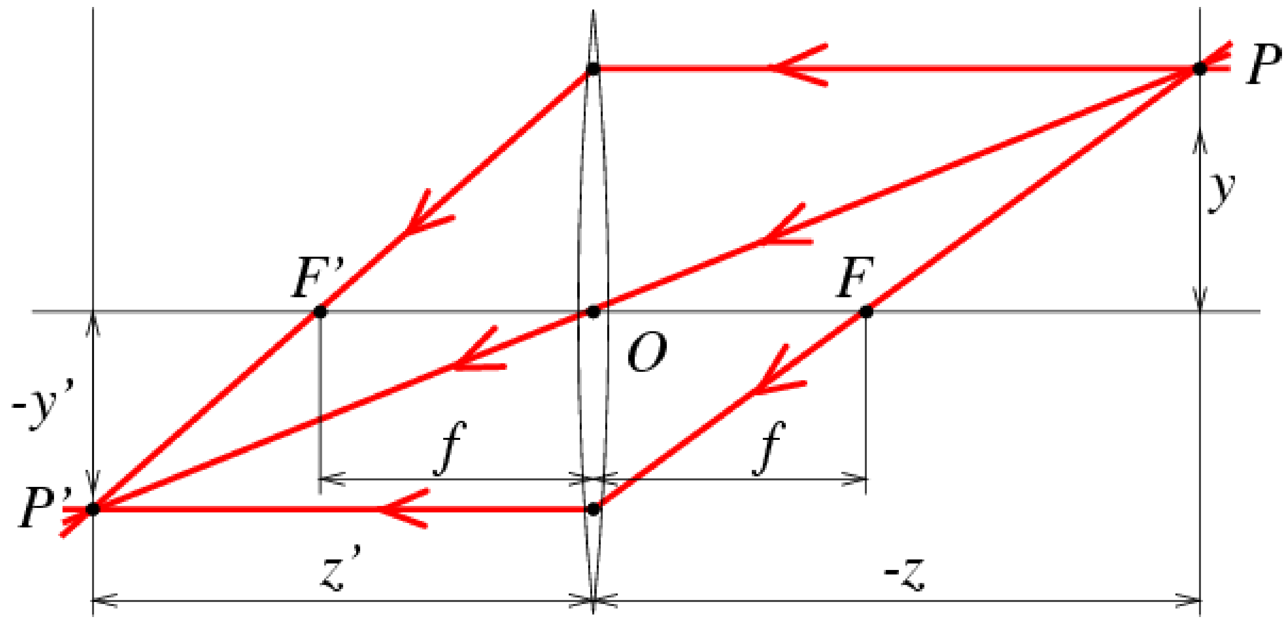
$$n_1 \alpha_1 \approx n_2 \alpha_2$$

$$n_1\left(\frac{h}{d_1} + \frac{h}{R}\right) = n_2\left(\frac{h}{R} - \frac{h}{d_2}\right)$$

$$\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}$$

Thin Lenses

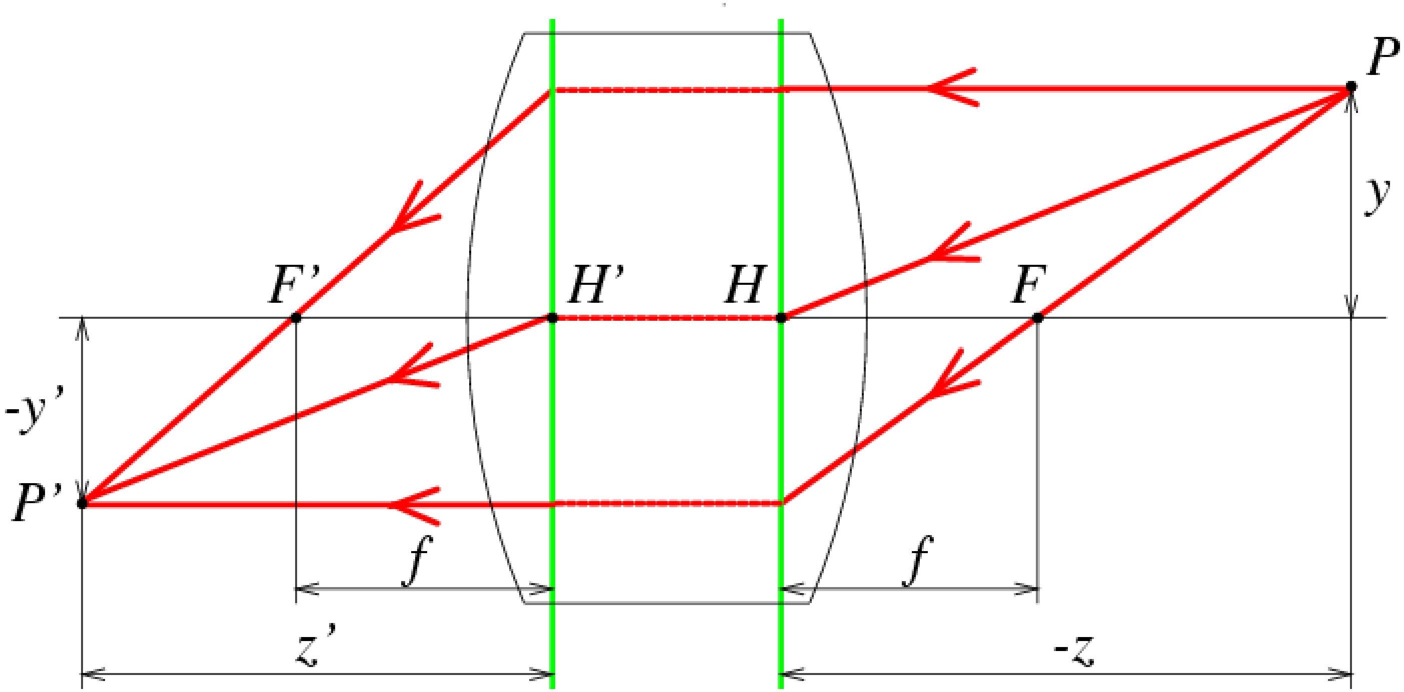
spherical lens surfaces; thickness << radii; same refractive index on both sides; all rays emerging from P and passing through the lens are focused at P’. Let $n_1=1$ (vacuum) and $n_2=n$.



$$\begin{cases} x' = z'\dfrac{x}{z} \\ y' = z'\dfrac{y}{z} \end{cases} \quad \text{where} \quad \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \quad \text{and} \quad f = \frac{R}{2(n-1)}$$

http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html

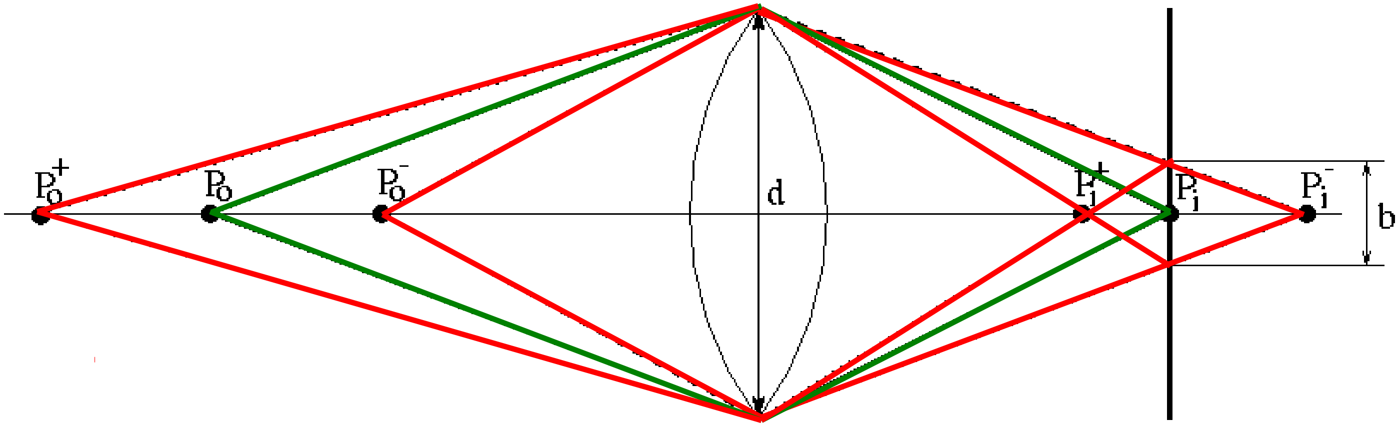
Thick Lens



Focus and depth of field

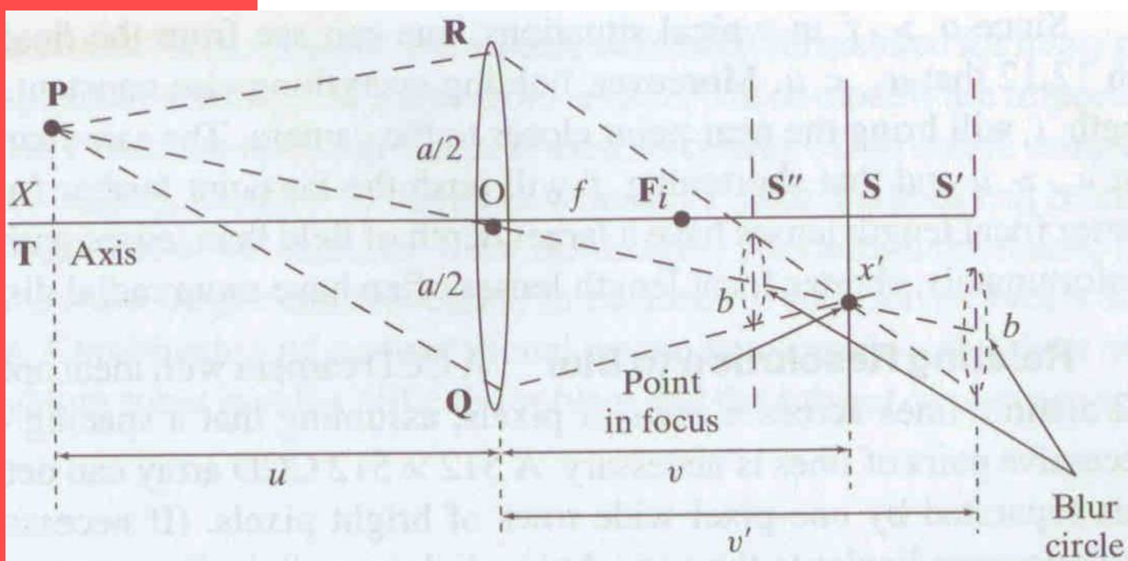
Image credit: cambridgeincolour.com

The depth-of-field



Focus and depth of field

- Depth of field: distance between image planes where blur is tolerable



Thin lens: scene points at distinct depths come in focus at different image planes.

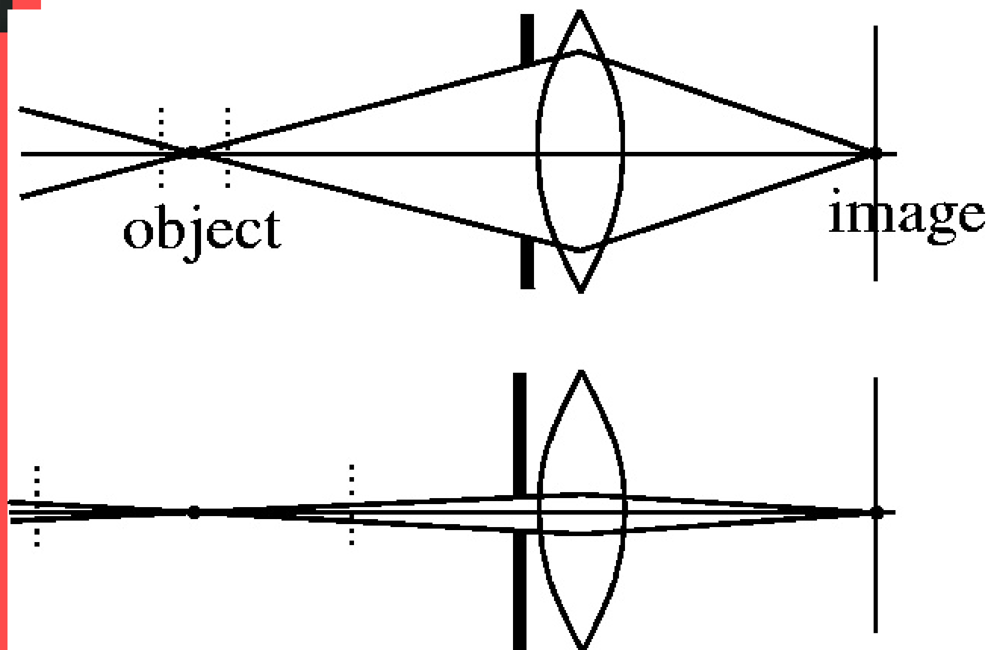
(Real camera lens systems have greater depth of field.)

← "circles of confusion" →

Shapiro and Stockman

Focus and depth of field

- How does the aperture affect the depth of field?

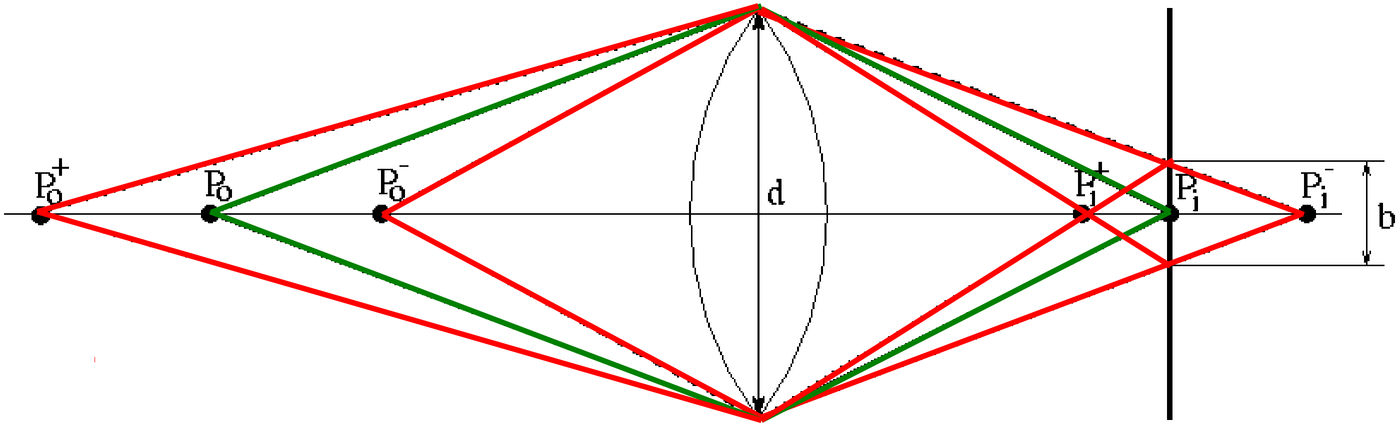


- A smaller aperture increases the range in which the object is approximately in focus

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

Slide from S. Seitz

The depth-of-field



The depth-of-field

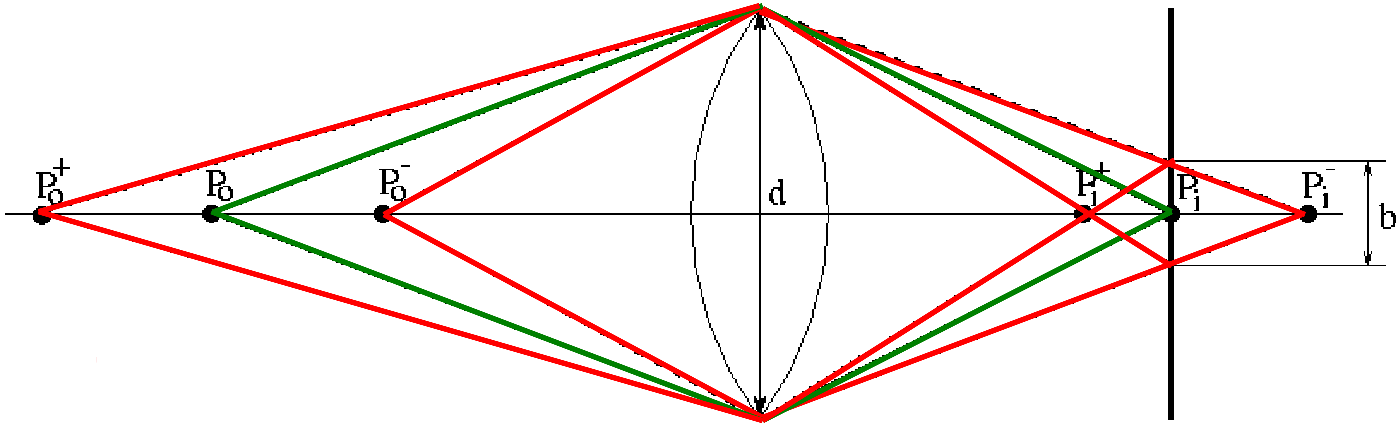
yields $$Z_o^- = f \frac{|Z_i^-|}{|Z_i^-| - f}$$

$$Z_o^- = f \frac{d\, Z_o}{b\, Z_0 + f\,(d - b)}$$

$$\Delta Z_o^- = Z_o - Z_o^- = \frac{Z_o\,(Z_o - f)}{Z_0 + f\, d/b - f}$$

Similar formula for $\Delta Z_o^+ = Z_o^+ - Z_o$

The depth-of-field



$$\Delta Z_0^- = Z_0 - Z_0^- = \frac{Z_0(Z_0 - f)}{Z_0 + f\, d/b - f}$$

decreases with d+, increases with Z_0+

strike a balance between incoming light and sharp depth range

Deviations from the lens model

3 assumptions :

1. all rays from a point are focused onto 1 image point

2. all image points in a single plane

3. magnification is constant

deviations from this ideal are *aberrations*

Aberrations

2 types :

1. geometrical

2. chromatic

geometrical : small for paraxial rays

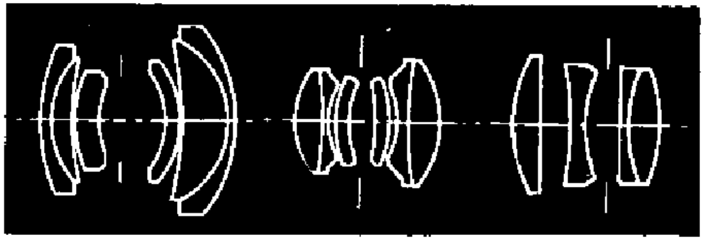
study through 3rd order optics $\sin(\theta) \approx \theta - \frac{\theta^3}{6}$

chromatic : refractive index function of wavelength

Geometrical aberrations

- spherical aberration
- astigmatism
- distortion
- coma

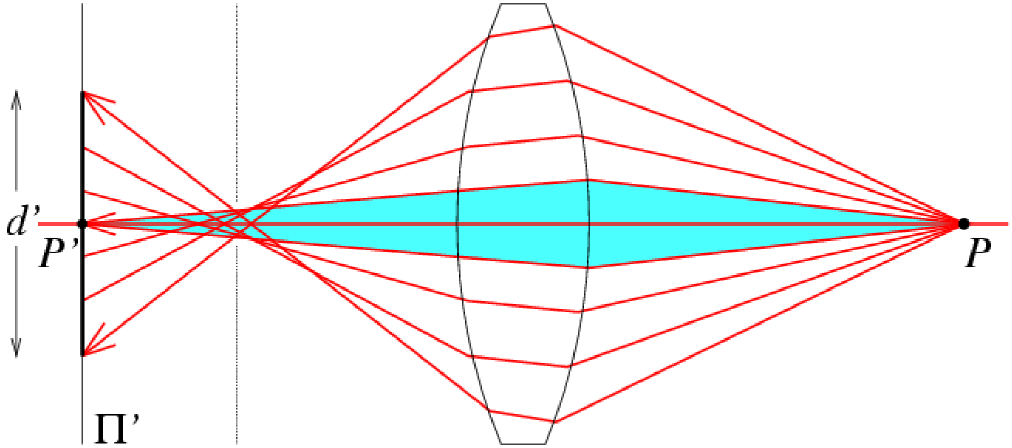
aberrations are reduced by combining lenses



Spherical aberration

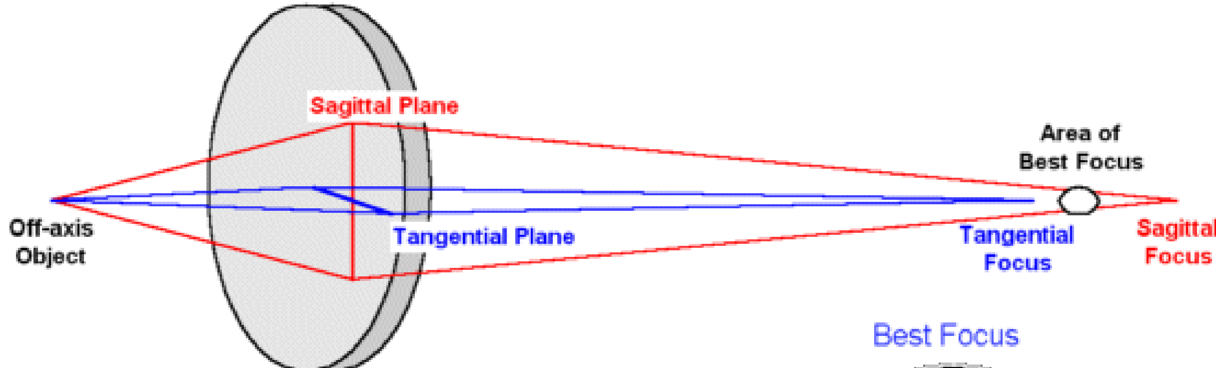
rays parallel to the axis do not converge

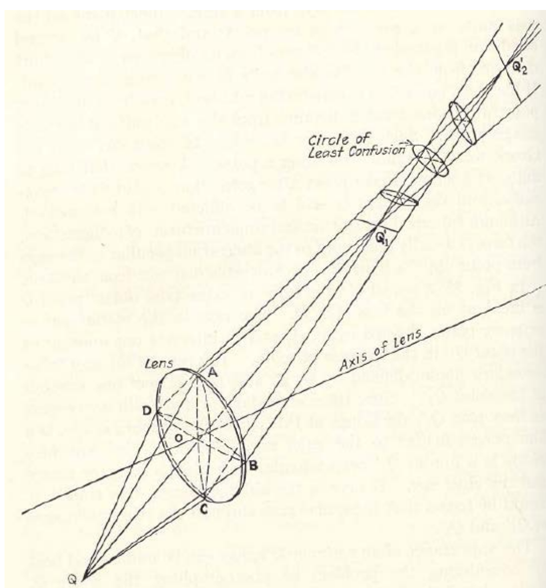
outer portions of the lens yield smaller focal lenghts

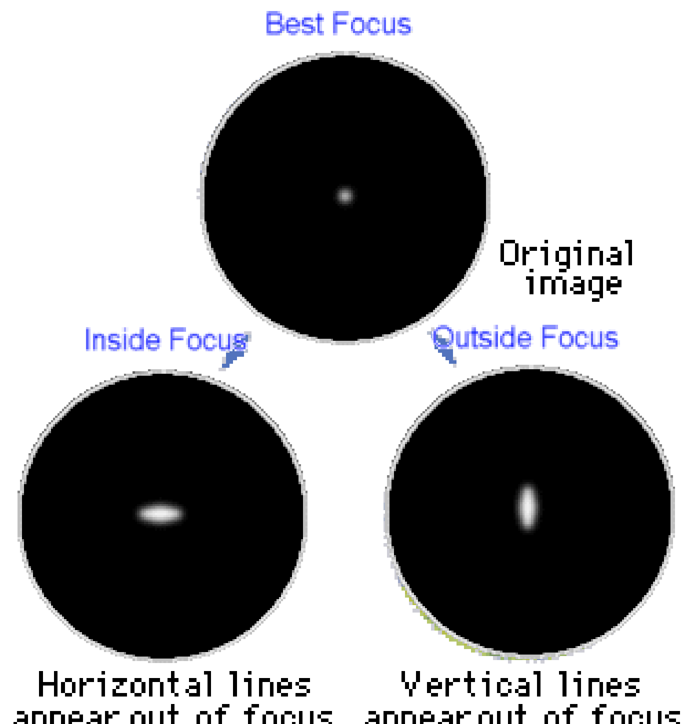


Astigmatism

Different focal length for inclined rays

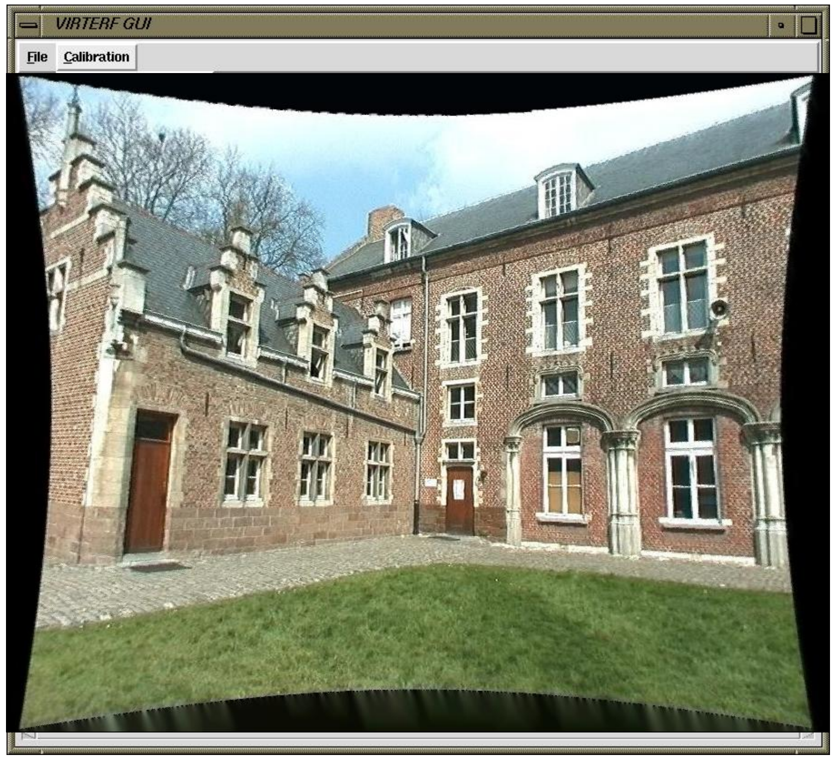




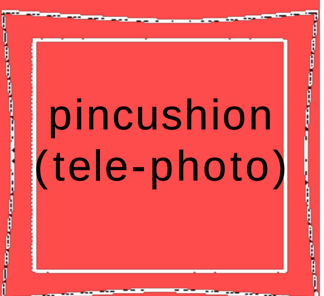


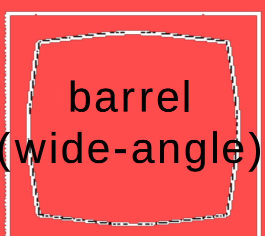
Distortion

magnification/focal length different
for different angles of inclination



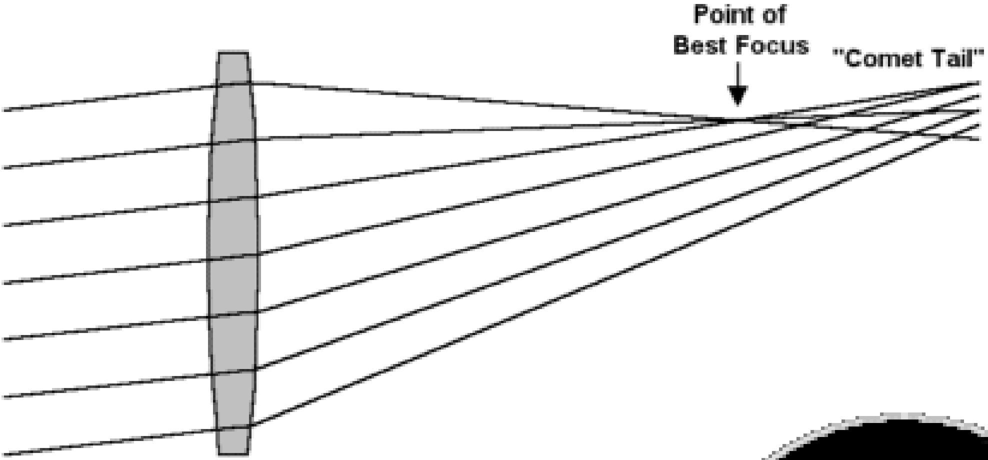
Can be corrected! (if parameters are know)

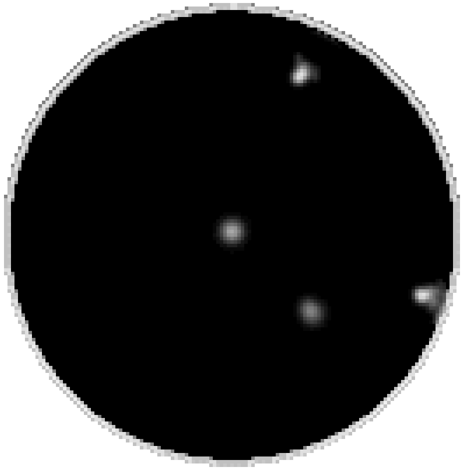




Coma

point off the axis depicted as comet shaped blob





Chromatic aberration

rays of different wavelengths focused in different planes

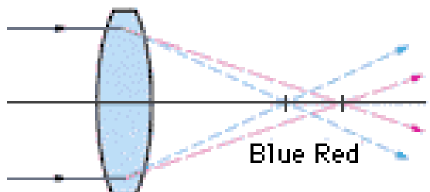


Fig.1
Axial chromatic aderration

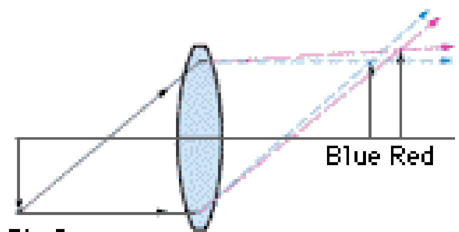
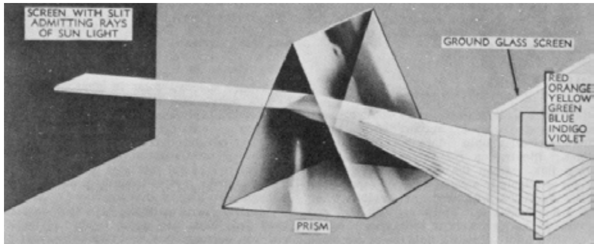


Fig.2
Magnification chromatic aderration

The image is blurred and appears colored at the fringe.

cannot be removed completely

sometimes *achromatization* is achieved for more than 2 wavelengths



Vignetting